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ABSTRACT

This guide provides a structured program for the slow learner in mathematics grades 6-11. Suggestions for implementing the program are included. The guide is divided into the following major areas of mathematical competency: Fundamental Operations, Geometry, Measurement, Graphing, Algebra, Probability and Statistics, and Logic. Recreation is the last section of the book. Each of the areas of mathematical competency contains: master chart of mathematics content, grade level chart of mathematics content, list of behavioral objectives, and student activity descriptions and materials. This curriculum guide is one of several prepared for secondary school mathematics instruction by Baltimore County Public Schools. (JG)

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BALTIMORE COUNTY PUBLIC SCHOOLS

Mathematics for Basic Education

Grade 9

A Tentative Guide

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INTRODUCTION

A. PHILOSOPHY

The rapid trends toward greater automation, use of computers, and the increased technological skills demanded of workers have dramatically reduced the market for unskilled and semi-skilled laborers. Twenty percent of our population consists of persons, who, according to their academic talents, are termed "slow learners." It becomes apparent that young people of limited ability, who are potential unskilled and semi-skilled workers, must be prepared for a useful place in our society. Unless these youths are taught salable skills, they must be supported by tax money. This situation could lead to a society composed of one segment which is overworked to contribute tax dollars and services, and another segment which is unemployed and consumes the wealth, yet produces nothing.

The magnitude and urgency of this problem demand that schools develop appropriate educational opportunities for slow learning students throughout their school experiences. The schools are thus faced with the problem of training students for jobs and services which may be outmoded by the time they enter the business world. Equally disturbing is the fact that no one can foretell the many new jobs and products which will be created for which no training has been provided. It is generally conceded that the service occupations hold the greatest promise of employment for the slow learner. Functional competence in mathematics is essential for all persons entering these service occupations. Industry is retaining many semi-skilled and unskilled workers who have been displaced by automation through extensive retraining. Reports indicate that greater success is obtained in retraining workers who have more mathematics background than those who do not. Furthermore, training in mathematics provides youth with broader choices of vocational employment. It is imperative that the student be given a sound foundation in mathematics if he is to function effectively as a producer and consumer, and a citizen in his community.

It is axiomatic that the slow learner should be educated in his own right and to the maximum of his ability. Any adaptation of an academically oriented program must surely fail. A program of mathematics for the slow learner should be based upon the latest developments and research in learning theory, an appropriate selection and reorganization of mathematical topics, and the inclusion of new materials as well as new techniques for presenting mathematical concepts and developing skills. Proper pacing of these concepts and skills must underlie the entire structure. All the human resources of the educational system - the mathematics teacher, the principal, the mathematics supervisor, resource teachers, the guidance counselor, and other specialists - must be brought to bear on this problem.

Probably the most important factor in the success of a mathematics program for the slow learner is the teacher. Such a teacher should be prepared psychologically to teach students of limited ability. This implies an acceptance of the student for what he is, and an awareness of the operational level of the student. Furthermore, the teacher should have such characteristics as emotional maturity, a broad background of mathematics and a curiosity for more, a liking for young people, patience, and above all, a sense of humor. Such a teacher can do much to enhance the usually poor self-image of the slow learner, and convince the student that he is indeed a person worthy of dignity and respect in this society.

B. IDENTIFICATION OF THE SLOW LEARNER

The most obvious characteristic of the slow learner is his inability to keep pace with those students who are average in their rate of academic growth. However, other psychological, social, cultural, and physical factors may be considered in identifying these students. The following criteria, which are divided into two categories, may be used to form a basis for the selection of the students who may be classified as slow learners. The two criteria - measurable and traits - should receive equal consideration when the student is being identified.

Measurable Criteria

1. I.Q. Range 75 - 90 resulting from at least two group tests or an individual test.
2. Percentiles on group tests of mental ability and achievement ranging from 0 - 19 (approximately two or more years below grade level in reading comprehension and arithmetic.)
3. Teacher grades - consistently below average, as indicated by "ability" C's and D's as well as E's.

Traits Criteria

1. Limited academic interest
2. Difficulties in planning and in carrying out work without supervision
3. Limited creativity and intellectual curiosity
4. Indications of short attention span
5. Severe limitations in the ability to communicate orally or in writing.

C. BEHAVIORAL OBJECTIVES

An integral part of any collection of instructional materials is a statement of the objectives. This bulletin is no exception. The objectives stated here are stated in behavioral terms. That is, each objective is stated in terms of the desired student behaviors.

To clarify, consider the following example of a behavioral objective which is taken from the Grade 7 geometry section of this bulletin.

> The student should be able to construct a drawing of a quadrilateral using a straightedge.

The characteristics of this objective are that it tells who is to perform, how he is to perform, and what constitutes an acceptable performance.

To assess the acquisition of the above stated behavior it is only necessary to give a student paper, pencil, and straightedge and instruct him to make a drawing of a quadrilateral. In response, the student can either make such a drawing or he cannot. In any event, it is possible to decide whether or not the stated objective has been realized. Any well stated behavioral objective should point clearly to the type of performance task necessary to assess its attainment.

The clarity of a behavioral objective such as the one stated above is in clear contrast to the vagueness of comparable objectives which state that the student should "understand the concept of quadrilateral" or that the teacher should "develop the concept of quadrilateral." These and other objectives such as "developing appreciations and attitudes" do not lend themselves well to evaluation. Indeed, the assessment of these qualities have always posed difficulties for researchers.

Behavioral scientists such as Jean Piaget and Robert Gagne have asserted that true learning involves a change on the part of the learner so that he no longer reacts as he did before. His whole being views similar situations in a new light. If our instructional program is to effect such changes in slow learning students, then the objectives such be so constructed that they specifically state the desired behavioral responses which are observable and hence can be assessed.

D. ACTION WORDS USED IN STATING BEHAVIORAL OBJECTIVES

The action words which are used to construct behavorial objectives are:

1. IDENTIFY

The student selects by pointing to, touching, picking up, or circling the correct object or class name. This class of performances also includes identifying object properties such as rough, smooth, straight, curved.

e. g. The student should be able to identify the prime numbers from a given set containing prime and composite numbers.

2. DISTINGUISH

The student identifies objects or events which are potentially confusable. This is a more difficult identification.

e. g. The student should be able to distinguish between ordered pairs such as (a, b) and (b, a).

3. CONSTRUCT

The student generates a construction using instruments, a freehand drawing, or by building a model.

e. g. The student should be able to construct a copy of an angle given a straight edge and a compass.

The student constructs an answer or example. The teacher is concerned only with the student's ability to construct the answer or example, not the method or procedure he uses in arriving at the solution.

e. g. The student should be able to construct the product of a fraction and a whole number.

4. NAME

The student supplies the correct name for a class of objects or events orally or in written form.

e. g. The student should be able to name the associative property of addition in the set of whole numbers.

The student names the correct solution to a problem. This is different from construct in that an immediate response is expected. In this sense Name is used in relation to the basic arithmetic facts which students should commit to memory.

e. g. The student should be able to name the addition facts through 9.

5. ORDER

The student arranges or classifies two or more objects or events in proper order in accordance with a stated category. This word is used when the student arranges something from largest to smallest, most to least, or fastest to slowest.

e.g. The student should be able to order a set of whole numbers from largest to smallest.

6. DESCRIBE

The student states all the necessary categories or properties relevant to the description of a designated situation. The student's description must be stated so clearly that any other individual could use the description to do a task, identify an object, or perform an operation. The description is mostly verbal, however a model, hand motions, or a written example could be used to aid in the description.

The teacher must be willing to accept more than one response. For example, the student might describe something in terms of his surroundings, by using example or by stating a definition. The description may include color, size, shape, etc.

e.g. The student should be able to describe sample spaces as ordered arrangements, listing all possible outcomes.

7. STATE A PRINCIPLE OR RULE

The student makes a verbal statement which conveys a rule or principle. This is more limiting than describing in that only one basic response is acceptable. Students may use their own words in stating the rule. For example, when asked the question, "How do you find the area of a square?" A student may respond, "To find the area of a square, measure the length of a side and multiply this number by itself," or "$A = S^2$"-- both are acceptable answers. Any formula, theorem, or definition is a statement of a rule.

e. g. The student should be able to state the principle that the circumference of a circle equals pi times diameter, ($C = \pi D$).

8. APPLY THE RULE

The student uses a rule or principle to derive an answer to a question. The question is stated in such a way that the student must employ a rational process to arrive at the solution. Students might not be able to state the rule, however, he may still be able to apply it.

e. g. The student should be able to apply the principle of casting nines to check addition problems involving whole numbers.

9. DEMONSTRATE

The student shows a procedure or test for the application of a rule or principle. The teacher wants the student to show how he arrived at an answer, not just the answer alone. This usually involves some action, other than verbal, on the part of the student

e. g. The student should be able to demonstrate a procedure for finding the least common multiple of a given pair of numbers.

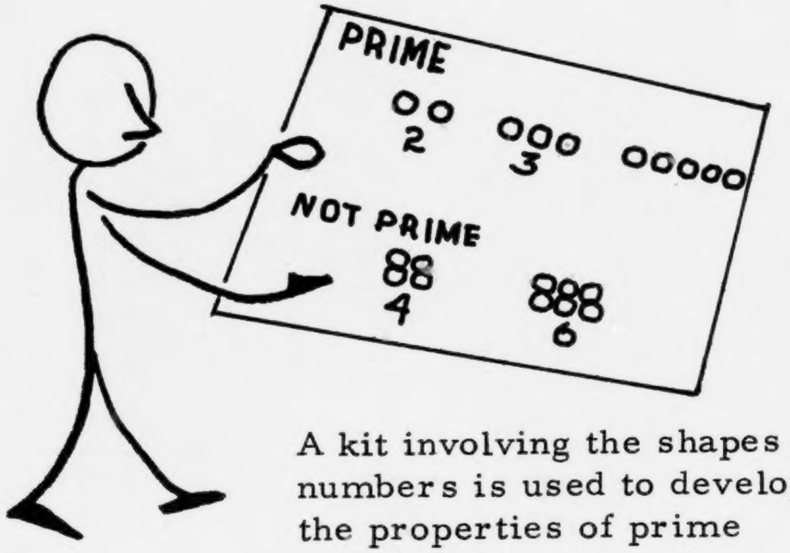
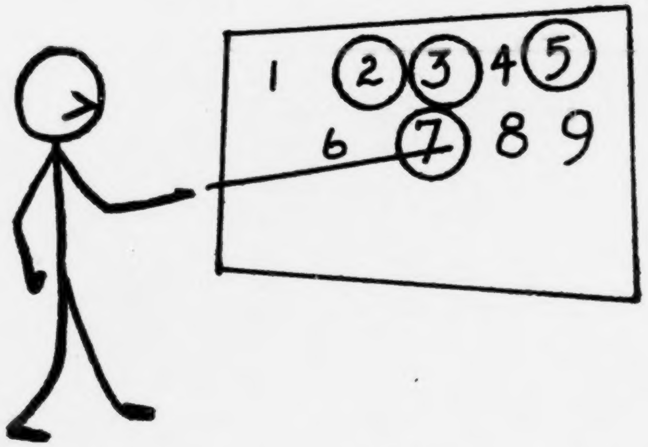
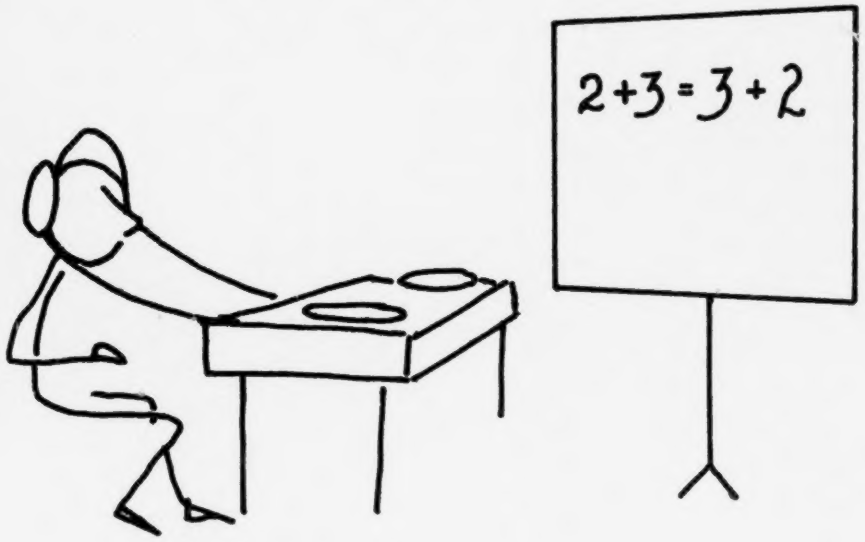
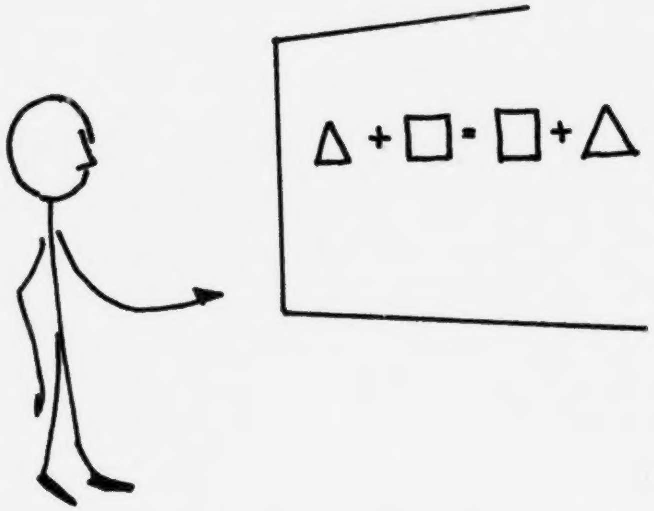
10. INTERPRET

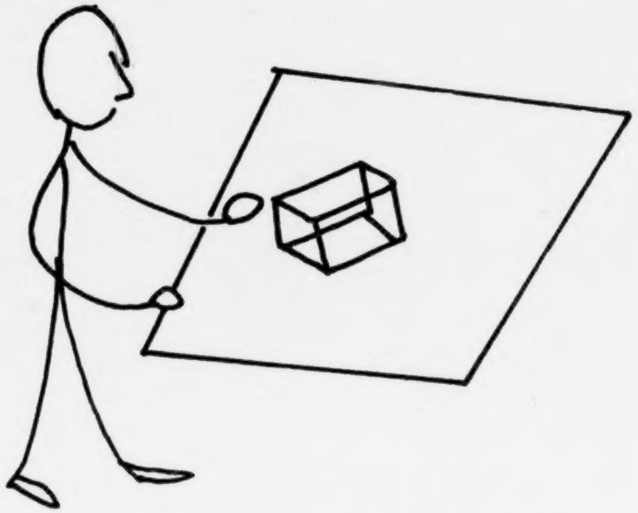
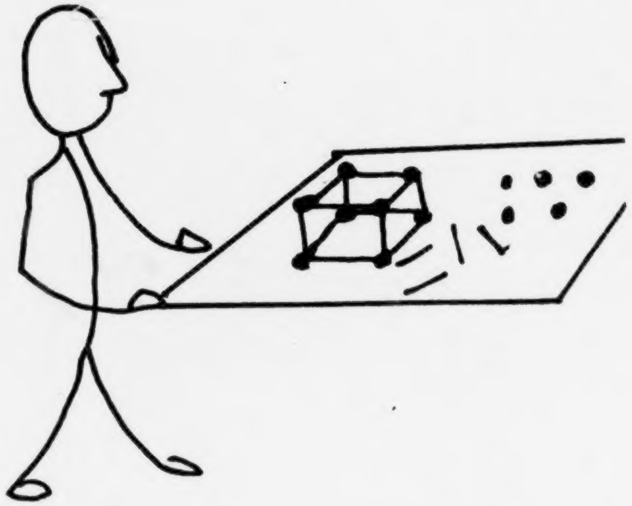
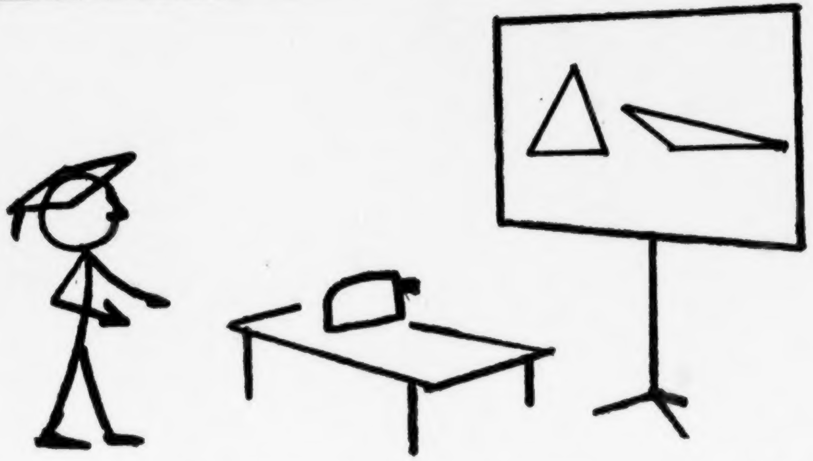
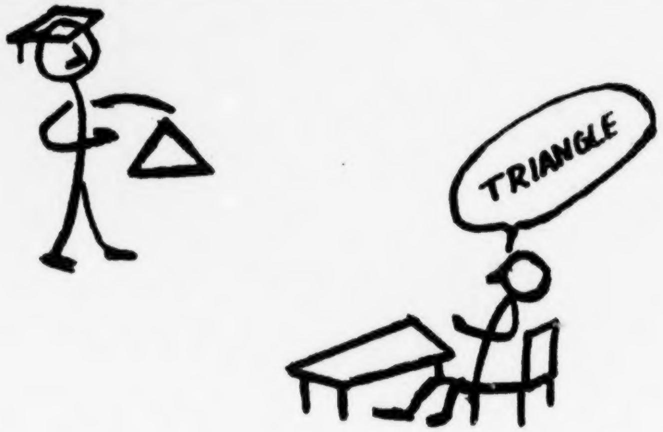
The student uses several rules or principles to draw a conclusion, or identifies objects and/or events in terms of their consequences. This constitutes a high level of learning since the student must see various relationships in order to arrive at the desired conclusion.

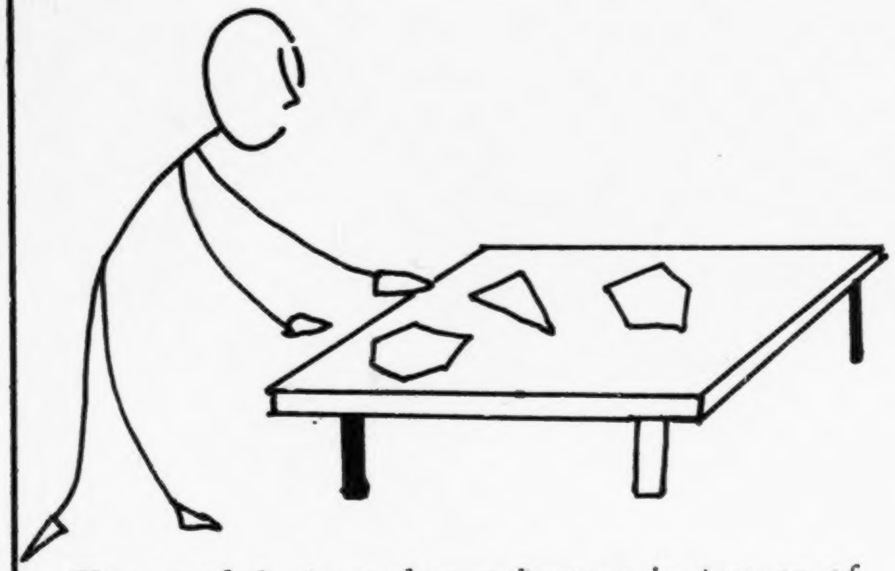
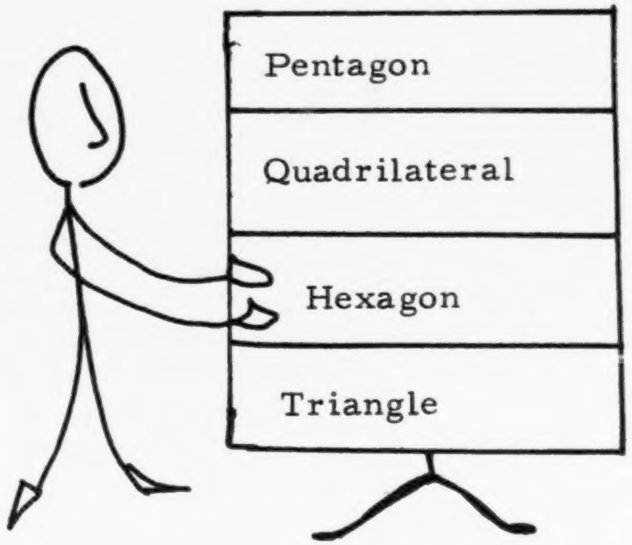
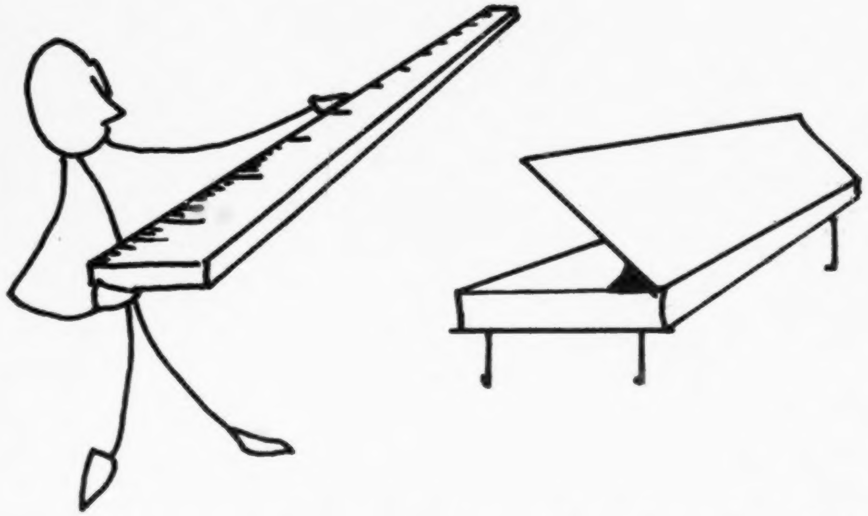
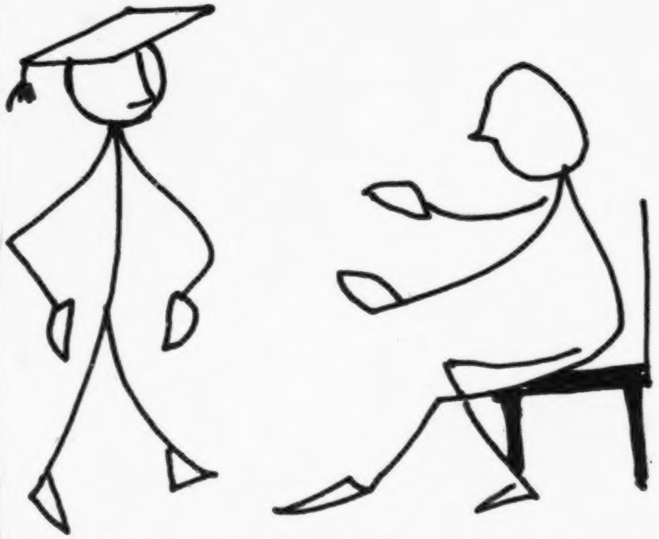
e. g. The student should be able to interpret the principles of angle measure by measuring and then classifying angles as right, acute, obtuse, straight, complementary and supplementary.

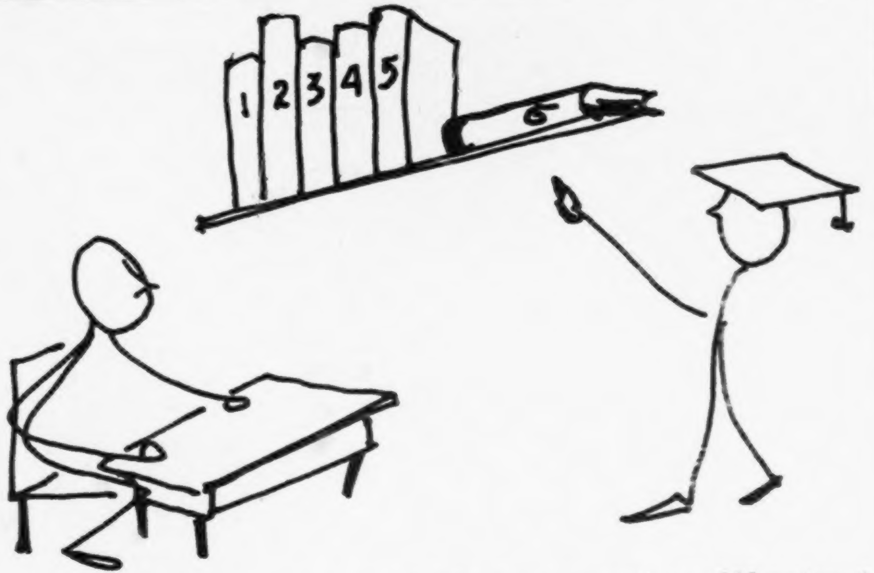
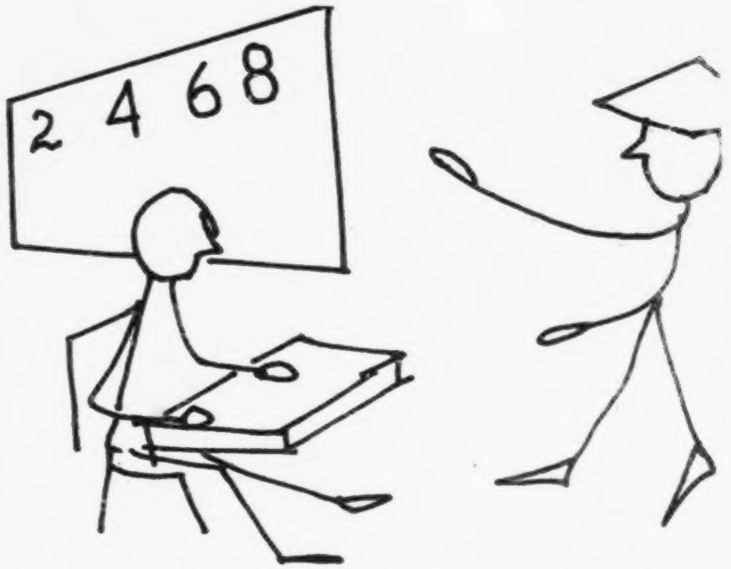
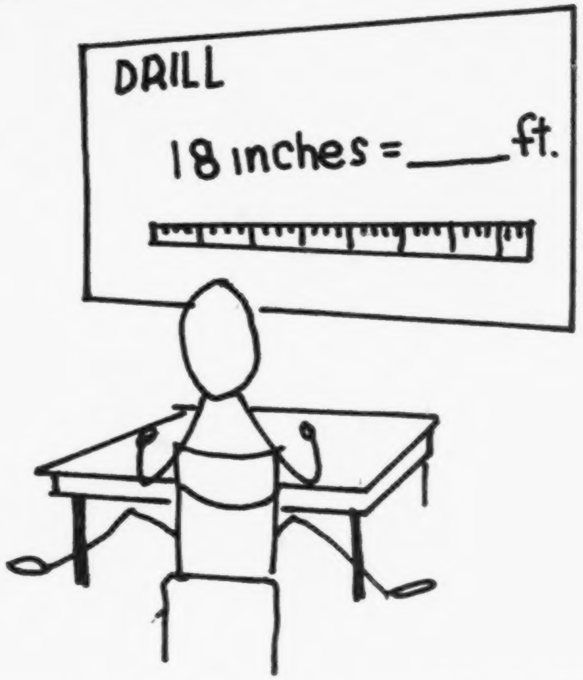
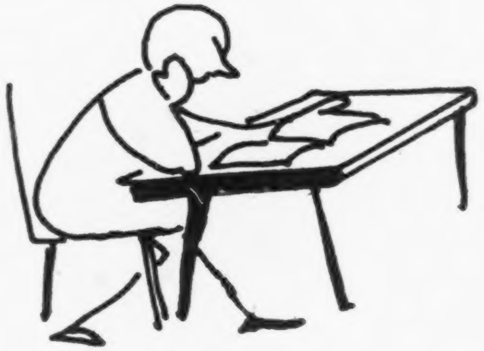
E. OBJECTIVES - INSTRUCTION - ASSESSMENT

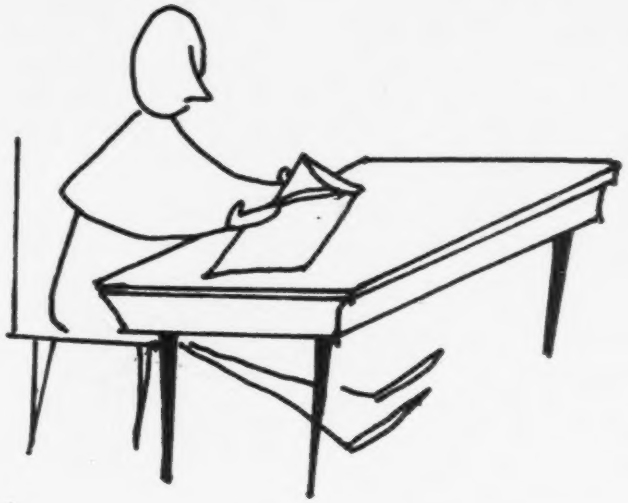
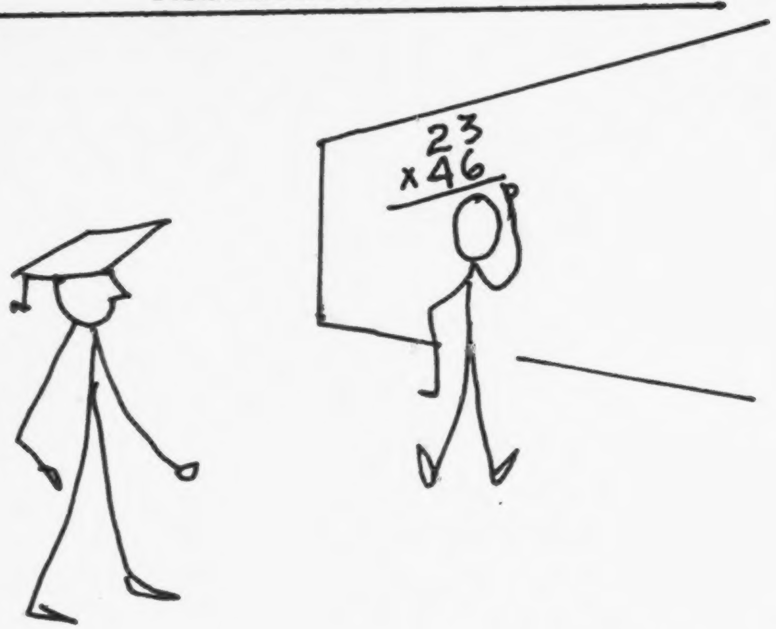
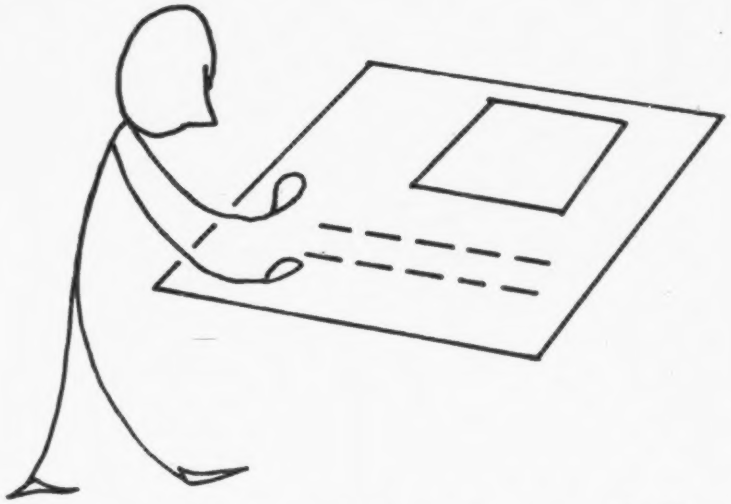
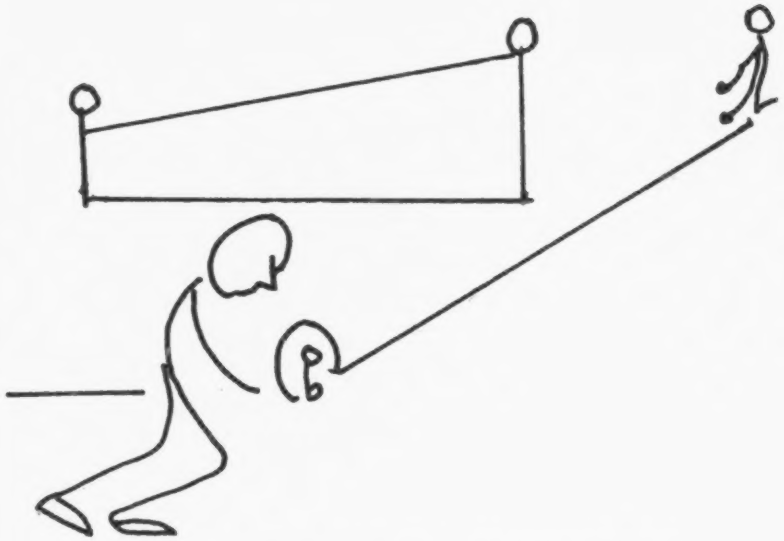
In each of the instructional activities included in this grade the focal point of most comments is the student. First the objectives of the activities are stated in terms of the desired behavioral outcomes on the part of the student. Secondly, the lessons are devoted primarily to student activities. Finally, the suggested assessment procedures indicate ways in which the student shows whether or not he has acquired the desired behavior. In effect, there should be a one-to-one correspondence between the set of objectives, the set of learning activities, and the set of assessment items. The following examples should clarify the relationship.

OBJECTIVE	INSTRUCTION	ASSESSMENT
1. The student should be able to identify a prime number less than 25.	 A kit involving the shapes of numbers is used to develop the properties of prime numbers.	 Students are to circle the numbers which are prime.
2. The student should be able to distinguish among the commutative, associative and distributive properties of addition.	 A tape and filmstrip are used to present the properties.	 Students are given examples on the board and instructed to identify each property.

OBJECTIVE	INSTRUCTION	ASSESSMENT
3. The student should be able to construct a model of a cube given appropriate materials.	 Given straws and string make a model of a cube.	 Use toothpicks and gumdrops to build a model in the shape of a cube
4. The student should be able to name a triangle.	 Filmstrip is used initially to present the name triangle.	 A model is used to assess the student's ability to name a triangle.

OBJECTIVE	INSTRUCTION	ASSESSMENT
5. The student should be able to order polygons in terms of increasing number of sides.	 Use models to order polygons in terms of increasing number of sides.	 Use the flannel board to order polygc in terms of increasing number of sid
6. The student should be able to describe a foot.	 Using kits students are introduced to the foot as a unit of measure.	 Student may describe a foot by a hand motion.

OBJECTIVE	INSTRUCTION	ASSESSMENT
7. The student should be able to state the principle that events which are certain to happen have a probability of 1.	 Six math books on a shelf are used to illustrate what is the probability of picking a math book if blindfolded.	 Student states "The probability of selecting an even number is certain t happen, so it has a probability of 1."
8. The student should be able to apply the principle that 12 inches equals 1 foot to convert a measure expressed in one unit to the other unit.	 DRILL 18 inches = ___ ft. A drill on converting inches to feet is used to introduce the topic.	 A written test is used for assessment purposes.

OBJECTIVE	INSTRUCTION	ASSESSMENT
9. The student should be able to demonstrate a procedure for constructing the product of two whole numbers.	 Student uses book to work problems at his seat.	 Students show work at the board.
10. The student should be able to interpret the rules for finding the areas of rectangles and computing in order to solve related problems.	 Students are asked to find the number of tiles necessary to cover the sheet of paper, and then compute to check their results.	 Students measure the dimensions of the tennis court, then compute to find its area.

F. THE BANDED APPROACH

Teachers who participated in the experimental program last year developed a method of teaching which seems to be effective with the slow learner. The key to the success of this method was that a variety of mathematical topics were incorporated into each lesson. Naturally, small group instruction, individual lab work, extensive use of audio-visual aids, games and the like provided the variety of activities within the lesson which is necessary to change the pace when working with this ability student. Thus, the student was exposed to a program of instruction which provided a variety of activities as well as a variety of mathematical content within a given period. This method of teaching will be referred to as the "banded approach."

To elaborate further the "banded approach" is a flexible way of organizing instructional activities in the class period. Normally the lesson is divided into three bands, although sometimes it may be divided into one, two, or even four bands depending on the nature of the activity. All bands are not necessarily concerned with the same mathematical topic. For example, a unit on Geometry might be taught along with related activities on Fundamental Operations. Thus, the unit on Geometry is split into smaller parcels and presented over a longer period of time rather than being presented as a two week concentrated unit. Since basic students typically have short attention spans, the material must be presented within a smaller time interval. Thus, the major portion of the lesson might be presented during a 25 minute segment since this seems to be about the maximum length of time these students can concentrate on any one activity.

Description of Bands

Band I is usually a short activity of about 5 - 10 minutes duration. For example, students may review their addition facts using the Math Builder, have an oral number puzzle, or complete a number pattern. The variety of activities which might be used is numerous.

Band II usually contains the major topic for the day. It is about 25 minutes in length. For this activity, specific behavioral objectives are stated. Students are exposed to instructional activities which are designed to enable them to acquire the desired behaviors. Assessment procedures might also be employed here to determine whether students have acquired some of the behaviors specified in the objectives. Remaining objectives may be assessed in other bands of subsequent lessons.

Band III is usually a short activity of about 5 - 10 minutes. This band can be handled two ways. First, all the students might begin work at the same time on a class activity. Secondly, as each student completes his work in Band II he begins some planned individual or small group activity. For example, after a student has completed his work from Band II, he may go to a specified place in the room and pick up an interesting puzzle or game, work on one of the SRA kits, or listen to a tape at the listening post. This approach keeps students actively involved in learning activities rather than just waiting for the class to finish an assignment. Thus, a more efficient use of the student's time is made.

The teacher should realize that the above descriptions indicate a general outline of what constitutes a banded approach. Flexibility is the key. Teachers should vary the number of bands as well as the length of time devoted to each depending upon what is being presented.

To illustrate how this approach could be implemented with your students, a sample two week unit is included in section H at the end of the Introduction. This unit contains:

1. A block plan indicating the topics to be presented each day.
2. Detailed lesson plans indicating the materials to be used, the behavioral objectives, suggested methods for presentation, student work sheets, and assessment items.
3. A series of inventory tests designed to indicate areas of difficulty.

This two week unit should be taught near the beginning of the school year. It is hoped that this unit will provide a model from which the teacher can create other units utilizing the same approach.

G. HOW TO USE

This guide provides a structured program of instruction for the slow learner in mathematics grades 6 - 11. Suggestions for implementing the program are included. The teacher is urged to read this section carefully. Familiarity with the materials included and suggestions for their use should be of great assistance in determining the best program of instruction for basic students.

The guide is divided into the following major areas of mathematical competency: Fundamental Operations, Geometry, Measurement, Graphing, Algebra, Probability and Statistics, and Logic. Recreation is the last section of the book.

Each of the areas of mathematical competency contains the following items:

MASTER CHARTS

These charts give an overview of the mathematical content and the behaviors students are to acquire in grades 6 - 11. The teacher can use these charts to get a picture of the total mathematics program for the slow learning student. Furthermore, the teacher can see which behaviors the students should have acquired prior to entering this grade, which behaviors will be developed during this grade, as well as those to be developed later.

GRADE LEVEL CHART

These charts are identical to the master charts except they contain only the information for a specific grade. They can be used to get an overview of those behaviors which should be acquired by the student during the school year.

LIST OF BEHAVIORAL OBJECTIVES

A list of behavioral objectives for this grade should enable the teacher to interpret the details omitted in the chart. The teacher can use these objectives when planning lessons, since they state precisely what is expected of the student. The teacher should realize that these objectives should not necessarily be taught in the order they are presented, rather objectives from several areas might be used in order to present a logical development of the topic. However, by the end of the year the students should be able to exhibit all the behaviors mentioned.

Also included in this section are references to the student activities which have been developed. These activities have been specifically designed to bring about the desired behavioral changes indicated in the objectives. This should assist the teacher in identifying the type of activity which might be used when developing a particular topic.

STUDENT ACTIVITIES

This section contains a series of suggested activities.

For each activity a Teacher Commentary printed on yellow paper is included. This commentary indicates the title of the activity, the unit, the behavioral objectives, necessary materials, a procedure for implementation and suggested assessment items. Student work sheets are printed on white paper and immediately follow the Teacher Commentary. The teacher can reproduce these work sheets by taking the master copy out of the guide and making a thermal spirit master. The spirit master can then be used to run off copies for the students. Be sure to place the original copy back in the guide so it can be used again at a later date.

If color is desired it may be added by using colored masters before duplication.

The Student Activities section also includes references to:

1. Kits. These kits are effective devices for use in small groups or with individual students. Students perform various experiments and as a result of this experimentation, are lead to generalizations. The teacher is supplied with all the necessary instructions for constructing the kit as well as the accompanying student work sheets. It is suggested that the teacher use student help in the construction of the kits.

2. Tapes. Some tapes and their related student work sheets are included. These tapes cover a variety of topics on each grade level. It is suggested that these tapes be used with small groups of students using listening posts, rather than as a class activity.
3. Programed Instruction. Several programed booklets are included in the guide. These can be used with individual students or small groups for remedial purposes or when the teacher feels additional development might be necessary. The programed booklets can be reproduced using the same procedures outlined for the student work sheets. Again, it is suggested that student help be employed in assembling the programed booklets.
4. Films. Several films are included in the Teacher Commentary. These films can be obtained from the Baltimore County Central Film Library. They can be used with the banded approach since the average running time is between 10 - 15 minutes.

RECREATIONAL ACTIVITIES

The Recreation section of the guide is significantly different from the sections dealing with mathematical competencies. The activities described in this section are designed to develop a positive attitude towards mathematics. There are no behavioral objectives specified in this section. Games and puzzles play an important role in the teaching of mathematics. These activities are to be used throughout the year for motivational purposes. When using the banded approach, the recreational activities are used extensively since they help provide the variety which is necessary to the success of this method of teaching.

In the period of time allocated to produce this guide it was impossible to create activities in each of the areas. Therefore, provisions were made to supplement this guide as other activities are developed. Teachers are requested to send activities which they have found to be successful to The Office of Mathematics so that they may be added to this guide.

H. A SAMPLE UNIT USING THE BANDED APPROACH

SAMPLE UNIT OF BANDED LESSONS - Grade 9

OUTLINE OF TOPICS

LESSON	BAND I	BAND II	BAND III
1	Flash cards-addition of whole numbers	Introduction to counting problems	Choice of puzzles and games
2	Survey Test Addition of whole numbers	More counting problems	Choice of puzzles and games
3	Number patterns	Fundamental principle of counting	Choice of puzzles and games
4	Math Builder	The Box Diagram student boardwork	Oral number puzzle
5	Survey Test Subtraction of whole numbers	The Tree Diagram	Discussion of puzzles and games
6	Quiz I	Counting problems	Choice of puzzles and games
7	Oral number puzzle	Discussion of quiz results	Introduction to arrangement problems
8	Factorial notation		Choice of puzzles and games
9	GROUP WORK INVOLVING THE SRA COMPUTATIONAL SKILLS KIT		
10	Flash cards-addition and subtraction of whole numbers	Applying the factorial notation to arrangement problems	SRA Kit and cross number puzzles
11	Math Builder	Arranging n things m at a time	SRA Kit and cross number puzzles
12	Oral drill on multiplication	Summary of arrangement problems	Choice of puzzles and games

LESSON 1

I. Unit: Probability

II. Materials:

- A. Flash cards on basic addition facts
- B. Student work sheets entitled, "Gino's" and "Ameche's"
- C. Student work sheets entitled, "Did You Read It Correctly?"
- D. Student work sheets entitled, "Do You Know Your Number Facts?"
- E. Student work sheets entitled, "How Well Can You Think?"
- F. Student work sheets entitled, "Geometrical Puzzles"
- G. Games: Heads Up, Take 12 and Equations

III. Procedure:

A. Band I

1. Use flash cards as an oral exercise for reviewing the basic addition facts. Each student should have the opportunity to answer at least one problem.

B. Band II

1. Tell the class that they are going to begin a study of some interesting types of problems.
2. Distribute the work sheet entitled, "Gino's - Snack Bar Mathematics." Discuss the problem and have students complete the work sheet with your help.
3. Distribute the work sheet entitled, "Ameche's - Snack Bar Mathematics." Have the students begin work on these in class.

C. Band III

The work sheets in the materials section C - F should be reproduced and placed in some convenient place in the room. The games listed should also be placed in a convenient location for the students. The students should have the opportunity to choose the activity which interests them most after completing the work in Band II. This should be a continuing process for the duration of the unit. The students do not necessarily need to complete all the problems on one work sheet before choosing another activity. Also the students should be encouraged to keep their results since there will be opportunities later in the unit to discuss their findings.

Note that the purpose of this activity is to allow students to work at their own rate. The faster students may work on the puzzles and games while the slower students are finishing previous assignments or work sheets. On occasion time should be allowed so that even the slowest student will have a chance to work a puzzle.

Solution: "Gino's - Snack Bar Mathematics"

1. Hamburger _or_ Cheeseburger
2. Coke _or_ Orange _or_ Root Beer
3. 2
4. 3
5. 3
6. a. Coke
   b. Orange
   c. Root Beer
7. 3
8. a. Coke
   b. Orange
   c. Root Beer
9. 6

Solution: "Ameche's - Snack Bar Mathematics"

1. Hamburger _or_ Cheeseburger _or_ Hot Dog
2. Coke _or_ Orange _or_ Root Beer _or_ Grape
3. 3
4. 4
5. 4
6. a. Coke
   b. Orange
   c. Root Beer
   d. Grape
7. 4

8. a. Coke
   b. Orange
   c. Root Beer
   d. Grape

9. 4

10. a. Coke
    b. Orange
    c. Root Beer
    d. Grape

11. 12

Solutions:

Did You Read It Correctly?

1. 9
2. Neither, 8 and 7 is 15
3. The man's picture
4. 900 pounds
5. Neither
6. 2 apples

Do You Know Your Number Facts?

1. 29 minutes
2. 5
3. 1
4. 36
5. South Pole
6. $1 + 2 + 3 + 4 + 5 + 6 + 7 + (8 \times 9) = 100$
7. 14

How Well Can You Think?

1. TOT, MADAM I'M ADAM, POP, DAD, TOOT, etc.
2. Alphabetically
3. Nickel, quarter (the quarter is not a nickel)
4. 0
5.

8	1	6
3	5	7
4	9	2

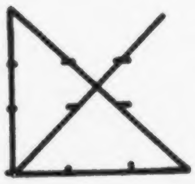
Not unique

6. (c) 15 minutes
7. None, the others flew away

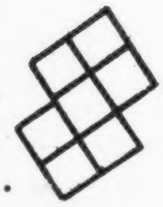
Geometrical Puzzles

1. Only b, c, and d can be drawn

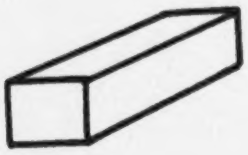
2.



3.

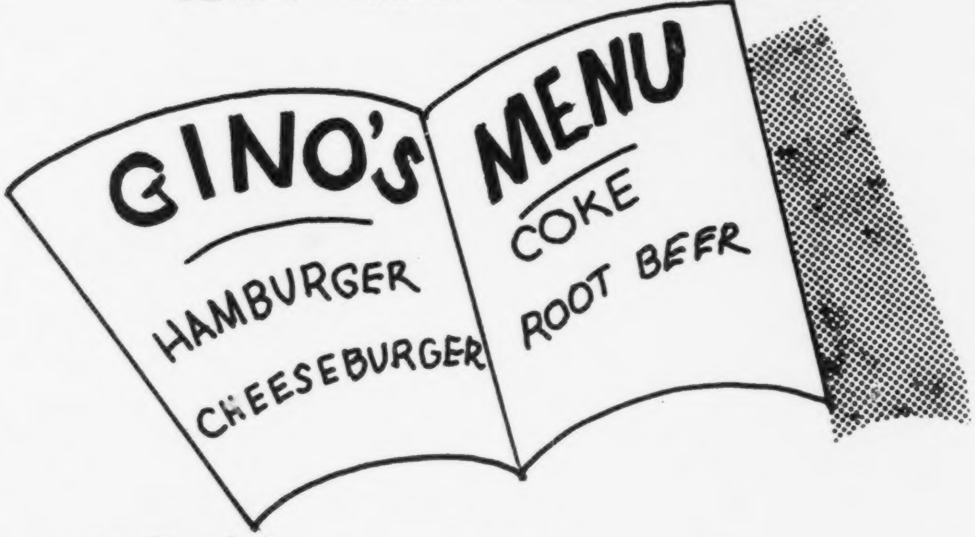


4.



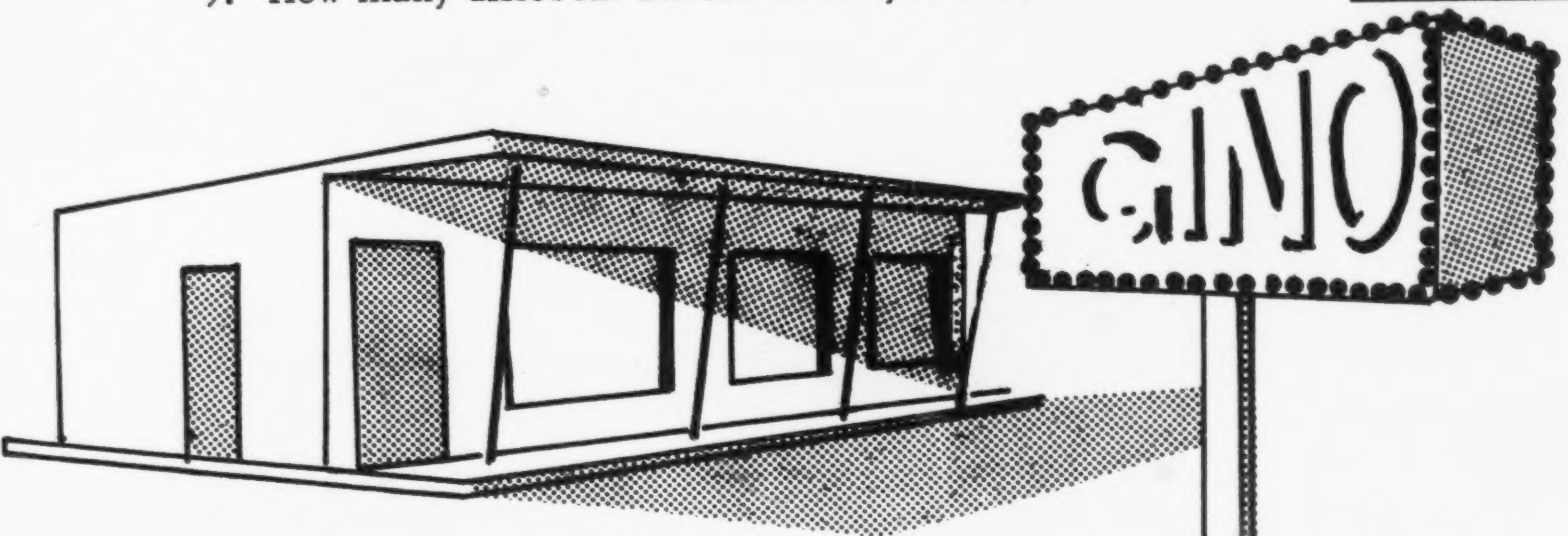
5. 11 squares; 26 triangles

GINO'S - SNACK BAR MATHEMATICS

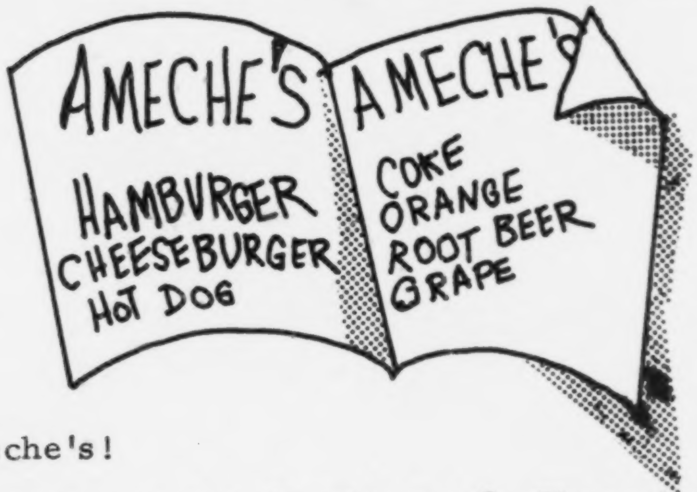

GINO'S
MENU
HAMBURGER
CHEESEBURGER
COKE
ROOT BEER

Let's go to Gino's!

1. What will you have to eat, Hamburger? Cheeseburger? ____________
2. What will you have to drink, Coke? Orange? Root beer? ____________
3. How many choices did you have to eat? ____________
4. How many choices did you have to drink? ____________
5. How many drinks could you choose with a hamburger? ____________
6. List them: a. (hamburger, ____________)
   b. (hamburger, ____________)
   c. (hamburger, ____________)
7. How many drinks could you choose with a cheeseburger? ____________
8. List them: a. (cheeseburger, ____________)
   b. (cheeseburger, ____________)
   c. (cheeseburger, ____________)
9. How many different snacks could you choose? ____________



AMECHE'S - SNACK BAR MATHEMATICS


AMECHE'S
HAMBURGER
CHEESEBURGER
HOT DOG
AMECHE'S
COKE
ORANGE
ROOT BEER
GRAPE

Let's go to Ameche's!

1. What will you have to eat, Hamburger? Cheeseburger? Hot Dog? ______________
2. What will you have to drink, Coke? Orange? Root beer? Grape? ______________
3. How many choices did you have to eat? ______________
4. How many choices did you have to drink? ______________
5. How many drinks could you choose with a hamburger? ______________
6. List them: a. (hamburger, ______________)
   b. (hamburger, ______________)
   c. (hamburger, ______________)
   d. (hamburger, ______________)
7. How many drinks could you choose with a cheeseburger? ______________
8. List them: a. (cheeseburger, ______________)
   b. (cheeseburger, ______________)
   c. (cheeseburger, ______________)
   d. (cheeseburger, ______________)
9. How many drinks could you choose with a hot dog? ______________
10. List them: a. (hot dog, ______________)
    b. (hot dog, ______________)
    c. (hot dog, ______________)
    d. (hot dog, ______________)
11. How many different snacks could you have at Ameche's? ______________

DID YOU READ IT CORRECTLY?

1. A man had 35 head of cattle. All but 9 died. How many did he have left?
2. Which is correct: 8 and 7 are 13, or 8 and 7 is 13?
3. A man observing a portrait says, "Brothers and sisters I have none, but that man's father is my father's son." How is this possible?
4. A steer on three legs weighs 900 pounds. What will the steer weigh when it stands on all four legs?
5. Train A leaves San Francisco and Train B leaves New York at the same time. Train A averages 50 miles per hour and Train B averages 75 miles per hour. Assume it is 3000 miles by rail from San Francisco to New York. Which train is the nearer to New York when they pass?
6. If you take 2 apples from 3 apples, how many do you have?

DO YOU KNOW YOUR NUMBER FACTS?

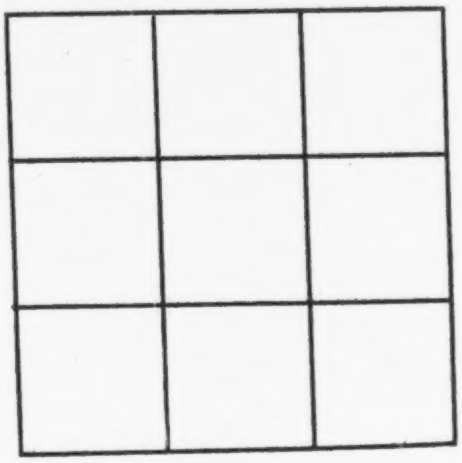
1. The number of eggs in a basket doubles every minute. The basket is full of eggs in half an hour. When was the basket half full?
2. In making change from a dollar bill for a 28¢ sale, what would be the least number of coins that could be used?
3. What is the quotient of any number divided by itself?
4. A man fenced in a square plot of land. When he had finished, there were ten fence posts on each side. How many posts did he use?
5. Where on earth could a man walk five miles due north, five miles due west, five miles due south, only to find himself back where he started.
6. Insert mathematical symbols to make a true statement:

   1 2 3 4 5 6 7 8 9 100
7. Which answer is correct? 6 + 2 x 4 = 32 or 14

HOW WELL CAN YOU THINK?

1. A palindrome is a word or group of words which spells the same thing backwards as it does forward. Here are some examples:
    1. MADAM
    2. OTTO
    3. WOW
    4. MOM
    5. RADAR

    Now try to write some more palindromes.
2. A secretary said that the digit symbols would naturally be arranged in the following order: 8 5 4 9 1 7 6 3 2 0. Why?
3. I have two current United States coins. Together they total 30¢. One of the coins is not a nickel. What are the coins?
4. Two times what number is the same as three times that number?
5. Complete the following magic square using the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 so that the sum of any row, column, or diagonal is equal to 15.



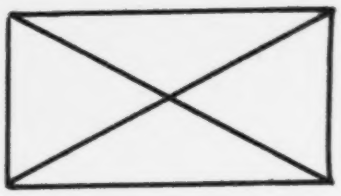
6. To walk a mile without fooling around would take about:

    (a) 4 minutes (b) 8 minutes (c) 15 minutes (d) $\frac{1}{2}$ hour
7. Twelve doves were sitting in a tree. A hunter shot four of them with his first shoot. How many remained?

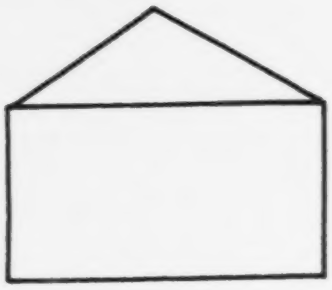
GEOMETRICAL PUZZLES

1. Try to draw the following figures without lifting your pencil and without retracing any part of a line.

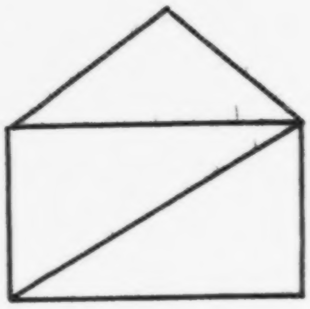
a)



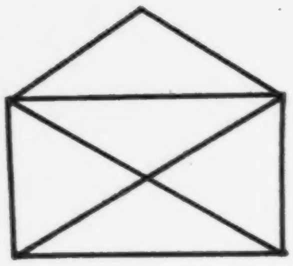
b)



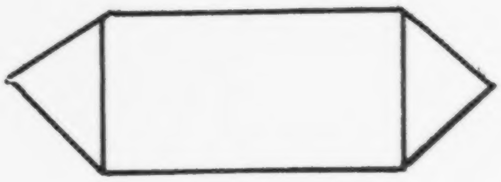
c)



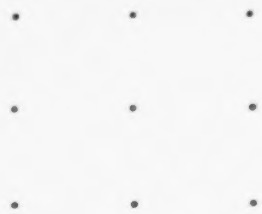
d)



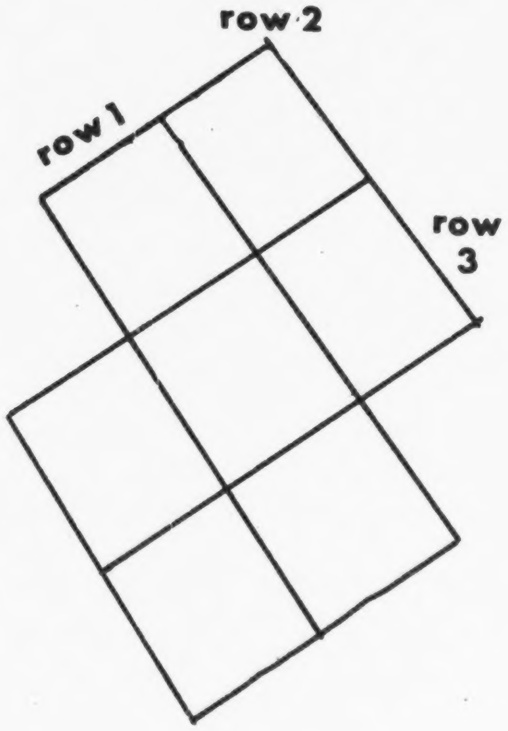
e)



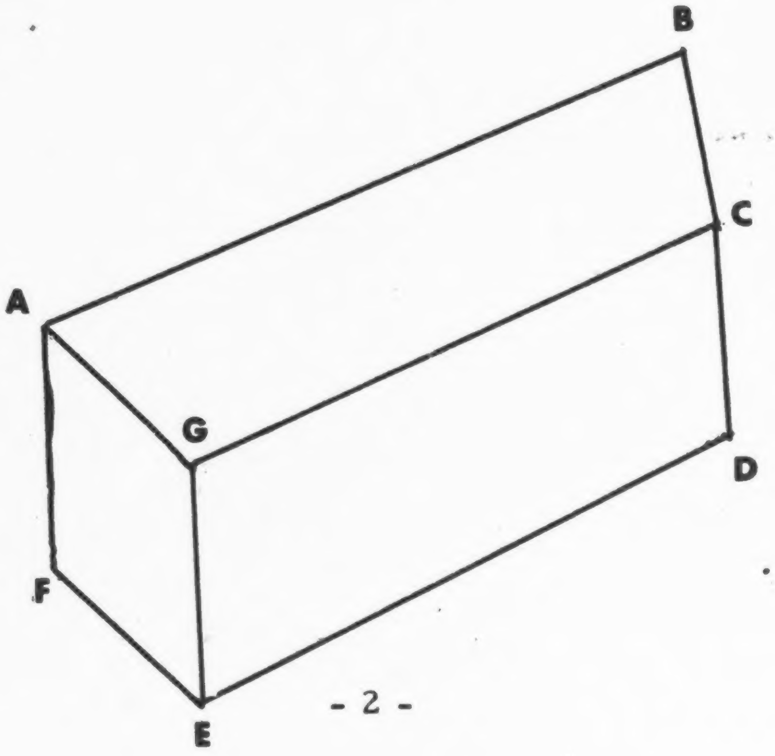
2. Try to draw four straight line segments without lifting your pencil. When you have drawn the four segments, each dot must be covered by at least one of these segments.



3. Use the numerals 1, 2, 3, 4, 5, 6, and 7 to complete the following diagram. Write a numeral in each of the seven diamonds. The sum of each of the three rows of three diamonds must add up to 10. Each number must be used only once.

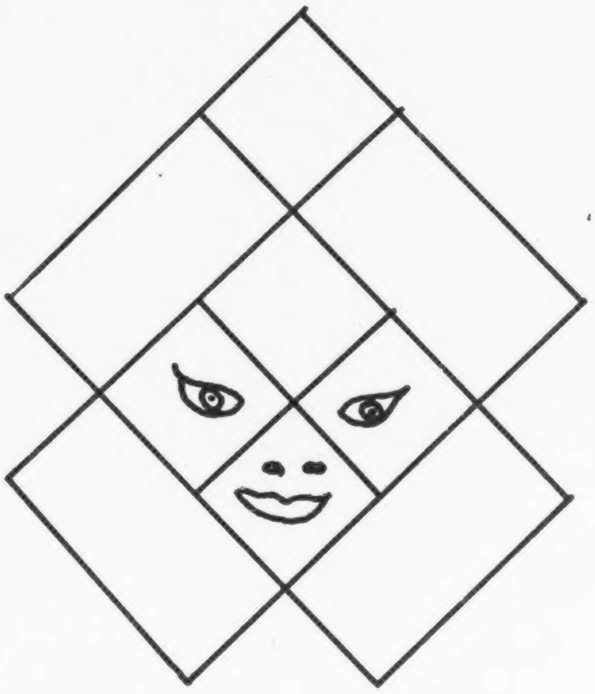


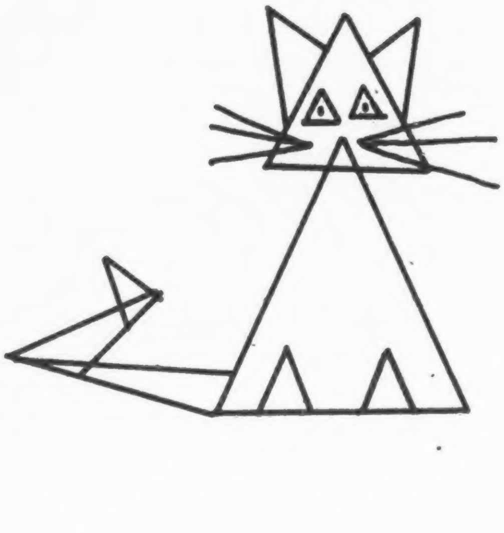
4. What is wrong with this picture? Can you redraw it correctly?



5. The Hindu and The Cat

How many different squares can you count in the picture of the Hindu boy? How many different triangles can you count in the picture of the cat? Look carefully. The problems are not as easy as you might think.





LESSON 2

I. Unit: Probability

II. Materials: Work sheets entitled "Big Boy's" and "How Many?"

III. Procedure:

A. Band I

1. Write the following problems on the board. Have students complete these as they enter the room. They are to be used as a survey test on addition of whole numbers.

$$\text{a.}\ \begin{array}{r}16\\+29\\\hline 45\end{array}\qquad \text{b.}\ \begin{array}{r}47\\+70\\\hline 117\end{array}\qquad \text{c.}\ \begin{array}{r}18\\215\\+\ 30\\\hline 263\end{array}\qquad \text{d.}\ \begin{array}{r}418\\375\\+296\\\hline 1089\end{array}\qquad \text{e.}\ \begin{array}{r}4198\\+5673\\\hline 9871\end{array}\qquad \text{f.}\ \begin{array}{r}426\\371\\18\\+9175\\\hline 9990\end{array}$$

2. These problems should be checked in class and collected. The teacher should keep an accurate record of the results since they will be used later as a basis for grouping.

B. Band II

1. Discuss the results of the work sheet entitled "Ameche's - Snack Bar Mathematics."

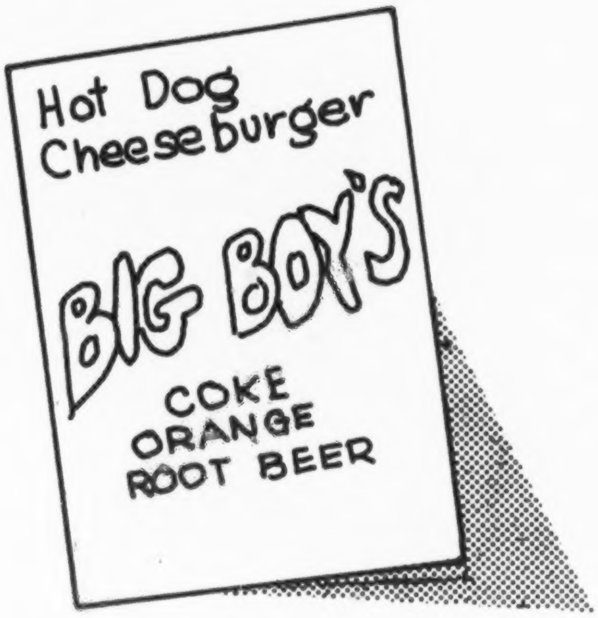
2. Distribute the work sheet entitled "Big Boy's." Have the students complete this quickly. Tell them to look for a pattern as they work.

3. Discuss the results of the "Big Boy's" work sheet. Do not formally reveal the Fundamental Principle of Counting yet.

4. Distribute the work sheet entitled "How Many?" Students should complete this at their own rate! Instruct them to be particularly conscious of patterns in their work.

C. Band III

As students complete the work sheet entitled, "How Many?" they should select their own puzzle or game for the remainder of the period.


Hot Dog
Cheeseburger
BIG BOY'S
COKE
ORANGE
ROOT BEER

Let's go to Big Boy's!

How many choices will you have to eat? ____________

How many choices will you have to drink? ____________

How many drinks could you have with a hot dog? ____________

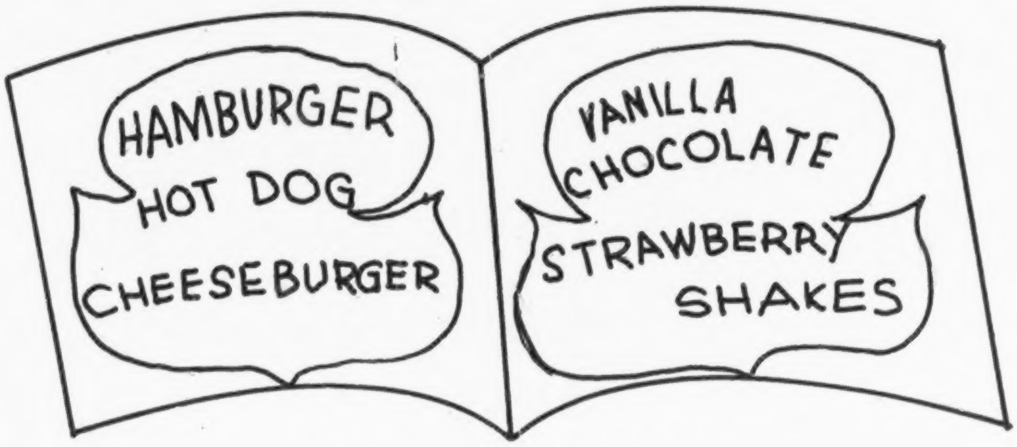
How many drinks could you have with a cheeseburger? ____________

How many snacks could you have at Big Boy's? ____________

How did you get your answer? ____________

HOW MANY?

1. How many snacks?


HAMBURGER
HOT DOG
CHEESEBURGER
VANILLA
CHOCOLATE
STRAWBERRY
SHAKES

2. How many outfits?

    2 sweaters - one red, one white

    3 skirts - one gray, one blue, one black

3. How many records?

    3 singers - James Brown, Petula Clark, Elvis Presley

    4 songs

4. How many sundaes?

    4 flavors of ice cream

    4 toppings

5. Sally has 8 outfits of skirts and sweaters to wear.

    She has 2 sweaters.

    How many skirts?

LESSON 3

I. Unit: Probability

II. Objectives: The student should be able to:

A. Describe the Fundamental Principle of Counting by using specific examples

B. Apply the Fundamental Principle of Counting to solve related problems

III. Materials: Work sheets entitled, "More Meal Menus" and "A Trip to Paris"

IV. Procedure:

A. Band I - Written Drill on Number Patterns

1. Have this drill on the board for the students to complete as they enter the room.

What Comes Next?

a. 1, 3, 5, 7, ___, ___, ___ (Answer: 9, 11, 13)

b. 1, 2, 4, 7, ___, ___, ___ (Answer: 11, 16, 22)

c. 1, 2, 1, 3, 1, ___, ___, ___ (Answer: 4, 1, 5)

Discuss the student solutions. Point out that there could be many possible solutions so long as a pattern is maintained.

B. Band II

1. Discuss the results of the work sheet entitled, "How Many?"

2. Ask - Is there an easier and faster way of finding answers to these problems than listing all the possibilities?

Hopefully, by this time, most students will have discovered the basic idea of the Fundamental Principle of Counting. Use some of the problems from the work sheet to discuss their ideas and label their rule as the Fundamental Principle of Counting, i.e. If a first thing can be done in $\underline{m}$ ways, and if after that, a second thing can be done in n ways and if, after that, a third thing can be done in p ways, then the total number of ways in which all three things can be done together is $m \cdot n \cdot p$.

3. Ask the students to make up some problems which involve the fundamental principle. Have them present their problems to the class and discuss.

4. Distribute work sheets entitled, "More Meal Menus" and "A Trip to Paris." Have students complete these sheets individually.

C. Band III - As students complete the work sheets from Band II let them continue working on their puzzles and games.

MORE MEAL MENUS

Sunday Dinner

Jim took Margie to dinner. They saw the following menu.

Meat	Vegetable	Dessert
roast beef	potatoes	ice cream
steak	peas	cake
chicken	carrots	
	string beans	

Margie said, "Everything looks so good. I would like to have as many different meals as I could." Jim laughed. He knew that Margie could not eat that many meals. Margie asked, "Why are you laughing?" Jim replied, "Do you know how many different meals that would be?" "No," said Margie. Jim showed Margie how she could find out how many different meals there were. A different meal is made of a meat, a vegetable and a dessert.

Place choices in the proper box

meat choices ☐

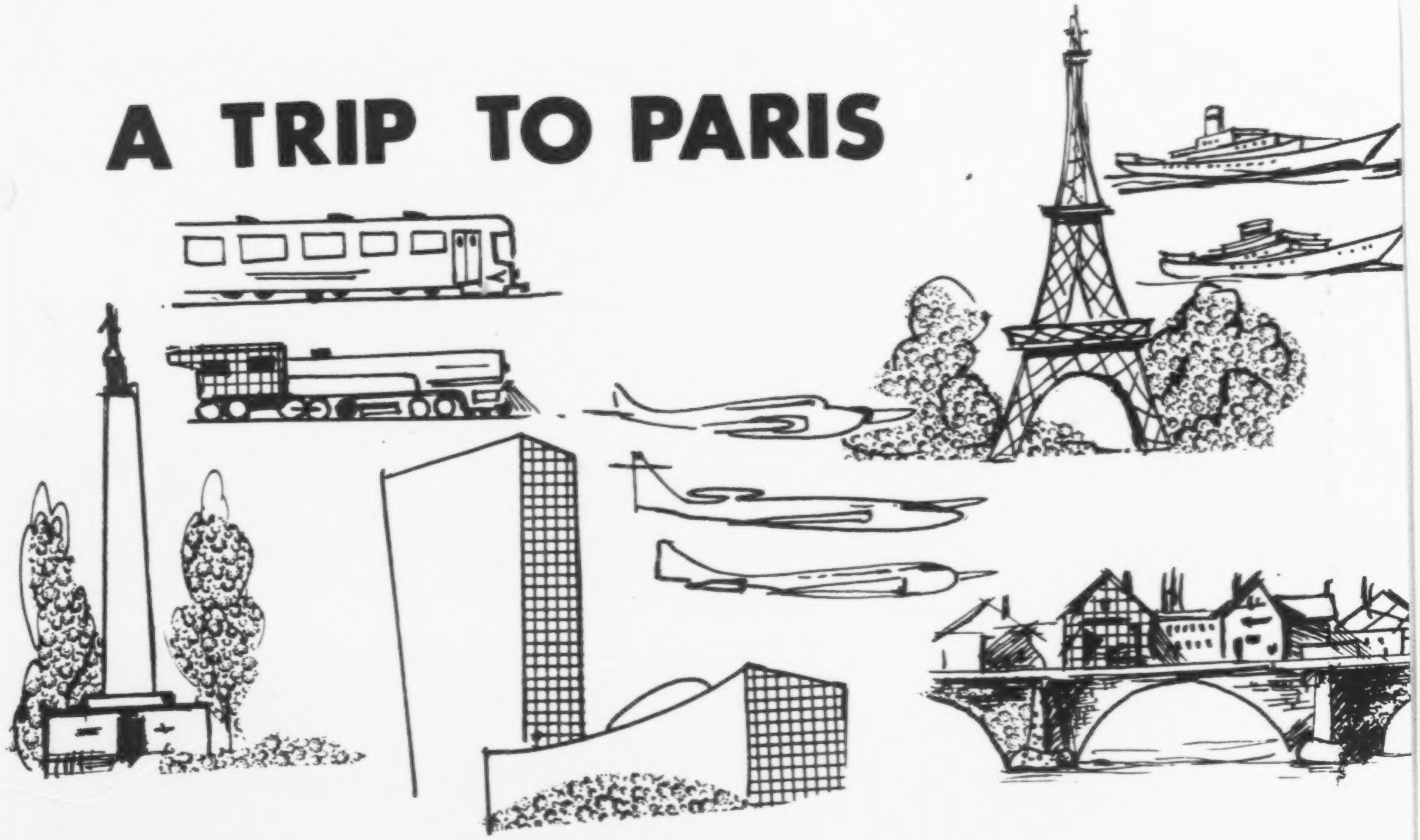
vegetable choices ☐

dessert choices ☐

Then Jim showed how the number of different meals can be found by multiplying the numbers in the boxes.

☐ ☐ ☐ ☐

A TRIP TO PARIS



Betty was going to Paris to visit her aunt. On the way she wanted to stop in New York to see the Empire State Building and other attractions. She took a train to New York. From the picture you can see that there are two railroads to New York.

Betty flew to Paris after enjoying herself in New York. Three airlines fly directly to Paris. Her visit made her aunt very happy. She took Betty for a tour of London. They went to London by ship. There were two choices for ships.

How many different ways could Betty have gone from Baltimore to London?

Trains		Planes		Ships		Different ways to travel from Baltimore to London
	x		x		=	

LESSON 4

I. Unit: Probability

II. Objectives: The student should be able to

- A. Construct a box diagram given appropriate data
- B. Describe a box diagram
- C. Apply the principles of the box diagram to solve related problems

III. Materials:

- A. Math Builder and Filmstrip 9, Subtraction, Minuend 7-10, Set AR-FX
- B. Paper and pencil

IV. Procedure:

- A. Band I
    1. Get the Math Builder and filmstrip 9, Subtraction, Minuends 7-10, from the set AR-FX.
    2. Preview the filmstrip.
    3. The students should have pencil and paper on their desk.
    4. Give the first 15 or 20 subtraction problems orally using the left-to-right scanning technique, and a speed setting of approximately 20. This speed will vary depending upon the class ability. If you find the students are waiting for the answer to appear before they respond, change the frame from left-to-right scanning to the full line frame and mask out the answer. If errors are made with the answers masked, use the pause button to give the students a longer look at the problem.
    5. After 15 or 20 problems have been done orally, advance the filmstrip to part B. Use the full line frame setting.
    6. Have the students number from 1 to 20 on their paper.
    7. Have the students write the answers to the problems on their paper.

8. Start with the speed control at a slow setting and gradually increase it. You will probably not want the setting to be greater than 80 when the fastest speed is obtained.

9. To check the problems turn the filmstrip back to frame 1 and go through the problems again having a student read his answers. This should be done at a slow pace.

B. Band II

1. Have some students describe the Fundamental Principle of Counting using examples that will serve as a review and also partial assessment of the objectives of Lesson 3.

2. Discuss the work sheets entitled, "More Meal Menus" and "A Trip to Paris." Point out the box diagram used in the solution. Introduce the name "box diagram." Emphasize that a box diagram is an easy way to find the total number of things that can happen in a counting problem.

3. Have the following two problems written on the board. Let the students solve them at their seats.

   Use a box diagram to solve these problems.

   1. Tony wanted to dress well. He liked to wear something different each day of the week. Tony had 2 sport coats, 3 pairs of pants, and 4 neckties. How many different ways can Tony dress?

   2. Jim wanted to go to Hawaii. He also wanted to stop in California to see a friend. Jim could get to California on any one of 3 trains. He could get from California to Hawaii on one of two different boats. How many different ways would Jim travel from his home to Hawaii?

4. Have different students put their solutions on the board. Try to get every student at the board for one of the two problems. This will help assess Objectives A and C of the lesson.

C. Band III - Oral Number Puzzle Involving Division by 9

Have each student:	Example:
1. Write a number with 3 or more digits.	1. 1234
2. Divide by 9.	2. $\begin{array}{r} 137\text{ R}1 \\ 9\overline{)1234} \end{array}$
3. Circle the remainder.	3. 1
4. Erase any one digit of the dividend other than zero.	4. $12\cancel{3}4$
5. Divide the new number by 9.	5. $\begin{array}{r} 13\text{ R}7 \\ 9\overline{)124} \end{array}$
6. Circle the remainder.	6. 7
7. You can now tell him the number from the dividend which he erased.	

Solution:

a. Ask the student his first remainder, then his second remainder.

b. Subtract the second remainder from the first.

c. If the second remainder is larger than the first one, add 9 to the first remainder, then subtract.

d. From the example problem:

$$\begin{array}{lr} 1 \text{ first remainder} & +9 = 10 \\ 7 \text{ second remainder} & -\underline{\ 7\ } \\ \quad\text{the number erased} & 3 \end{array}$$

LESSON 5

I. Unit: Probability

II. Objectives: The student should be able to:

- A. Consturct a tree diagram given appropriate data
- B. Describe a tree diagram
- C. Apply the principle of the tree diagram to solve related problems

III. Materials: None

IV. Procedure:

A. Band I

1. Write these problems on the board. They are to be used as and inventory test on subtraction of whole numbers.

$$\text{a.}\ \begin{array}{r} 47 \\ -32 \\ \hline 15 \end{array} \qquad \text{b.}\ \begin{array}{r} 51 \\ -13 \\ \hline 38 \end{array} \qquad \text{c.}\ \begin{array}{r} 506 \\ -394 \\ \hline 112 \end{array} \qquad \text{d.}\ \begin{array}{r} 653 \\ -\ 94 \\ \hline 559 \end{array} \qquad \text{e.}\ \begin{array}{r} 800 \\ -453 \\ \hline 347 \end{array} \qquad \text{f.}\ \begin{array}{r} 644 \\ -408 \\ \hline 236 \end{array}$$

2. These problems should be checked in class and collected. The teacher should keep an accurate record of the results since they will be used in conjunction with the inventory test for addition of whole numbers as a basis of grouping the students for extra work in these areas.

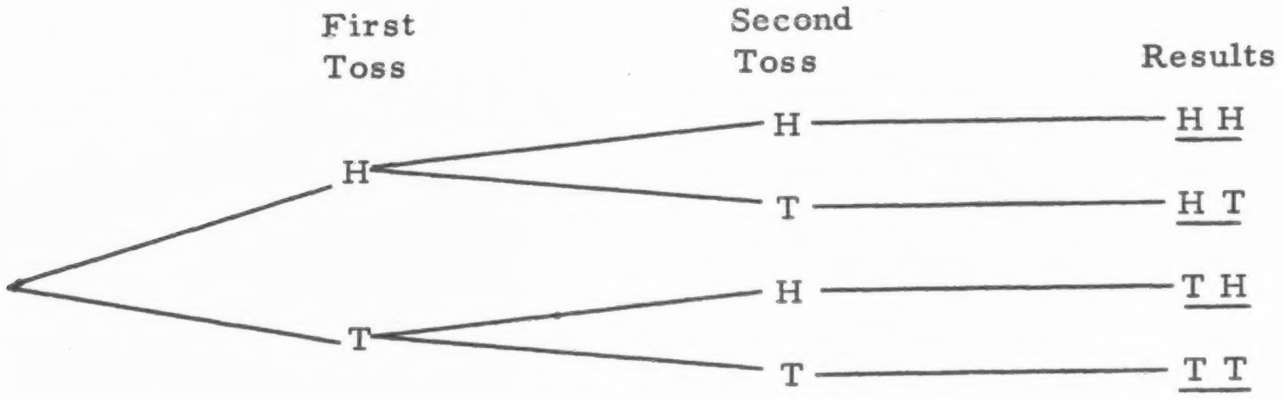
B. Band II

1. Discuss the fundamental principle and the box diagram. Have students describe each. This will serve as a review and help assess the objectives of Lessons 3 and 4.

2. Pose the following problem:

> If a coin is tossed, it can land heads or tails. Let H stand for heads and T stand for tails. Suppose we have two coins and we toss each in turn. One result could be indicated HT. This tells us that the first coin landed heads and the second landed tails. List the different ways in which the two coins can land.

3. Point out that this problem doesn't just ask for the number of ways, but asks for a list of the ways. Construct a tree diagram for the students which shows how to arrive at the list.



Introduce the name "tree diagram."

4. Have the students construct a tree diagram to solve the following problem. Have as many students as possible place their solutions on the chalkboard. This will help assess Objective A.

   List the different meals that can be made of: a meat and a vegetable. The choices are listed below.

Meat	Vegetables
chicken	potato
meat loaf	beets
	corn

5. Use the same problem as above except complicate it by including choices of a tossed salad or a lettuce and tomato salad. Have the students construct a tree diagram and place their solutions on the chalkboard.

C. Band III

Give the students a chance to discuss some of their findings from the games and puzzles. Try to have one problem discussed from each work sheet.

LESSON 6

I. Unit: Probability

II. Materials: Quiz I

III. Procedure:

A. Band I

1. Distribute the Quiz sheet and permit students to work at their own rate.

2. Solutions:

	First Coin		Second Coin		Third Coin	
1.	3	x	2	x	1	= 6

2. PVC, PCV, VCP, VPC, CPV, CVP

3. 12

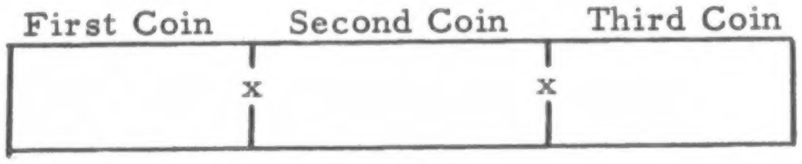
4. SDU, SUD, DSU, DUS, USD, UDS

B. Band II

As students complete the quiz let them work on their choice of games and puzzles.

QUIZ I

1. Tom has 3 coins in his pocket - a penny, a nickel and a dime. He takes the coins from his pocket one at a time. Construct a box diagram to show how many different ways he can take the coins from his pocket.



2. There are three chairs on the stage. The principal, the vice-principal and the coach will use them in an assembly. Construct a tree diagram to show the different ways they can arrange the chairs on the stage.

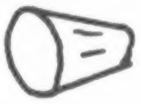
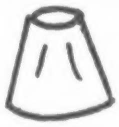
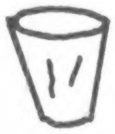
First Chair	Second Chair	Third Chair	Arrangement

3. The school cafeteria has the following menu:

Dinners	Beverage	Dessert
Soup plate	Milk	Ice Cream
Hot plate	Orange Drink	Cake
		Jello

A meal consists of a dinner, beverage and dessert. How many different meals could be selected?

4. If a toothpaste cap is tossed it can land on its side, small side up, or small side down as shown.

Let S stand for side, U stand for up and D stand for down. Suppose we have three caps and toss each one in turn. One result could be SDU. List the different ways in which the three caps can land.

LESSON 7

I. Unit: Probability

II. Objectives: The student should be able to:

   A. Construct the number of arrangements of N things taken N at a time by applying the principle of the box diagram

   B. State the principle that the number of arrangements of N things taken N at a time is the product of the first N counting numbers

III. Materials:

   A. Work sheets entitled "The Bookshelf," "A Pocketful of Money" and "Problems to Solve"

   B. Four different books

IV. Procedure:

   A. Band I - Oral puzzle - Guessing a Birthday

      1. Have each student do the following:

         a. Multiply the number of the month in which you were born by five.

         b. Add six.

         c. Multiply by four.

         d. Add nine.

         e. Multiply by five.

         f. Add the number of the day on which you were born.

      2. Ask individual students for their final results.

      3. Mentally subtract 165 from the final result.

      4. After subtracting 165 the last two digits tell you the day, and the other digits tell you the month of the birthday. For example, if his result is 666, when your subtract 165 you get 501. Therefore, this person's birthday is on May 1.

   B. Band II

      Return and discuss "Quiz I" on arrangements.

   C. Band III

      1. Use four different books for the following problem.

      2. Discuss the number of ways one book can be arranged on a shelf.

3. Continue the discussion by increasing the number of books to two, then three and eventually to four. Each time have students arrange the books and list the arrangements on the board.

4. Organize the results in a chart on the board.

Number of Books	Number of Arrangements
1	1
2	$2 \cdot 1 = 2$
3	$3 \cdot 2 \cdot 1 = 6$
4	$4 \cdot 3 \cdot 2 \cdot 1 = 24$

5. Distribute work sheets entitled "The Bookshelf" and "A Pocketful of Money." Have the students complete these in class quickly.

6. Discuss the results. Use the results to formulate the rule concerning the product of the first N counting numbers.

7. Assessment - Objective A - Distribute work sheet entitled "Problems to Solve."

Solutions:

The Bookshelf

1. 4
2. 3
3. 2
4. 1
5. 24

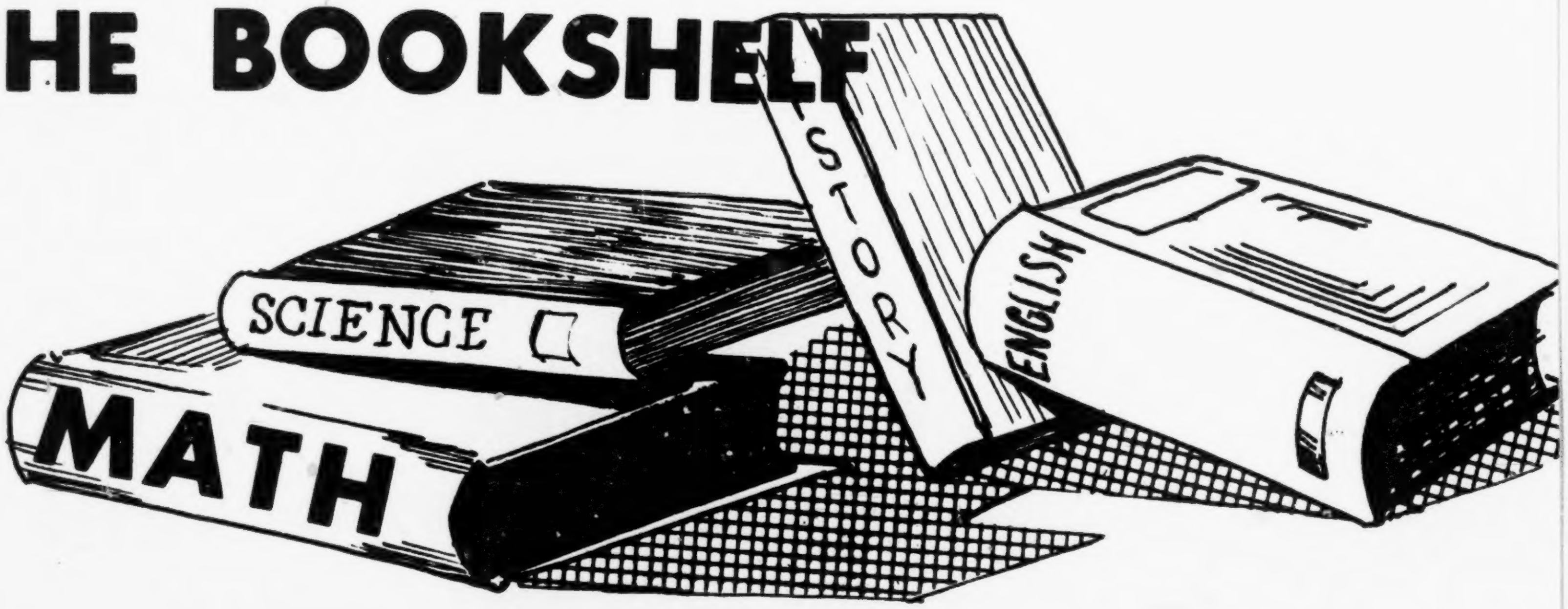
A Pocketful of Money

1. 5
2. 4
3. 3
4. 2
5. 120

Problems to Solve

1. 6
2. 720
3. 120

THE BOOKSHELF



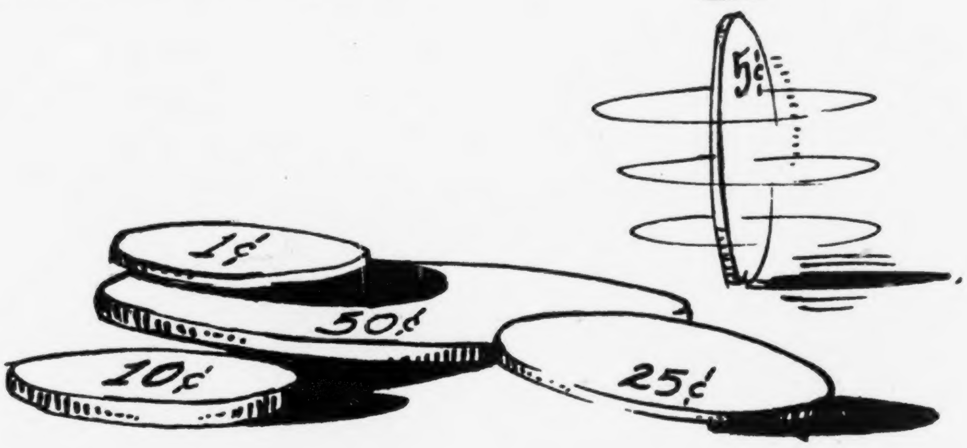
Four books are to be arranged on a shelf.

1. How many choices do you have for the first position? ☐
   (cross out the book you chose)
2. How many choices do you have for the second position? ☐
   (cross out the second choice)
3. How many choices are left for the third choice? ☐
   (cross out the third book)
4. How many are left for the fourth choice? ☐
5. How many different ways can the books be arranged on the shelf?

1st Choice		2nd Choice		3rd Choice		4th Choice		All Ways
☐	x	☐	x	☐	x	☐	=	☐

POCKETFUL OF MONEY

Joe has 5 different coins in his pocket.



In how many different orders can the coins be taken from his pocket, one at a time?

1. How many choices does Joe have for the first coin? ☐
2. He picks a 2nd coin. How many choices? ☐
3. He picks a 3rd coin. How many choices? ☐
4. He picks a 4th coin. How many choices? ☐
5. In how many different orders can Joe take the coins from his pocket?

☐ x ☐ x ☐ x ☐ x ☐ = ☐

PROBLEMS TO SOLVE

1. In how many ways can 3 people occupy the back seat of a taxi?

2. In how many ways can six students line up to buy lunch in the cafeteria?

3. In how many ways can 5 people fill the offices of President, Vice president, Secretary, Treasurer, and Parliamentarian of the Student Council?

LESSON 8

I. Unit: Probability

II. Objectives: The student should be able to:

- A. Name and identify the factorial symbol
- B. Construct products indicated by factorial notation
- C. Describe factorial notation as a short way of indicating products of the first n counting numbers

III. Materials: Work sheet entitled "Factorial"

IV. Procedure:

- A. Band I
  1. Discuss the results of the work sheet entitled "Problems to Solve." Have several students state the rule concerning the product of the first n counting number.
  2. Introduce and discuss the problem about the picture of the basketball team which is found on page 15 in Book 5, Experiences in Mathematical Discovery: Arrangements and Selections, National Council of Teachers of Mathematics.
  3. Introduce the factorial symbol.
  4. Start the following chart with the students and have them complete it.

Factorial Notation	Indicated Product	Product
1!	1	1
2!	2· 1	2
3!	3· 2· 1	6
4!	4· 3· 2· 1	24
5!	5· 4· 3· 2· 1	120
6!	6· 5· 4· 3· 2· 1	720
7!	7· 6· 5· 4· 3· 2· 1	5, 040
8!	8· 7· 6· 5· 4· 3· 2· 1	40, 320
9!	9· 8· 7· 6· 5· 4· 3· 2· 1	362, 880
10!	10· 9· 8· 7· 6· 5· 4· 3· 2· 1	3, 628, 800

5. Explain: Factorial notation is used as a short way of indicating the product of the first n counting numbers.

6. Work the following examples on the board.

   a. $2! + 3! = 8$

   b. $4! - 2! = 22$

   c. $2! \cdot 3! = 12$

   d. $4! \div 2! = 12$

7. Assessment - Objective B - Distribute the work sheet entitled "Factorial." Have students begin work on this sheet.

B. Band II

Students should be given the opportunity to pursue their interests in the games and puzzles as they complete the work sheet.

Solutions to: Factorial Worksheet

1. a. 3
   b. 8
   c. 4
   d. 26
   e. 126

2. a. 1
   b. 18
   c. 2
   d. 0
   e. 22

3. a. 2
   b. 12
   c. 4
   d. 48
   e. 720

4. a. 2
   b. 12
   c. 3
   d. 20
   e. 1

5. a. 1
   b. 2
   c. 6
   d. 24
   e. 120

6. a. 2
   b. 3
   c. 4
   d. 5
   e. 6

7. a. 8
   b. 5!
   c. 3!
   d. 6!
   e. 4!

FACTORIAL

1. a. $1! + 2! =$ ______

   b. $2! + 3! =$ ______

   c. $2! + 2! =$ ______

   d. $4! + 2! =$ ______

   e. $5! + 3! =$ ______

2. a. $2! - 1! =$ ______

   b. $4! - 3! =$ ______

   c. $3! - 2! =$ ______

   d. $5! - 5! =$ ______

   e. $4! - 2! =$ ______

3. a. $1! \cdot 2! =$ ______

   b. $3! \cdot 2! =$ ______

   c. $2! \cdot 2! =$ ______

   d. $4! \cdot 2! =$ ______

   e. $5! \cdot 3! =$ ______

4. a. $2! \div 1! =$ ______

   b. $4! \div 2! =$ ______

   c. $3! \div 2! =$ ______

   d. $5! \div 3! =$ ______

   e. $3! \div 3! =$ ______

5. a. $1 \cdot 1! =$ ______

   b. $2 \cdot 1! =$ ______

   c. $3 \cdot 2! =$ ______

   d. $4 \cdot 3! =$ ______

   e. $5 \cdot 4! =$ ______

6. a. $2! \div 1! =$ ______

   b. $3! \div 2! =$ ______

   c. $4! \div 3! =$ ______

   d. $5! \div 4! =$ ______

   e. $6! \div 5! =$ ______

7. a. $8! =$ ______ $\cdot\ 7!$

   b. $6! = 6 \cdot$ ______!

   c. $5! = 5 \cdot 4 \cdot$ ______

   d. $7! \div$ ______ $= 7$

   e. ______ $\div\ 3! = 4$

LESSON 9

I. Unit: Probability

II. Materials:

Science Research Associates Kit

III. Procedure:

A. Divide the class into three groups according to how well they were able to perform on the survey test on addition and subtraction of whole numbers. Explain to the class that you will work with the group which had the most difficulty. The middle group will use the SRA computational skills development kit and the third group, which did best on the tests, will solve cross number puzzles found in the SRA kit of Cross Number Puzzles.

B. Prior to this time the teacher should acquaint himself with the kits. The teacher's guide offers many helpful suggestions.

C. Inform students that the purpose of the kits is for them to locate skills on which they need practice. Encourage them to take their time and work carefully. Explain that if they have difficulty with the diagnostic tests it will not affect their grade so long as they work hard. If this tone is established at the outset, you should find these kits successful.

LESSON 10

I. Unit: Probability

II. Objectives: The student should be able to:

Apply the principle of factorial notation to solve problems concerned with arrangements of n things taken n at a time

III. Materials:

A. Flash cards on addition and subtraction of whole numbers

B. Work sheet entitled, "Arrangements"

C. SRA Computational Skills Kit

IV. Procedure:

A. Band I

Use the flash cards as an oral exercise for reviewing the basic addition and subtraction facts. Each student should have the opportunity to answer at least one problem.

B. Band II

1. Have the following problem written on the board:

   Which of the following is the factorial symbol?

   (a) X (b) ! (c) ? (d) $\sqrt{\quad}$

2. Have students write the answer on their papers. Discuss the correct answer and have students describe the meaning of the factorial symbol using examples. This will assess objectives A and C of Lesson 8.

3. Discuss the results of the work sheet entitled, "Factorial." Point out the following patterns:

   a. $n \cdot (n - 1)! = n!$

   b. $n! \div (n - 1)! = n$

4. Have the following two arrangement problems written on the board.

   a. How many ways can seven people line up to buy tickets to a baseball game?

   b. In how many ways can a baseball manager arrange the batting order of nine players?

5. Discuss the solution of these problems using a box diagram. Show how the factorial symbol can be used to express the solutions.

6. Assessment - Distribute work sheet entitled, "Arrangements." Have the students complete these as a home assignment.

C. Band III

1. The slower students should be introduced to the SRA Kit under the direction of the teacher.

2. The middle group should continue working with the SRA Kit individually.

3. The group which had no problems working with addition and subtraction of whole numbers should have the option of working on SRA cross number puzzles, games, or puzzles.

Solution - "Arrangements"

1. 120
2. 85 !
3. 37 !
4. n !

ARRANGEMENTS

1. How many ways can the five players on a basketball team be arranged?
2. In how many ways can a freight train of 85 cars be coupled together?
3. How many ways can 37 floates be arranged in a parade?
4. How many ways can the people in this room be seated?

LESSON 11

I. Unit: Probability

II. Objectives: The student should be able to:

- A. Construct the number arrangements of n things taken r at a time
- B. Apply the Fundamental Principle of Counting by solving related problems

III. Materials:

- A. Work sheet entitled, "Related Problems"
- B. SRA Kit

IV. Procedure:

- A. Band I
    - 1. Use the Math Builder and filmstrip 14, Multiplication, Multipliers from 6 through 9, from the set AR-FX.
    - 2. Preview the filmstrip.
    - 3. Give the first 10 to 15 problems orally in Part A. Use the left-to-right scanning technique. Start at a rather low speed.
    - 4. Change from the left-to-right scanning to the full line frame and mask out the answers. Complete Part A.
    - 5. If students make errors use the pause button to give the students a longer look at the problem.
    - 6. Start at a low speed and gradually increase it. You will probably want the speed below 90 when the fastest speed is obtained.
- B. Band II
    - 1. Introduce arrangements of n things taken r at a time by discussing the problem about Coach Johnson which is found on page 16 in Book 5, Experiences in Mathematical Discovery: Arrangements and Selections, National Council of Teachers of Mathematics. Discuss the related problems at the top of page 17 placing emphasis on the Fundamental Principle of Counting.
    - 2. Distribute work sheet entitled, "Related Problems" and permit students to work at their own rate.

C. Band III

The students should have the opportunity to work on the SRA kits or cross number puzzles as they finish the work sheet from Band II.

D. Solutions: Related Problems

1. 210
2. 336
3. 1680
4. 30

RELATED PROBLEMS

1. A stock broker receives seven orders to sell stock at the same time. In how many ways can he select the first three orders to fill?
2. Eight customers came into a store at the same time. In how many ways can the clerk choose the first 3 to wait on?
3. Four persons enter a bus in which there are eight vacant seats. In how many different ways can they pick their seats?
4. Six persons have been nominated for 2 offices, president and vice president. In how many ways might 2 persons be elected?

LESSON 12

I. Unit: Probability

II. Materials: Work sheets entitled, "How Many License Tags" and "Let's Review"

III. Procedure:

A. Band I

This should consist of oral exercises dictated by the teacher. The problems should be the basic multiplication facts. Each student should have the opportunity to answer at least one problem.

B. Band II

1. Use a summary similar to the one found on pages 18-19 in Book 5, Experiences in Mathematical Discovery: Arrangements and Selections, National Council of Teachers of Mathematics for distinguishing between different types of arrangement problems.
2. Distribute work sheets entitled, "How Many License Tags" and "Let's Review." Permit students to work at their own rate.

C. Band III

As students complete the work of Band II they should have the opportunity to work on their selection of games and puzzles.

D. Solutions to: How Many License Tags

1. 26
2. 26
3. 10
4. 10
5. 10
6. 10
7. 26 x 26 x 10 x 10 x 10 x 10 = 6, 760, 000

Let's Review

1. 36
2. 6
3. 120
4. 17, 576, 000
5. 120
6. 12
7. 56
8. 26
9. 114

HOW MANY LICENSE TAGS

Each state in the U. S. A. sells license tags to car owners. The state must make enough tags with different numbers so that each car has different tags. Suppose a state used tags with only 3 numbers. Then only 1000 different tags could be made. If there were more than 1000 cars registered in the state, then there would not be enough different tags.

There is a method of finding out how many different tags could be made. In Maryland the tags have two letters and four numerals. These may be used more than once in a tag. For example: AA 4464. This is the method of finding out how many cars tags are different. (The first numeral may be zero.)

1. How many choices are there for the first letter?

2. How many choices are there for the second letter?

3. How many choices are there for the first numeral?

4. How many choices are there for the second numeral?

5. How many choices are there for the third numeral?

6. How many choices are there for the fourth numeral?

7. What is the total number of different tags which can be made in Maryland?

LET'S REVIEW

EXERCISES

1. How many different meals can be made of: a salad, a meat, a vegetable, and a dessert? The choices are listed below.

Salad	Meat	Vegetables	Dessert
tossed green	chicken	potato	jello
lettuce and tomato	meat loaf	beets	cake
		corn	pie

2. To get to Hawaii Jim took a train to San Francisco. He had a choice of 3 trains. From San Francisco he took a ship to Hawaii. He had a choice of 2 ships. How many different ways could he travel from San Francisco to Hawaii?

3. Sally arranged 5 dolls on a shelf. How many different ways could she arrange the dolls?

4. Maine has 3 letters and 3 numerals in its license tags. How many different tags could be made?

5. A chorus concert consists of 6 selections. The first part of the concert is composed of 3 of these selections. In how many ways can the first part of the concert be planned?

6. $2! \cdot 3! =$

7. $8! \div 6! =$

8. $4! + 2! =$

9. $5! - 3! =$

RESOURCE MATERIALS

A. Books

Adler, Irving. The Giant Golden Book of Mathematics. New York: Golden Press. 1958

Bergamini, David. Mathematics, Life Science Library. New York, N. Y.: Time, Inc. 1963

Heddens, James M. Today's Mathematics. Chicago: Science Research Associates, Inc. 1963

Highland, H. J. The How and Why Wonder Book of Mathematics. New York: Wonder Books. 1963

Hughes, Toni. How to Make Shapes in Space. New York: E. P. Dutton and Co., Inc. 1955

Johnson, Pauline. Creating With Paper. Washington: University of Washington Press. 1958

Morris, Dennis and Topfer, Henry. Advancing in Mathematics, Grade 7. Chicago: Science Research Associates, Inc. 1963

Morris, Dennis and Topfer, Henry. Advancing in Mathematics, Grade 8. Chicago: Science Research Associates, Inc. 1963

Northrop, Eugene P. Riddles in Mathematics. New York: D. Van Nostrand Co., Inc. 1944

Wirtz, Robert; Botel, Morton and Nunley, B. G. Discovery in Elementary School Mathematics. Chicago: Encyclopaedia Britannica Press, Inc. 1963

Young, Mary. Singing Windows. New York: Abingdon Press. 1962

B. Pamphlets and Periodicals

Amir-Mo-Az. Ruler, Compasses and Fun. New York: Ginn and Co. 1966

Bazdon, Jack and Murtin, Mark. Cross Number Puzzle Boxes. Chicago: Science Research Associates, Inc. 1966

Criflinski, Henry. Modern Mathematics, Ditto Workbooks. Washington, D. C.: Hayes School Publishing Co. 1964

Herrick, Marian C. *Modern Mathematics for Achievement.* New York: Houghton Mifflin Co. 1966

Johnson, Donovan. *Paper Folding for the Mathematics Class.* Washington, D. C.: National Council of Teachers of Mathematics. 1957

Johnson, Donovan A. and Glenn, William H. *Topology: The Rubber-Sheet Geometry.* Atlanta: Webster Publishing Co. 1960

Larson, Harold. *Enrichment Program for Arithmetic Grades 3-8:* Elmsford, New York: Harper and Row Publishers. 1963

Murray, William D. and Rigney, Francis. *Paper Folding for Beginners.* New York: Dover Publications, Inc. 1960

Potter, Mary and Mallory, Virgil. *Education in Mathematics for the Slow Learner.* Washington, D. C.: National Council of Teachers of Mathematics. 1958

Proctor, Charles and Johnson, Patricia. *Computational Developmental Skills Kit.* Chicago: Science Research Associates, Inc. 1965

School Mathematics Study Group. *Conference on Mathematics Education for Below Average Achiever.* Pasadena, California: Vroman Co. 1964

Topics in Mathematics for Elementary School Teachers. Washington, D. C.: National Council of Teachers of Mathematics. 1964

Wirtz, Robert and Botel, Morton. *Math Workshop, Levels A-F.* Chicago: Encyclopaedia Britannica Press, Inc. 1961

Woodby, Lauren. *The Low Achiever in Mathematics.* Washington, D. C.: U. S. Office of Education. 1964

C. Games

Milton Bradley Co., Springfield, Mass.
"Primary Peg Board #474X"
Pegs #472X or #475X

Edmund Scientific, Barrington, New Jersey 08007
"Dr. Nim" ($2.98)
"Probability Kit" ($4.00)
"Soma" ($2.00)

Ideal Supply Co., 11315 Watertown Plank Road, Milwaukee, Wis. 53201
"Geometric Wire Forms and Patterns #794" ($3.00)

Kohner Bros., Inc., 155 Wooster Street, New York, N. Y. 10012
"Euclid" ($1.00)
"Hexed" ($1.00)
"Hi-Q" ($1.00)
"Kwazy Quilt" ($1.00)
"Pythagoras" ($1.00)
"Tormentor" ($1.00)
"Voodoo" ($1.00)

Krypto Corporation, 2 Pine Street, San Francisco, California 94111
"Krypto" ($3.95)

Parker Bros., Inc., P. O. Box 900, Salem, Mass.
"Take Twelve" ($3.00)

Science Research Associates, 259 East Erie Street, Chicago, Illinois 60611
"Equations" ($3.00)
"Cross Number Puzzles" ($22.75)

D. Films

Baltimore County Central Film Library

Probability, McGraw Hill Book Co.

Mean, Median and Mode, McGraw Hill Book Co.

NUMBERS, OPERATIONS AND ALGORITHMS

NUMBERS, OPERATIONS AND ALGORITHMS

I. Master Chart - Grades Six through Eleven

II. Grade Nine Chart and Behavioral Objectives
   - A. Whole Numbers
   - B. Fractional Numbers - Preliminary Topics
   - C. Multiplication of Fractional Numbers
   - D. Division of Fractional Numbers
   - E. Addition of Fractional Numbers
   - F. Subtraction of Fractional Numbers
   - G. Decimal Numerals
   - H. Percent by Ratio and Proportion
   - I. Square Root

III. Activities

UNIT **WHOLE NUMBERS** GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Numerals to Millions	6	6	6						6	6
Numerals to Billions	7	7	7						7	7
Rounding to Millions	6	6	6				6			6
Rounding to Billions	7	7	7				7			7
Expanded Notation to Millions	6	6	6			8				
Expanded Notation to Billions	7	7	7			8	7			
Renaming Numbers	6	6	6			8	6			
Vocabulary	6	6								6
Place Value to Millions	6	6								
Place Value to Billions	7	7								
Denominate Numbers		6	6		6	7	6			
Verbal Problems			6, 7, 8	6, 7, 8				6, 7, 8		
Betweenness			6, 7		6					
Symbol(s)	6, 7, 8	6, 7, 8		6, 7, 8						
Number Sentences	6	6	6		6		6			
Vertical Form Addition			6, 7, 8	6, 7, 8						
Using Number Line	6		6	6	6		6			
Basic Facts	6	6	6							

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UNIT WHOLE NUMBERS GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Number Patterns			6	6	6		6			
Estimation			7	7	7		7			
Casting Out Nines	8		8		8		8			
Closure	8				6	9				
Commutative Property	8	6	6		6	9	6			8
Associative Property	8	6	6		6	9	6			8
Identity Element	9	6	6		6	7	6			
Inverse Operations		7	6		7	10	6			
Vertical Form Subtraction			6, 7, 8	6, 7, 8						
Checking			6		6					
Role of Zero			6		6	7	6			
Order of Operations			6, 8		6, 8	10	6, 8			
Vertical Form Multiplication			6, 7, 8	6, 7, 8						
Factoring		6	6							
Divisors	6	6	6							
Rules for Divisibility						6, 7, 8	6, 7, 8			
Prime Numbers	7	7	7							7
Composite Numbers	7	7	7		7					

UNIT WHOLE NUMBERS GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Prime Factorization	7	7	7	7	7					7
Multiples	7	7		7	7					8
Power	7	7	7	7	7				8	
Base	7	7	7		7					
Exponent	7	7	7		7					
Distributive Property	9	9	9			10	6			
Division			6, 7, 8	6, 7, 8						
Remainders	6	6			6					
Role of One			6		6	7	6			

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UNIT PRELIMINARY TOPICS OF FRACTIONAL NUMBERS

GRADE(S) Six through Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning of Fractions	6	6	6		6	8				
Numerator and Denominator	6	6			6					
Symbols (Fraction Bar)	6	6			6	6				
Number Line	6, 7, 8	6, 7, 8	6, 7, 8	7, 8	8					
Comparing Fractions			6	6					6	
Divisibility Rules		6, 7, 8	6, 7, 8			6, 7, 8	6, 7, 8, 9			6, 7, 8
Fractional Names for One	6	6	6	6	6	8	7			
Greatest Common Factor	7, 8, 9	7, 8, 9	7, 8, 9	7, 8	7, 8, 9					7
Simplifying Fractions	6, 7, 8	6, 7, 8	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8			
Renaming Fractions in Higher Terms	6, 7, 8	6, 7, 8	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8			
Mixed Form	6, 7	6, 7	6, 7	6, 7	6, 7					
Rename Mixed Form as Fractions			6, 7, 8	6, 7, 8	6, 7, 8	8	6, 7, 8			
Rename Fractions as Mixed Form			6, 7, 8	6, 7, 8	6, 7, 8	8	6, 7, 8			
Equivalent Fractions	6, 7	6, 7	6, 7	6, 7		6, 7	6, 7, 8			6, 7
Nonequivalent Fractions		6, 7, 8	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8		6, 7, 8	

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Nine

Multiplication of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Expressing a Whole Number as a Fraction	6	6	6		6	6	6			
Fraction Times Whole Number, Meaning	6, 7	6, 7	6	7	7					
Fraction Times Whole Number	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Fraction Times Fraction, Meaning	6, 7	6, 7	6, 7	7						
Fraction Times Fraction	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Mixed Form Times Whole Number	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Mixed Form Times Fraction	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Mixed Form Times Mixed Form	6	6	6, 7, 8	6, 7, 8	7, 8	8	6, 7, 8, 9			
Closure Property					7	8, 9				
Commutative Property	7	6, 7	7	6, 7, 8, 9		7, 8				
Associative Property	8	8	8	8		8, 9	9			8, 9
Distributive Property	9	9	9			9	9			9
Identity Element	6	6	6	6	6	6	6			
Estimation	6	6	6	6, 7, 8, 9						
Translating Verbal Problems into Number Sentences	6, 7 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9			6, 7, 8, 9		

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Nine

Division of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning	6	6	6, 7	6, 7	7					
Reciprocals	7	7	7, 8	8	8		7			
Complex Fractions	7	7	7, 8, 9	7, 8, 9	8, 9	9	7, 8, 9			
Fraction Divided by a Whole Number	7	7	7, 8, 9	7, 8, 9	8, 9	9	7, 8, 9			
Whole Number Divided by a Fraction	7	7	7, 8, 9	7, 8, 9	8, 9		7, 8, 9			
Fraction Divided by a Fraction	7	7	7, 8, 9	7, 8, 9	8, 9	9	7, 8, 9			
Mixed Form Divided by a Whole Number	8	8	8, 9	8, 9	9		8, 9			
Whole Number Divided by a Mixed Form	8	8	8, 9	8, 9	9		8, 9			
Mixed Form Divided by a Fraction	8	8	8, 9	8, 9	9		8, 9			
Fraction Divided by a Mixed Form	8	8	8, 9	8, 9	9		8, 9			
Mixed Form Divided by a Mixed Form	9	9	9	9	9	9	9			
Closure Property					8	8				
Non-Commutativity		7, 8	7, 8	8, 9						
Non-Associativity	9	9	9							
Identity Element	8	8		8						
Inverse Operation	8	8	8	8	9		9			
Translating Verbal Problems into Number Sentences	7, 8, 9	7, 8, 9	7, 8, 9	7, 8, 9	7, 8, 9			7, 8, 9		

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Nine

Addition of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning of Addition of Fractions	6	6	6	6	7					
Addition of Fractions, Like Denominators	6	6	6	6		6	6,7			
Least Common Multiple	6	6	6,7,8,9	6,7,8,9	7	6,7,8,9	6,7,8,9			6,7,8,9
Addition of Fractions, Unlike Denominators	7,8	7,8	7,8	7,8,9			7,8,9			
Whole Number Plus Fraction	6	6	6	6	6,7		6,7			
Whole Number Plus Mixed Form	6	6	6	6	6,7		6,7			
Fraction Plus Mixed Form, Like Denominators	6	6	6	6	6,7	7	6,7			
Mixed Form Plus Fraction, Unlike Denominators	7	7	7	7			7			
Mixed Form Plus Mixed Form, Like Denominators	6	6	6,7	6,7	6,7	7	6,7			
Mixed Form Plus Mixed Form, Unlike Denominators	7	7	7	7	7		7			
Miscellaneous Problems of Adding Fraction Expressions			8	8,9		9	8,9			
Closure Property	7	7				6,7,8,9				
Commutative Property	7	6,7	6,7	6,8,9		7,8	6,7,8			8,9
Associative Property	8	8	8	8,9		8	8,9			8,9
Identity Element	6	6	6	6	6	6	6			
Estimation	6	6	6	6,7,8,9	6		6,8,9			
Translating Verbal Problems into Number Sentences	6,7 8,9	6,7,8,9	6,7,8,9	6,7,8,9	6,7,8,9			6,7,8,9		

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Ten

Subtraction of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Relationship of Addition and Subtraction, Meaning	6	6	6	6	6					
Subtraction, Like Denominators	6	6	6	6	6	7	6, 7			
Subtraction, Unlike Denominators	7	7	7	7, 8	7	8	7, 8			
Fractions from Mixed Form, Like Denom., No Renaming	6	6	6	6, 7	6	7	6, 7			
Fractions from Whole Numbers	6	6	6	6, 7	6	7	6, 7			
Renaming Whole Numbers as Mixed Forms	6	6	6	6	6	7	6, 7			
Fractions from Mixed Forms, Like Denom., Renaming	6	6	6	6, 7, 8	7	8	6, 7, 8			
Fractions from Mixed Forms, Unlike Denom., No Renaming	7	7	7	7	7	8	7, 8			
Fractions from Mixed Forms, Unlike Denom., Renaming	7	7	7	7	7	8	7, 8			7, 8
Whole Number from Mixed Forms	6	6	6	6	6	7	6, 7			
Mixed Forms from Mixed Forms, No Renaming	6	6	6	6, 7	6	7	6, 7			
Mixed Form from Whole Number	6	6	6	6	6	7	6, 7			
Mixed Forms from Mixed Form, Like Denom., Renaming	6	6	6	6, 7, 8	7	8	6, 7, 8			6, 7, 8
Mixed Form from Mixed Form, Unlike Denom., No Renaming	7	7	7	7, 8	7	8	7, 8			
Mixed Form from Mixed Form, Unlike Denom., Renaming	7	7	7	7, 8	7	8	7, 8			7, 8
Closure Property		6	7	6, 7, 8		7, 8				
Non-Commutativity	7	6	7, 8							
Non-Associativity	7	6		6, 7, 8						

UNIT FRACTIONAL NUMBERS GRADE(S) Six through Ten

Subtraction of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Identity Element		6	6		6	6, 7, 8	6			
Inverse Operation	8	8		8	9	10	9			
Translating Verbal Problem to Number Sentences	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9					

UNIT DECIMAL NUMERALS GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Place Value	6, 7, 8	6, 7, 8				6, 7, 8				6, 7, 8
Numbers to a Million	6	6	6						6	
Numbers to a Billion	7, 8	7, 8	7, 8						7, 8	
Number Patterns				6, 7, 8		6, 7, 8	6, 7, 8			
Equivalent Powers of Ten						6, 7, 8				
Expanded Notation			6, 7, 8							
Decimal Equivalence	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8						
Annexing Zeros	6, 7, 8, 9	6, 7, 8, 9	6, 7, 8, 9							
Number Line	6, 7	6, 7	6, 7			6, 7			6, 7	
Comparing Numbers			6, 7, 8						6, 7, 8	
Betweenness	6, 7, 8, 9		6, 7, 8, 9							
Rounded Numbers	6, 7, 8, 9	6, 7, 8, 9					6, 7, 8, 9			
Decimal Simplifying Numerals	6, 7, 8	6, 7, 8		6, 7, 8						
Equivalence Decimal Charts			8	8						
Column Form Addition			6, 7, 8, 9	6, 7, 8, 9						
Arrange Addends in Column Form			6, 7, 8, 9			6, 7, 8, 9				
Estimating Sums			8	8	8					
Subtraction			6, 7, 8, 9	6, 7, 8, 9						

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UNIT DECIMAL NUMERALS GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Division of a Decimal by a Decimal			10	10						
Quotients Rounded			10	10	10					
Decimal Equivalents	10	10	10	10						

FO-12

UNIT **PERCENT BY RATIO AND PROPORTION** GRADE(S) **Eight through Ten**

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Meaning of Ratio	8	8	8	8	8					
Meaning of Rate Pair	8	8	8	8	8					
Simplifying a Rate Pair	8	8	8			8	8			
Translating Verbal Problems into Rate Pairs		8	8	8	8					
Meaning of Percent by Rate Pairs		8	8	8	8					
Changing Percents in Verbal Problems to Rate Pairs		8		8						
Equivalent Rate Pairs						9	9			
Proportion	9	9	9	9						
Solving Multiplication Equation	9	9	9			9	9, 10			
Translating Verbal Problems into Equivalent Rate Pairs		9	9	9	9					
Solving Proportions		9	9			9	9, 10			
Finding What Percent One Number is of Another		9	9				9, 10			
Translating Verbal Problems into Equivalent Rate Pairs		9	9	9	9					
Finding a Percent of a Number		10	10			10	10			10
Finding a Number When a Percent of it is Known		10	10			10	10			10
Translating Verbal Problems into Equal Rate Pairs		10	10	10	10					10

UNIT SQUARE ROOT GRADE(S) Seven through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Square Numbers	7	7	7	7	7					7
Vocabulary	7, 8	7, 8								8
Symbol	8	8		8						8
Expansion of Powers	8	8	8	8	8					8
Use of Square Root Table	9	9	9	9	9					
Extracting Square Roots			10, 11	10, 11	10, 11		10, 11			

UNIT COMPUTING DEVICES GRADE(S) Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Vocabulary	10	10								10
Calculator			10	10	10					
Addition, Subtraction, Multiplication, Division			10	10						
Accuracy			10		10					
Combined Operations			10	10				10		

UNIT SLIDE RULE GRADE(S) Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Vocabulary	11	11								11
Construction of Slide Rules			11	11	11					
Reading Scales			11		11					
Multiplication			11	11						
Division			11	11						
Combined Operations			11	11	11			11		
Estimating Solutions			11	11			11			
Square Root			11	11						
Squaring			11	11						

UNIT WHOLE NUMBERS GRADE(S) Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Closure						9				
Commutative Property						9				
Associative Property						9				
Identity Element	9									
Distributive Property	9	9	9							

NUMBERS, OPERATIONS AND ALGORITHMS
WHOLE NUMBERS - Grade 9

Note: In the following list of objectives, we shall agree to use phrases such as "a three digit number" to mean "a number named by a numeral containing three digit symbols."

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Properties of Addition of Whole Numbers

The student should be able to:

1. Name the additive identity FO-56
2. State the principles of closure, commutativity, and associativity in addition

Properties of Subtraction of Whole Numbers

The student should be able to:

1. State the principles of non-closure, non-commutativity, and non-associativity of subtraction

Properties of Multiplication of Whole Numbers

The student should be able to:

1. State the principles of closure, commutativity, and associativity in multiplication FO-58
2. Name and identify the distributive principle FO-62
3. Demonstrate the distributive principle using parentheses

Properties of Division of Whole Numbers

The student should be able to:

1. State the principles of non-closure, non-commutativity, and non-associativity in division FO-70

UNIT FRACTIONAL NUMBERS GRADE(S) Nine

Preliminary Topics

TOPIC	NAME	IDENTIFY	DEMONSTRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTINGUISHING
Divisibility Rules							9			
Greatest Common Factor	9	9	9		9					

NUMBERS, OPERATIONS AND ALGORITHMS
FRACTIONAL NUMBERS - Grade 9

Preliminary Topics

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Divisibility Rules

The student should be able to:

1. Apply the principle of divisibility rules to solve related problems

Greatest Common Factor

The student should be able to:

1. Name and identify the greatest common factor for a pair of numbers
2. Demonstrate how to find the greatest common factor using Euclid's Algorithm
3. Describe Euclid's Algorithm for finding the greatest common factor of two numbers

UNIT FRACTIONAL NUMBERS GRADE(S) Nine

Multiplication of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Fraction Times Whole Number							9			
Fraction Times Fraction							9			
Mixed Form Times Whole Number							9			
Mixed Form Times Fraction							9			
Mixed Form Times Mixed Form							9			
Closure Property						9				
Commutative Property				9						
Associative Property						9	9			9
Distributive Property	9	9	9			9	9			9
Estimation				9						
Translating Verbal Problems into Number Sentences	9	9	9	9	9			9		

NUMBERS, OPERATIONS AND ALGORITHMS
FRACTIONAL NUMBERS - Grade 9

Multiplication of Fractions

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The Product of a Fraction and a Whole Number

The student should be able to:

1. Apply the following principles for multiplying a fraction by a whole number:
    - a. Rename the whole number as a fraction
    - b. Use the "short-cut" method before multiplying or multiply the numerators and denominators, expressing the answer in simplest terms

The Product of a Fraction Times a Fraction

The student should be able to:

1. Apply the following principles for multiplying a fraction by a fraction:
    - a. Use the "short-cut" method before multiplying, or multiply the numerators and denominators, expressing the answer in simplest terms

The Product of a Mixed Form and a Whole Number

The student should be able to:

1. Apply the following principles for multiplying a mixed form by a whole number:
    - a. Rename the mixed form as a fraction
    - b. Rename the whole number as a fraction
    - c. Use the "short-cut" method before multiplying or multiply the numerators and denominators, expressing the answer in simplest terms

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The Product of a Mixed Form and a Fraction

The student should be able to:

1. Apply the following principles for multiplying a mixed form by a fraction:
    a. Rename the mixed form as a fraction
    b. Use the "short-cut" method before multiplying or multiply the numerators and denominators, expressing the answer in simplest terms

The Product of a Mixed Form and a Mixed Form

The student should be able to:

1. Apply the following principles for multiplying a mixed form by a mixed form
    a. Rename the mixed forms a fraction
    b. Use the "short-cut" method before multiplying or multiply the numerators and denominators, expressing the answer in simplest terms

Closure Property under Multiplication

The student should be able to:

1. State the principle of closure for multiplication of fractions

Commutative Property

The student should be able to:

1. Construct a number sentence to illustrate the commutative property of multiplication of fractions

Associative Property

The student should be able to:

1. State the principle of associativity for multiplication of fractions
2. Apply the principle of associativity for multiplication of fractions to solve related problems
3. Distinguish between the associative and commutative properties for multiplication of fractions

Page

Distributive Property

(Examples should be limited to mixed forms times whole numbers and mixed forms times fractions.)

The student should be able to:

1. Name and identify number sentences which illustrate the distributive property of multiplication of fractions
2. State the principle of distributivity for multiplication of fractions
3. Apply the principle of distributivity for multiplication of fractions to solve related problems
4. Distinguish among the distributive, associative, and commutative properties as they apply to multiplication of fractions

Translating Verbal Problems into Number Sentences

The student should be able to:

1. Name and identify verbal problems whose solutions require multiplication of fractions
2. Demonstrate a procedure for translating verbal problems into number sentences
3. Construct a number sentence from the information contained in a verbal problem
4. Describe the steps needed to translate verbal problems into number sentences, using specific examples
5. Interpret the information contained in a verbal problem in order to construct a corresponding number sentence

UNIT FRACTIONAL NUMBERS GRADE(S) Nine

Division of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Complex Fractions			9	9	9	9	9			
Fraction Divided by a Whole Number			9	9	9	9	9			
Whole Number Divided by a Fraction			9	9	9		9			
Fraction Divided by a Fraction			9	9	9	9	9			
Mixed Form Divided by a Whole Number			9	9	9		9			
Whole Number Divided by a Mixed Form			9	9	9		9			
Mixed Form Divided by a Fraction			9	9	9		9			
Fraction Divided by a Mixed Form			9	9	9		9			
Mixed Form Divided by a Mixed Form	9	9	9	9	9	9	9			
Non-Commutativity				9						
Non-Associativity	9	9	9							
Inverse Operation					9		9			
Translating Verbal Problems into Number Sentences	9	9	9	9	9			9		

NUMBERS, OPERATIONS AND ALGORITHMS
FRACTIONAL NUMBERS - Grade 9

Division of Fractions

Page

Complex Fractions

(Listed below are some representative examples)

$$\frac{\frac{1}{2}}{6},\quad \frac{9}{\frac{1}{3}},\quad \frac{\frac{1}{3}}{\frac{1}{4}},\quad \frac{3\frac{1}{2}}{6},\quad \frac{8}{2\frac{1}{2}},\quad \frac{4\frac{2}{3}}{\frac{1}{3}},\quad \frac{\frac{2}{3}}{6\frac{3}{4}},\quad \frac{8\frac{1}{2}}{10\frac{2}{3}}$$

The student should be able to:

1. Demonstrate how to write a complex fraction given a division problem of the form $3\frac{1}{2} \div 6\frac{2}{3}$, etc.
2. Construct a complex fraction given a division problem of the form $5 \div \frac{1}{5}$, etc.
3. Describe how to rename a division problem as a complex fraction by using specific examples
4. State the principle of renaming a division problem as a complex fraction
5. Apply the principle to rename a division example as a complex fraction to solve related problems

Fraction Divided by a Whole Number

(Some representative examples will be found in Chart I below)

The student should be able to:

1. Demonstrate how to construct the quotient of a fraction and a whole number
2. Construct the quotient for a fraction and a whole number
3. Describe how to divide a fraction by a whole number by using specific examples
4. State the principle of dividing a fraction by a whole number by definition of division of fractions

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5. Apply the following principles for dividing a fraction by a whole number:
    a. Rename the division problem as a multiplication problem
    b. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answer in simplest terms

CHART I

Fraction Divided by a Whole Number - Grade 9

$\frac{1}{2} \div 8$

$\frac{3}{5} \div 8$

$\frac{7}{8} \div 7$

$\frac{4}{5} \div 2$

$\frac{2}{3} \div 4$

$\frac{9}{10} \div 6$

$\frac{6}{10} \div 9$

$\frac{1}{3} \div 12$

$\frac{4}{5} \div 21$

$\frac{9}{15} \div 9$

$\frac{18}{21} \div 2$

$\frac{6}{10} \div 12$

$\frac{12}{15} \div 8$

$\frac{8}{15} \div 12$

$\frac{1}{14} \div 18$

$\frac{19}{25} \div 32$

$\frac{18}{19} \div 18$

$\frac{50}{64} \div 25$

$\frac{12}{25} \div 24$

$\frac{21}{25} \div 18$

$\frac{14}{18} \div 21$

Whole Number Divided by a Fraction

(Some representative examples will be found in Chart II)

The student should be able to:

1. Demonstrate how to construct the quotient of a whole number and a fraction
2. Construct the quotient for a whole number and a fraction
3. Describe how to divide a whole number by a fraction by using specific examples

Page

4. Apply the following principles for dividing a whole number by a fraction:
    a. Rename the division problem as a multiplication problem
    b. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answer in simplest terms

CHART II

Whole Number Divided by a Fraction - Grade 9

$6 \div \frac{1}{3}$

$7 \div \frac{2}{3}$

$4 \div \frac{4}{5}$

$8 \div \frac{4}{5}$

$2 \div \frac{4}{5}$

$2 \div \frac{4}{6}$

$6 \div \frac{9}{10}$

$9 \div \frac{6}{10}$

$16 \div \frac{1}{3}$

$17 \div \frac{2}{3}$

$4 \div \frac{4}{15}$

$10 \div \frac{5}{6}$

$6 \div \frac{12}{13}$

$2 \div \frac{14}{16}$

$8 \div \frac{12}{15}$

$12 \div \frac{8}{15}$

$16 \div \frac{1}{13}$

$17 \div \frac{12}{13}$

$14 \div \frac{14}{15}$

$20 \div \frac{10}{16}$

$10 \div \frac{20}{21}$

$12 \div \frac{24}{36}$

$12 \div \frac{16}{19}$

$16 \div \frac{12}{19}$

Fraction Divided by a Fraction

(Some representative examples will be found in Chart III)

The student should be able to:

1. Demonstrate how to construct the quotient of two fractions
2. Construct the quotient for pairs of two fractions

Page

3. Describe how to divide pairs of fractions by using specific examples
4. State the principle of dividing pairs of fractions by definition of division of fractions
5. Apply the following principles for dividing pairs of fractions:
    a. Rename a multiplication problem as a division problem
    b. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answer in simplest terms

CHART III

Fraction Divided by a Fraction - Grade 9

$\frac{1}{4} \div \frac{1}{3}$
$\frac{3}{4} \div \frac{1}{3}$
$\frac{3}{4} \div \frac{2}{3}$
$\frac{1}{4} \div \frac{3}{4}$
$\frac{5}{8} \div \frac{5}{8}$
$\frac{4}{5} \div \frac{4}{7}$
$\frac{5}{6} \div \frac{5}{12}$
$\frac{4}{7} \div \frac{2}{7}$
$\frac{2}{5} \div \frac{3}{10}$
$\frac{3}{4} \div \frac{1}{2}$
$\frac{2}{3} \div \frac{4}{9}$
$\frac{6}{8} \div \frac{9}{12}$

$\frac{1}{4} \div \frac{1}{13}$
$\frac{3}{14} \div \frac{1}{3}$
$\frac{3}{14} \div \frac{2}{3}$
$\frac{1}{14} \div \frac{3}{14}$
$\frac{5}{18} \div \frac{5}{18}$
$\frac{4}{15} \div \frac{4}{17}$
$\frac{5}{12} \div \frac{5}{24}$
$\frac{4}{17} \div \frac{2}{17}$
$\frac{3}{10} \div \frac{16}{20}$
$\frac{3}{20} \div \frac{4}{10}$
$\frac{2}{3} \div \frac{14}{27}$
$\frac{6}{8} \div \frac{12}{18}$

$\frac{1}{14} \div \frac{1}{13}$
$\frac{13}{14} \div \frac{1}{13}$
$\frac{13}{14} \div \frac{2}{13}$
$\frac{11}{14} \div \frac{13}{14}$
$\frac{17}{18} \div \frac{17}{18}$
$\frac{14}{15} \div \frac{14}{17}$
$\frac{11}{12} \div \frac{11}{24}$
$\frac{14}{17} \div \frac{7}{17}$
$\frac{8}{10} \div \frac{16}{20}$
$\frac{19}{20} \div \frac{8}{10}$
$\frac{12}{15} \div \frac{24}{30}$
$\frac{16}{18} \div \frac{32}{45}$

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Mixed Form Divided by a Whole Number

(Some representative examples will be found in Chart IV below)

The student should be able to:

1. Demonstrate how to construct the quotient of a mixed form and a whole number
2. Construct the quotient of a mixed form and a whole number
3. Describe how to divide a mixed form by a whole number by using specific examples
4. Apply the following principles for dividing a mixed form by a whole number:
   a. Rename the mixed form as a fraction
   b. Rename the division problem as a multiplication problem
   c. Use the "short-cut" method before multiplying or multiply the fraction, expressing the answer in simplest terms

CHART IV

Mixed Form Divided by a Whole Number - Grade 9

$2\frac{1}{2} \div 2$

$1\frac{1}{2} \div 3$

$2\frac{2}{3} \div 4$

$1\frac{1}{3} \div 8$

$1\frac{1}{5} \div 9$

$1\frac{4}{5} \div 6$

$8\frac{1}{2} \div 14$

$6\frac{1}{2} \div 13$

$6\frac{2}{3} \div 10$

$2\frac{2}{5} \div 24$

$3\frac{1}{3} \div 14$

$4\frac{2}{3} \div 10$

$16\frac{2}{3} \div 13$

$12\frac{1}{3} \div 37$

$2\frac{7}{16} \div 13$

$2\frac{1}{16} \div 66$

$2\frac{3}{11} \div 65$

$4\frac{9}{14} \div 25$

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Whole Number Divided by a Mixed Form

(Some representative examples will be found in Chart V below)

The student should be able to:

1. Demonstrate how to construct the quotient of a whole number and a mixed form
2. Construct the quotient for a whole number and a mixed form
3. Describe how to divide a whole number and a mixed form by using specific examples
4. Apply the following principles for dividing a whole number and a mixed form:
   - a. Rename the mixed form as a fraction
   - b. Rename the division problem as a multiplication problem
   - c. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answer in simplest terms

CHART V

Whole Number Divided by a Mixed Form - Grade 9

$2 \div 1\frac{2}{3}$

$3 \div 1\frac{1}{2}$

$8 \div 1\frac{1}{3}$

$4 \div 2\frac{2}{3}$

$6 \div 2\frac{1}{4}$

$12 \div 1\frac{3}{5}$

$14 \div 7\frac{2}{3}$

$15 \div 3\frac{3}{4}$

$16 \div 1\frac{3}{5}$

$6 \div 2\frac{2}{5}$

$8 \div 2\frac{2}{5}$

$16 \div 1\frac{1}{11}$

$17\frac{2}{3} \div 18$

$39 \div 19\frac{1}{2}$

$64 \div 10\frac{2}{3}$

$16 \div 12\frac{4}{5}$

$14 \div 10\frac{1}{2}$

$21 \div 4\frac{2}{3}$

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Mixed Form Divided by a Fraction

(Some representative examples will be found in Chart VI below)

The student should be able to:

1. Demonstrate how to construct the quotient of a mixed form and a fraction
2. Construct the quotient for a mixed form and a fraction
3. Describe how to divide a mixed form and a fraction by using specific examples
4. Apply the following principles for dividing a mixed form by a fraction:
    a. Rename the mixed form as a fraction
    b. Rename the division problem as a multiplication problem
    c. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answer in simplest terms

CHART VI

Mixed Form Divided by a Fraction - Grade 9

$1\frac{1}{2} \div \frac{2}{3}$

$1\frac{1}{2} \div \frac{3}{5}$

$1\frac{1}{2} \div \frac{6}{7}$

$1\frac{1}{3} \div \frac{2}{7}$

$1\frac{1}{5} \div \frac{4}{7}$

$1\frac{1}{2} \div \frac{3}{4}$

$1\frac{1}{6} \div \frac{7}{8}$

$1\frac{1}{2} \div \frac{10}{13}$

$1\frac{1}{2} \div \frac{3}{17}$

$1\frac{1}{2} \div \frac{15}{16}$

$1\frac{1}{5} \div \frac{16}{17}$

$6\frac{2}{3} \div \frac{5}{9}$

$1\frac{3}{15} \div \frac{4}{7}$

$1\frac{1}{2} \div \frac{3}{16}$

$6\frac{1}{2} \div \frac{10}{13}$

$6\frac{1}{2} \div \frac{3}{17}$

$1\frac{3}{12} \div \frac{30}{48}$

$2\frac{2}{5} \div \frac{16}{17}$

$6\frac{1}{3} \div \frac{10}{17}$

$1\frac{3}{15} \div \frac{16}{17}$

$6\frac{1}{2} \div \frac{13}{16}$

CHART VI (Cont'd)

Mixed Form Divided by a Fraction - Grade 9

$2\frac{1}{4} \div \frac{3}{8}$

$2\frac{1}{4} \div \frac{3}{8}$

$1\frac{2}{3} \div \frac{1}{4}$

$1\frac{3}{4} \div \frac{1}{4}$

$1\frac{1}{4} \div \frac{1}{8}$

$1\frac{1}{4} \div \frac{1}{2}$

$1\frac{1}{6} \div \frac{1}{9}$

$1\frac{1}{6} \div \frac{1}{4}$

$1\frac{3}{4} \div \frac{7}{18}$

$2\frac{1}{4} \div \frac{3}{16}$

$3\frac{3}{4} \div \frac{6}{8}$

$2\frac{3}{5} \div \frac{1}{14}$

$4\frac{1}{4} \div \frac{1}{4}$

$3\frac{3}{4} \div \frac{1}{8}$

$3\frac{3}{4} \div \frac{1}{2}$

$2\frac{5}{6} \div \frac{1}{9}$

$4\frac{1}{4} \div \frac{17}{18}$

$3\frac{1}{8} \div \frac{15}{16}$

$1\frac{1}{14} \div \frac{9}{28}$

$1\frac{17}{18} \div \frac{15}{32}$

$3\frac{1}{5} \div \frac{1}{14}$

$4\frac{1}{4} \div \frac{1}{14}$

$1\frac{6}{9} \div \frac{1}{18}$

$1\frac{3}{24} \div \frac{1}{12}$

Fraction Divided by a Mixed Form

(Some representative examples will be found in Chart VII)

The student should be able to:

1. Demonstrate how to construct the quotient of a fraction and a mixed form
2. Construct the quotient for a fraction and a mixed form
3. Describe how to divide a fraction and a mixed form by using specific examples
4. Apply the following principles for dividing a fraction by a mixed form:
   a. Rename the mixed form as a fraction
   b. Rename the division problem as a multiplication problem

c. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answer in simplest terms

CHART VII

Fraction Divided by a Mixed Form - Grade 9

$\frac{2}{3} \div 1\frac{1}{2}$

$\frac{7}{8} \div 1\frac{4}{3}$

$\frac{4}{5} \div 2\frac{2}{3}$

$\frac{6}{7} \div 1\frac{1}{2}$

$\frac{6}{9} \div 2\frac{1}{4}$

$\frac{3}{4} \div 1\frac{4}{8}$

$\frac{7}{8} \div 1\frac{1}{6}$

$\frac{6}{9} \div 1\frac{1}{3}$

$\frac{1}{4} \div 2\frac{1}{3}$

$\frac{1}{4} \div 1\frac{3}{4}$

$\frac{1}{4} \div 1\frac{1}{8}$

$\frac{1}{4} \div 1\frac{1}{2}$

$\frac{1}{6} \div 1\frac{1}{9}$

$\frac{2}{13} \div 1\frac{1}{2}$

$\frac{7}{18} \div 1\frac{4}{3}$

$\frac{4}{5} \div 1\frac{11}{17}$

$\frac{6}{17} \div 1\frac{1}{2}$

$\frac{6}{19} \div 2\frac{1}{4}$

$\frac{3}{4} \div 2\frac{14}{16}$

$\frac{17}{18} \div 1\frac{1}{16}$

$\frac{16}{21} \div 4\frac{2}{3}$

$\frac{1}{14} \div 2\frac{1}{3}$

$\frac{1}{14} \div 1\frac{3}{14}$

$\frac{1}{4} \div 1\frac{5}{16}$

$\frac{1}{14} \div 1\frac{1}{2}$

$\frac{1}{16} \div 1\frac{1}{24}$

$\frac{12}{13} \div 1\frac{1}{2}$

$\frac{17}{18} \div 12\frac{4}{3}$

$\frac{4}{15} \div 1\frac{11}{17}$

$\frac{18}{19} \div 4\frac{1}{2}$

$\frac{16}{19} \div 5\frac{3}{5}$

$\frac{13}{17} \div 1\frac{5}{34}$

$\frac{19}{21} \div 2\frac{5}{7}$

$\frac{25}{36} \div 2\frac{3}{6}$

$\frac{1}{14} \div 3\frac{1}{18}$

$\frac{1}{14} \div 3\frac{5}{14}$

$\frac{1}{18} \div 3\frac{7}{36}$

$\frac{1}{24} \div 2\frac{1}{12}$

$\frac{1}{21} \div 10\frac{3}{14}$

Mixed Form Divided by a Mixed Form

(Some representative examples will be found in Chart VIII below)

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The student should be able to:

1. Name and identify a division problem whose parts are mixed forms
2. Demonstrate how to construct the quotient of a pair of mixed forms
3. Construct the quotient for a pair of mixed forms
4. Describe how to divide pairs of mixed forms by using specific examples
5. State the principle of dividing pairs of mixed forms by definition of division of fractions
6. Apply the following principles for dividing pairs of mixed forms:
    a. Rename the mixed forms as a fraction
    b. Rename the division problem as a multiplication problem
    c. Use the "short-cut" method before multiplying or multiply the fractions, expressing the answer in simplest terms

CHART VIII

Mixed Form Divided by a Mixed Form - Grade 9

$1\frac{1}{3} \div 1\frac{3}{4}$

$1\frac{2}{3} \div 1\frac{1}{4}$

$2\frac{1}{2} \div 1\frac{1}{4}$

$1\frac{2}{6} \div 1\frac{1}{3}$

$1\frac{1}{3} \div 1\frac{1}{3}$

$4\frac{2}{3} \div 4\frac{1}{4}$

$5\frac{2}{3} \div 4\frac{1}{4}$

$7\frac{1}{2} \div 3\frac{3}{4}$

$1\frac{13}{15} \div 1\frac{1}{3}$

$9\frac{1}{2} \div 9\frac{1}{2}$

$1\frac{1}{9} \div 1\frac{2}{6}$

$20\frac{7}{8} \div 18\frac{4}{5}$

$7\frac{3}{5} \div 12\frac{2}{3}$

$3\frac{3}{6} \div 1\frac{3}{18}$

$11\frac{1}{4} \div 2\frac{1}{2}$

$6\frac{13}{16} \div 6\frac{13}{16}$

$2\frac{6}{15} \div 2\frac{4}{10}$

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Non-Commutative Property of Division for Fractions

(Use examples found in Chart I through Chart VIII)

The student should be able to:

1. Construct number sentences which illustrate that division of fractions is not commutative

Non-Associative Property of Division for Fractions

The student should be able to:

1. Name and identify number sentences which illustrate that division of fractions is not associative
2. Demonstrate a procedure which shows that division of fractions is not associative
3. Construct number sentences which illustrate that division of fractions is not associative

Division as the Inverse Operation of Multiplication

The student should be able to:

1. Describe inverse relationship given either a multiplication or a division example
2. Apply the principle of the inverse relation to solve related examples

Translating Verbal Problems Involving Division of Fractions into Number Sentences

(Choose only those problems whose solution can be determined in two steps, each step requires a different operation)

The student should be able to:

1. Name and identify verbal problems whose solutions require division of fractions
2. Demonstrate a procedure for translating verbal problems into number sentences
3. Construct a number sentence from the information contained in a verbal problem
4. Describe the steps needed to translate verbal problems to number sentences, using specific examples
5. Interpret the information contained in a verbal problem in order to construct a corresponding number sentence

UNIT FRACTIONAL NUMBERS GRADE(S) Nine

Addition of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Least Common Multiple			9	9		9	9			9
Addition of Fractions, Unlike Denominators				9			9			
Miscellaneous Problems of Adding Fraction Expressions				9		9	9			
Closure Property						9				
Commutative Property				9						9
Associative Property				9			9			9
Estimation				9			9			
Translating Verbal Problems into Number Sentences	9	9	9	9	9			9		

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FRACTIONAL NUMBERS - Grade 9

Addition of Fractions

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Least Common Multiple

The student should be able to:

1. Demonstrate how to find the least common multiple of two or more numbers
2. Construct the least common multiple for two or more numbers
3. State the principle that the least common multiple of two or more numbers is the smallest number divisible by all those numbers
4. Apply the principle for finding the least common multiple and solve related examples
5. Distinguish between the least common multiple and other multiples for a set of numbers

Addition of Fractions with Unlike Denominators

The student should be able to:

1. Construct sums, in simplest terms, for addition of fractions with unlike denominators
2. Apply the principle for adding fractions with unlike denominators to solve related problems

Addition of Numbers in Different Form with Unlike Denominators

(Listed below are some representative examples.
Note the denominators are 2, 3, 4, 5, 6, 8, 10, 16.)

$$8 + \frac{2}{3} + \frac{5}{6}, \quad 9 + \frac{5}{6} + 2, \quad 8\frac{1}{2} + 9\frac{2}{3} + 16\frac{5}{8}$$

$$18\frac{2}{5} + 9\frac{1}{3} + 6, \quad 25\frac{4}{5} + 11 + \frac{11}{16}, \quad 8 + 9\frac{1}{5} + 16$$

The student should be able to:

1. Construct sums, in simplest terms, of whole numbers and fractions with unlike denominators
2. State the principle for adding different fractional expressions with unlike denominators

3. Apply the principle for adding whole numbers and fractional expressions to solve related problems

Closure Property of Fractions under Addition

The student should be able to:

1. State the principle that the set of fractions is closed under addition

Commutative Property of Addition of Fractions

The student should be able to:

1. Construct a number sentence showing the commutative property of addition of fractions
2. Apply the associative property to add fractions and to solve related problems
3. Distinguish between the associative and commutative properties

Associative Property of Addition of Fractions

The student should be able to:

1. Construct a number sentence showing the associative property of fractions
2. State the associative property for adding fractional expressions
3. Apply the principle of associativity for adding fractions to solve related examples
4. Distinguish between the associative and commutative property

Estimation

The student should be able to:

1. Apply the principle for estimating sum of fractions by rounding the fractions to the nearest whole number

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Translating Verbal Problems Involving Addition of Fractions into Number Sentences

The student should be able to:

1. Name and identify verbal problems containing addition of fractional expressions
2. Demonstrate a procedure for translating verbal statements into number sentences
3. Construct a number sentence from the information given in the verbal problem
4. Describe how the number sentence was translated from the verbal problem
5. Interpret the information contained in a verbal problem in order to construct a number sentence

UNIT FRACTIONAL NUMBERS GRADE(S) Nine

Subtraction of Fractions

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Translating Verbal Problems to Number Sentences	9	9	9	9	9					

Subtraction of Fractions

Translate Verbal Problems into Number Sentences

Page

The student should be able to:

1. Name and identify verbal problems which contain subtraction of fractions
2. Demonstrate a procedure for writing a number sentence from the information contained in verbal problem
3. Construct a number sentence from the information contained in verbal problem
4. Describe how a verbal problem is translated into a number sentence by using specific examples

UNIT DECIMAL NUMERALS GRADE(S) Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Decimal Equivalence	9	9	9							
Annexing Zeros	9	9	9							
Betweenness	9		9							
Rounded Numbers	9	9					9			
Column Form Addition			9	9						
Arrange Addends in Column Form			9			9				
Subtraction			9	9						
Multiplication by Powers of Ten			9			9	9			
Products of Decimal and Whole Numbers			9	9						
Products of Two Decimals			9	9						
Division of a Decimal by a Whole Number			9	9						
Division of a Whole Number by a Decimal Fraction			9	9						
Division of a Decimal by a Decimal			9	9						
Quotients Rounded			9	9	9					
Estimating Quotients			9	9	9					
Decimal Equivalents	9	9	9	9						

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Decimal Equivalence

The student should be able to:

1. Name and identify the decimal equivalent of a fraction with denominator of 10, 000
2. Name and identify a fractional equivalent with denominator of 10, 000 given a decimal to the ten-thousandths place
3. Construct the decimal equivalent of a fraction with denominator of 10, 000
4. Construct the fractional equivalent with denominator of 10, 000 given a decimal to the ten-thousandths place

Annexing Zeros

The student should be able to:

1. Name and identify decimals having zeros annexed which are the same as a given decimal
2. Demonstrate how to rename decimal numerals by annexing zeros

Betweenness Using Ten Thousandths

The student should be able to:

1. Name a decimal between two consecutive given numbers
2. Name two consecutive whole numbers between which a given decimal lies
3. Demonstrate betweenness on a number line

Rounding Decimal Numbers to Nearest Thousandths

The student should be able to:

1. Name and identify a number which has been rounded to the nearest thousandth
2. Apply the following principles to construct a number rounded to the nearest thousandth:

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a. Consider the numbers named in the thousandths place and the ten-thousandths place. If the number named in the ten-thousandths place is less than 0.0005, then replace the digits right of the thousandths place by 0

b. If the number named in the ten-thousandths place is 0.0005 or greater replace the digits at the right of the thousandths place by 0 and increase the number named in the thousandths place by 0.001

Column Form Addition

The student should be able to:

1. Demonstrate how to construct sums of up to four five-digit addends each of which contains a ten-thousandths digit
2. Demonstrate how to construct sums of up to four five-digit addends, some of which contain a ten-thousandths digit
3. Construct sums of up to four five-digit addends using the ten-thousandths digit

Arrange Addends in Column Form

The student should be able to:

1. Demonstrate how to arrange a given number of addends in column form to preserve place value
2. State the principle that the decimal point must remain in a vertical line to preserve place value

Subtraction with No Renaming

The student should be able to:

1. Demonstrate how to construct the difference of two five-digit numbers each of which contains a ten-thousandths digit
2. Construct the difference of two five-digit numbers each of which contains a ten-thousandths digit

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Subtraction with Renaming Required

The student should be able to:

1. Demonstrate how to construct the difference of two five-digit numbers each of which contains a ten-thousandths digit
2. Construct the difference of two five-digit numbers each of which contains a ten-thousandths digit

Subtraction with No Renaming Required, Annexing Zeros

The student should be able to:

1. Demonstrate how to construct the difference of two numbers where the subtrahend contains a ten-thousandths digit
2. Construct the difference of two number where the subtrahend contains a ten-thousandths digit

Subtraction with Renaming Required, Annexing Zeros

The student should be able to:

1. Demonstrate how to construct the difference of two numbers where the subtrahend contains a ten-thousandths digit
2. Construct the difference of two numbers where the subtrahend contains a ten-thousandths digit

Arrange a Subtraction Problem in Vertical Form

The student should be able to:

1. Demonstrate how to arrange a subtraction problem in vertical form where the ten-thousandths digit is used in some numbers
2. State the principle that the decimal point is in a straight line to preserve place value

Multiplication by .0001

The student should be able to:

1. Demonstrate how to construct products using the factor .0001

2. State the principle that when .0001 is used as a factor the product is found by "moving" the decimal point one place to the left
3. Apply the principle to construct products using a factor of .000

The Product of a Decimal and a Whole Number

The student should be able to:

1. Demonstrate how to construct products using a four-digit decimal factor and a three-digit whole number factor
2. Demonstrate how to construct products, using a three-digit whole number factor and a four-digit decimal factor, which contains a thousandths digit
3. Construct products using a three-digit whole number factor and a three-digit decimal factor which contains a thousandths or ten-thousandths place digit

Products of Two Decimals

The student should be able to:

1. Demonstrate how to construct products using two three-digit decimal factors
2. Construct products using two three-digit decimal factors

Division of a Decimal by a Whole Number

The student should be able to:

1. Demonstrate how to construct quotients with no remainder using a three-digit divisor and a decimal dividend containing a ten-thousandths digit
2. Demonstrate how to construct quotients with no remainders using a three-digit divisor and a mixed decimal dividend which contains a ten-thousandths digit
3. Construct quotients using divisors of three digits and a decimal dividend which contains a ten-thousandths digit

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Division of a Whole Number by a Decimal Fraction

The student should be able to:

1. Demonstrate how to construct quotients with no remainder using a three-digit divisor containing a thousandths digit and a whole number dividend
2. Construct quotients with no remainder using a three-digit divisor containing a thousandths digit and a whole number dividend

Division of a Decimal by a Decimal

The student should be able to:

1. Demonstrate how to construct quotients with no remainders using a three-digit divisor containing a thousandths digit and a dividend which contains a thousandths digit
2. Construct quotients with no remainders using a three-digit divisor containing a thousandths digit and a dividend which contains a thousandths digit

Quotients Rounded to Nearest Hundredth

The student should be able to:

1. Demonstrate how to construct a quotient to the nearest hundredth by getting a thousandths digit in the quotient, then round to the nearest hundredth
2. Describe how to find a quotient to the nearest hundredth

Decimal Equivalents

The student should be able to:

1. Name and identify any fraction with a whole number as the numerator and a denominator of 3, 6, 9, 12, 15
2. Demonstrate how to use division to construct decimal numerals for fractions with denominator of 3, 6, 9, 12, 15

Page

3. Construct a decimal numeral when given a fraction with denominator of 3, 6, 9, 12, 15

Estimating Quotients to Nearest Whole Number

The student should be able to:

1. Demonstrate how to construct the estimation of a quotient to the nearest whole number by rounding each factor to the nearest whole number, then divide
2. Construct a quotient to the nearest whole number
3. Describe a procedure for constructing an estimation

UNIT PERCENT BY RATIO AND PROPORTION GRADE(S) Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Equivalent Rate Pairs						9	9			
Proportion	9	9	9	9						
Solving Multiplication Equations	9	9	9			9	9			
Translating Verbal Problems into Equivalent Rate Pairs		9	9	9	9					
Solving Proportion		9	9			9	9			
Finding What Percent One Number is of Another		9	9				9			
Translating Verbal Problems into Equivalent Rate Pairs		9	9	9	9					

NUMBERS, OPERATIONS AND ALGORITHMS
PERCENT BY RATIO AND PROPORTION - Grade 9

Page

Equivalent Rate Pairs

The student should be able to:

1. State the principle that two rate pairs, (a, b) and (c, d) are equivalent whenever ad = bc
2. Apply this principle to identify equivalent rate pairs

Proportion

The student should be able to:

1. Name and identify a proportion
2. Describe a proportion as a statement of equivalent rate pairs
3. Construct a proportion given equivalent rate pairs
4. Demonstrate how to construct a proportion given equivalent rate pairs

Solving Multiplication Equations

FO-66

(Listed below are some representative examples. Note that in the exercises the coefficients are whole numbers and the roots progress in the degree of difficulty. Do not use fractional coefficients.)

A	B	C
$2x = 4$	$3x = 5$	$2x = 1$
$9x = 81$	$5x = 17$	$3x = 2$
$6x = 132$	$4x = 29$	$5x = 3$

The student should be able to:

1. Name and identify multiplication equations
2. Demonstrate a procedure for solving multiplication equations
3. State the following principle: A number times its inverse equals one. (The identity element for multiplication.)
4. Apply this principle to construct solution sets for multiplication equations

Page

Translating Verbal Problems into Equivalent Rate Pairs

(Note that the verbal problems must contain all 4 terms.)

The student should be able to:

1. Identify verbal problems containing equivalent rate pairs
2. Demonstrate a procedure for translating the information contained in a problem into equivalent rate pairs
3. Construct equivalent rate pairs from the information contained in a verbal problem
4. Describe a procedure for translating the information contained in a verbal problem into equivalent rate pairs by using specific examples

Solving Proportions

FO-67

(Listed below are some representative examples. Note that the coefficients are whole numbers and the solutions progress in the degree of difficulty.)

$(2,\ 5) = (x,\ 100)$; $(x,\ 10) = (20,\ 30)$; $(4\frac{1}{2},\ x) = (10,\ 20)$

$(\frac{3}{5},\ 5) = (x,\ 15)$; $(x,\ 40) = (60,\ 100)$

The student should be able to:

1. Identify a proportion with an unknown term
2. Demonstrate a procedure for solving a proportion
3. State the following principles for solving a proportion:
   a. Use cross multiplication to construct a multiplication equation
   b. Solve the multiplication equation
4. Apply these principles to construct the solution for a proportion with an unknown term

Page
FO-65

Finding What Percent One Number Is of Another

(Listed below are some representative examples. Note that the examples progress in the degree of difficulty.)

a. 5 is what percent of 10?

b. 15 is what % of 20?

c. What percent of 30 is 10?

d. 10 is what % of 10?

e. 6 is what % of 2?

f. What % of 7 is 15?

The student should be able to:

1. Identify problems which require a percent to be found
2. Demonstrate a procedure for finding what percent one number is of another
3. Apply the following principles to construct the percent one number is of another
   a. Construct a proportion with an unknown term
   b. Use cross multiplication to construct a multiplication equation
   c. Solve the multiplication equation

Translating Percent Verbal Problems into Equivalent Rate Pairs

(The types of problems that the students should concentrate on are: Finding what percent one number is of another for rate of commission, interest, and discount.)

The student should be able to:

1. Identify verbal problems in which they are required to find what percent one number is of another
2. Demonstrate a procedure for writing equivalent rate pairs from the information contained in a verbal problem
3. Construct equivalent rate pairs from the information contained in the verbal problem
4. Describe a procedure for constructing equivalent rate pairs from the information contained in a verbal problem using specific examples

UNIT SQUARE ROOT GRADE(S) Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Use of Square Root Table	9	9	9	9	9					

NUMBERS, OPERATIONS AND ALGORITHMS
SQUARE ROOT - Grade 9

Use of Square Root Tables Page

The student should be able to:

1. Name and identify square root tables FO-64
2. Demonstrate how to use square root tables by reading entries
3. Construct a square root table for square numbers
4. Describe a square root table

PRACTICE IN ADDING MORE THAN TWO ADDENDS-INTERMEDIATE

Teacher Commentary

A Tape Recording of Addition Problems for Use in Grade 9

I. Materials:

   A. Tape recorder

   B. Eight-station listening post (optional)

   C. Tape entitled, "Practice In Adding More Than Two Addends-Intermediate"

   E. Pencil and eraser

II. Procedure:

   A. Preview the tape to determine if it will be used with a small group of students, or if it will be used as a class activity.

   B. The tape has twenty problem sets. Each problem set has five parts. Do not do all problem sets at the same time. The tape has been designed so a student could do two or three problem sets at one sitting. Each problem set is approximately $1\frac{1}{2}$ minutes long.

   C. Make an analysis of the problems done incorrectly to determine where students are having problems. A teacher copy has been supplied for this purpose.

   D. The total tape time is approximately 20 minutes. Ten second intervals separate each problem set.

   E. Provide one or more practice examples so that students will be familiar with the format of the answer sheet.

Teacher's Copy

PRACTICE IN ADDING MORE THAN TWO ADDENDS-INTERMEDIATE

	A	B	C	D	E
Example 1	2 + 1	3	5	7	6
	3	6	11	18	24
Example 2	1 + 8	0	3	5	2
	9	9	12	17	19
Problem 1	4 + 4	6	9	8	0
	8	14	23	31	31
Problem 2	9 + 7	4	6	7	1
	16	20	26	33	34
Problem 3	5 + 0	6	7	9	9
	5	11	18	27	36
Problem 4	3 + 4	0	2	8	5
	7	7	9	17	22
Problem 5	1 + 3	2	8	5	3
	4	6	14	19	22
Problem 6	2 + 0	9	2	0	4
	2	11	13	13	17
Problem 7	6 + 5	7	7	3	9
	11	18	25	28	37
Problem 8	1 + 1	4	8	8	6
	2	6	14	22	28
Problem 9	0 + 1	6	8	3	0
	1	7	15	18	18
Problem 10	9 + 3	1	4	2	6
	12	13	17	19	25
Problem 11	8 + 7	7	2	4	9
	15	22	24	28	37
Problem 12	5 + 5	3	8	4	5
	10	13	21	25	30
Problem 13	8 + 2	2	7	0	9
	10	12	19	19	28
Problem 14	7 + 3	5	6	9	1
	10	15	21	30	31
Problem 15	6 + 4	1	0	6	6
	10	11	11	17	23
Problem 16	0 + 0	2	9	5	8
	0	2	11	16	24
Problem 17	4 + 3	8	1	3	2
	7	15	16	19	21
Problem 18	1 + 7	9	5	7	4
	8	17	22	29	33
Problem 19	1 + 5	8	4	4	0
	6	14	18	22	22
Problem 20	7 + 8	2	2	9	6
	15	17	19	28	34

Name ______________________________

Section ____________________________

PRACTICE IN ADDING MORE THAN TWO ADDENDS-INTERMEDIATE

	A	B	C	D	E
Example 1					
Example 2					
Problem 1					
Problem 2					
Problem 3					
Problem 4					
Problem 5					
Problem 6					
Problem 7					
Problem 8					
Problem 9					
Problem 10					
Problem 11					
Problem 12					
Problem 13					
Problem 14					
Problem 15					
Problem 16					
Problem 17					
Problem 18					
Problem 19					
Problem 20					

NAPIER'S BONES

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

A. Construct the product of two whole numbers

B. Demonstrate how to find products of two whole numbers using Napier's Bones.

III. Materials:

A. A piece of cardboard (about $8\frac{1}{2}$ x 11)

B. Glue

C. Scissors

IV. Procedure:

A. Introduction

This lesson should be preceded by the lattice method of multiplication. The use of lattices in multiplication was very popular in the seventeenth century.

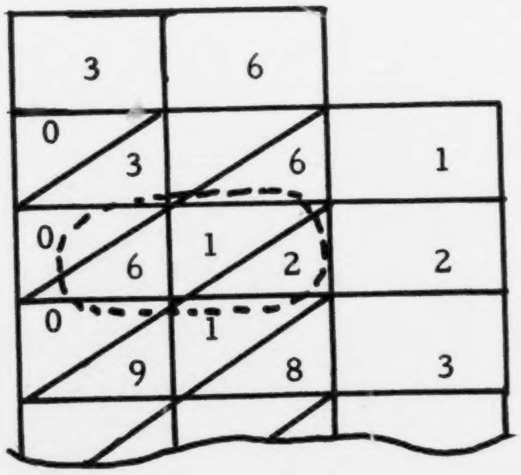
B. Have the students construct their own Napier's Bones as follows:

1. Make a spirit master of the chart on the following page.
2. Using the spirit master, run sheets of tag board through the duplicating machine.
3. Distribute the tag board to the students and have them cut out the computing rods by cutting along the dashed lines. They will have 11 strips in all.
4. The teacher can make transparencies of strips for use with the overhead projector by running an acetate sheet through the duplicating machine.

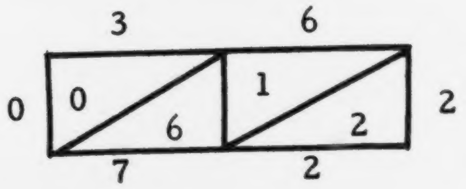
   NOTE: Run the acetate first while the master is fresh.
5. Once students are able to construct products using Napier's Bones the class can be divided into two teams. The teams can compete for speed and accuracy with one team multiplying by the standard algorithm and the other team using Napier's Bones. The method can then be reversed.

6. The following are examples of how these strips are used for multiplication.

Example 1 36 x 2 = 72

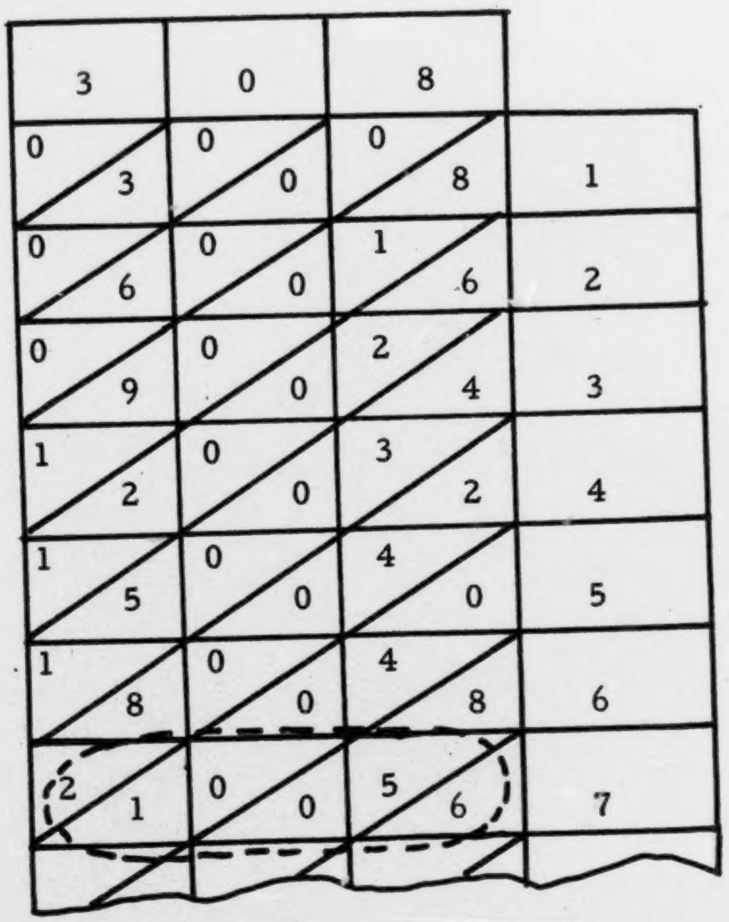

3 6
0 3 0 6 1
0 6 1 2 2
0 9 1 8 3

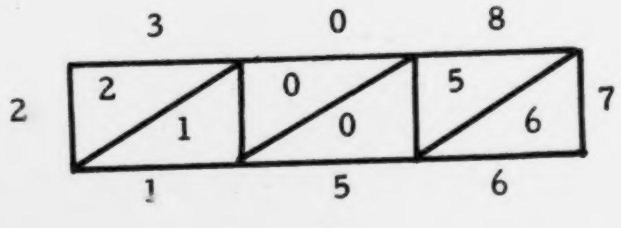
1. Place the necessary 3 strips as they are in this illustration.
2. Do the multiplication by the lattice method.


3 6
0
0 6
1 2
2
7 2

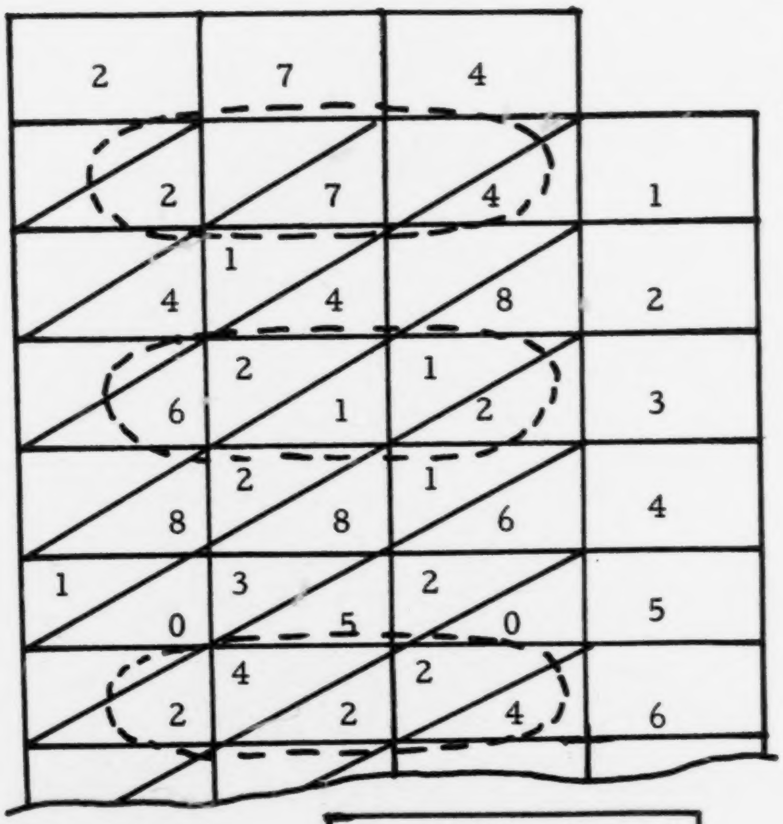
3. The answer is 72.

Example 2 308 x 7 = 2156


3 0 8
0 3 0 0 0 8 1
0 6 0 0 1 6 2
0 9 0 0 2 4 3
1 2 0 0 3 2 4
1 5 0 0 4 0 5
1 8 0 0 4 8 6
2 1 0 0 5 6 7


3 0 8
2
2 1
0 0
5 6
7
1 5 6

Example 3 274 x 316


2 7 4
2 7 4 1
4 1 4 8 2
6 2 1 1 2 3
8 2 8 1 6 4
1 0 3 5 2 0 5
2 4 2 2 4 6

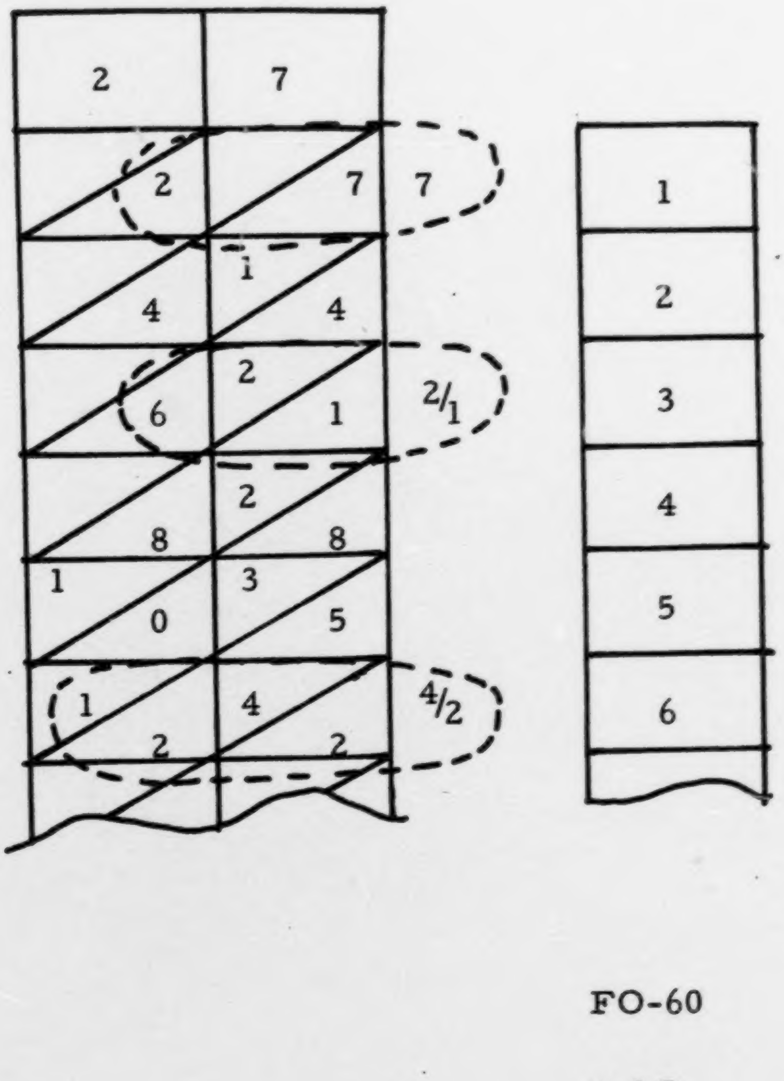
1 x 274 = 274 Since 1 is in the tens column, this product should be multiplied by 10. = 2740

3 x 274 = 822 Since 3 is in the hundreds column, this product should be multiplied by 100. =82200

6 x 274 = 1644 Since 6 is in the ones column this product should be multiplied by 1. = 1644

86584

Example 4 277 x 316 = 87, 532


2 7
2 7 7
4 1 4
6 2 1 2/1
8 2 8
1 0 3 5
1 2 4 2 4/2
1
2
3
4
5
6

1 x 277 = 277

3 x 277 = 831

6 x 277 = 1, 662

1662
277
831
87,532

C. Suggested Assessment Procedures

1. Have students multiply the following with the use of Napier's Bones. They may check by regular multiplication.

   6 x 7 = 42

   18 x 9 = 162

   26 x 14 = 364

   218 x 17 = 3,706

   397 x 416 = 165,152

   221 x 325 = 71,825

   666 x 222 = 147,852

   21456 x 72158 = 1,548,222,048

2. Students could demonstrate the use of Napier's Bones at the overhead projector or at their seats with their own cutouts.

   A Multiple Computation Board may be used for demonstration of lattices. A diagram and description are found in: Instructional Aids for Arithmetic. Holt, Rinehart, and Winston, Inc., Box 2334, Grand Central Station, New York, New York 10017.

1	2	3	4	5	6	7	8	9	0	
1	2	3	4	5	6	7	8	9	0	1
2	4	6	8	1 0	1 2	1 4	1 6	1 8	0	2
3	6	9	1 2	1 5	1 8	2 1	2 4	2 7	0	3
4	8	1 2	1 6	2 0	2 4	2 8	3 2	3 6	0	4
5	1 0	1 5	2 0	2 5	3 0	3 5	4 0	4 5	0	5
6	1 2	1 8	2 4	3 0	3 6	4 2	4 8	5 4	0	6
7	1 4	2 1	2 8	3 5	4 2	4 9	5 6	6 3	0	7
8	1 6	2 4	3 2	4 0	4 8	5 6	6 4	7 2	0	8
9	1 8	2 7	3 6	4 5	5 4	6 3	7 2	8 1	0	9
0	0	0	0	0	0	0	0	0	0	0

DUONOMOGRAPH
DISTRIBUTIVE PROPERTY
Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

Demonstrate how to construct the product of the sum of two numbers and a counting number by using the duonomograph

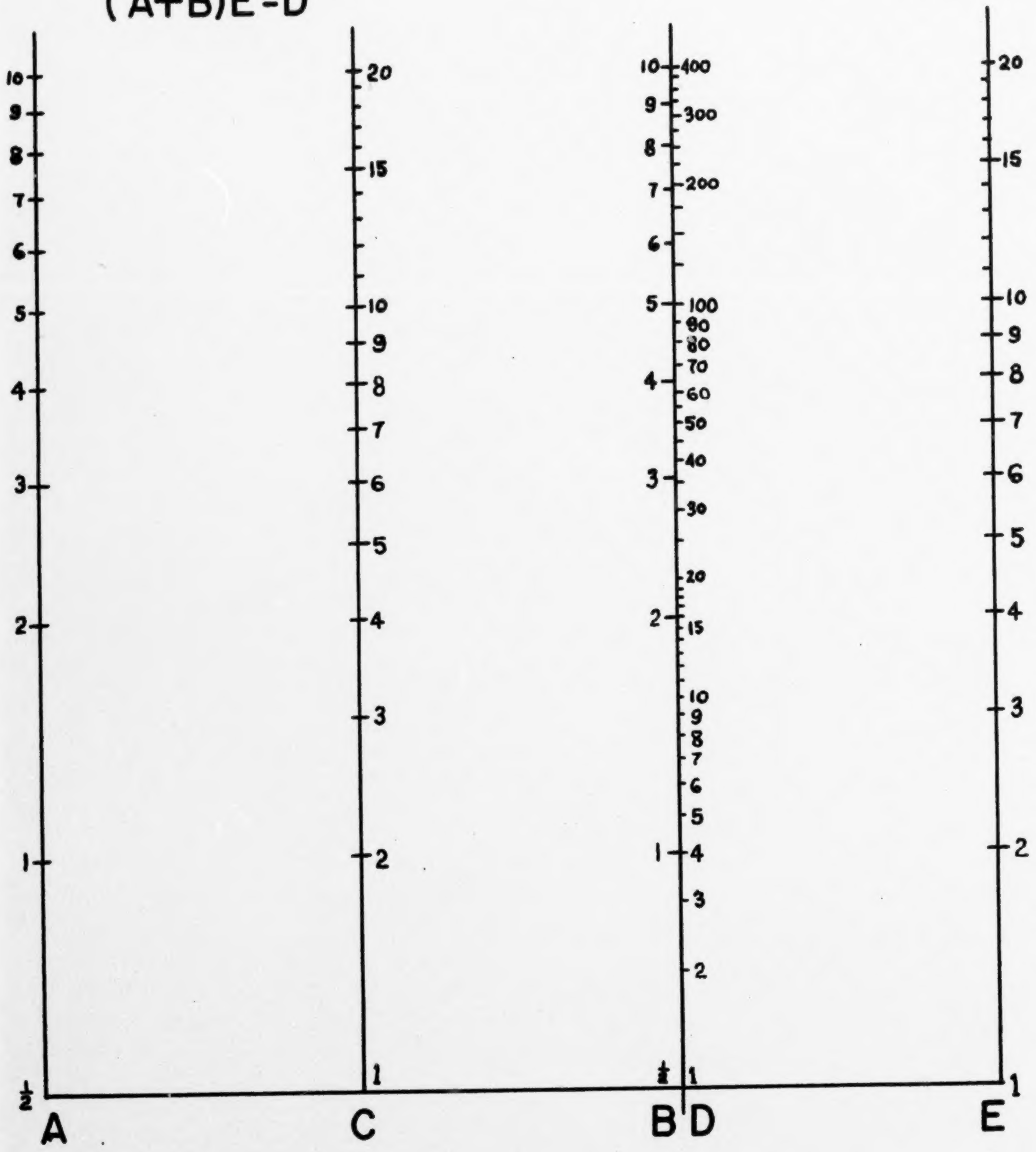
III. Materials:

- A. Student work sheet "Duonomograph"
- B. A twelve inch ruler

IV. Procedure:

- A. Distribute the materials to each student.
- B. Discuss the five scales A, B, C, D and E.
  1. Scales A and B begin with one-half and end with ten.
  2. Scales C and E begin with one and end with twenty.
  3. Scale D begins with one and ends with four hundred.
  4. Locate some points on the scales and have the students identify them.
  5. Have students locate points on the scales.
- C. In order to demonstrate the distributive principle, a problem will have to be done in two steps. Look at the following example, (3 + 2) 15.
  1. In order to solve this problem add 3 and 2. Multiply this sum by 15.
  2. On the duonomograph place your ruler on 3 on scale A and 2 on scale B. The line joining these two points will cross scale C at a point that represents the sum (5). Using 5 on scale C place the ruler on 15 on scale E. The line joining these two points will cross scale D at a point that represents the product (75).
  3. It is not possible to multiply (3) (15), (2) (15) and add the products using the duonomograph. This would have to be done as a written exercise.
- D. After the students have completed some written exercises, they could use the duonomograph to check their results.

Duonomograph
(A+B)E=D


A
½
1
2
3
4
5
6
7
8
9
10
C
1
2
3
4
5
6
7
8
9
10
15
20
B
½
1
2
3
4
5
6
7
8
9
10
D
1
2
3
4
5
6
7
8
9
10
15
20
30
40
50
60
70
80
90
100
200
300
400
E
1
2
3
4
5
6
7
8
9
10
15
20

NOMOGRAPH
DIVISION - MIXED FORMS
Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

Demonstrate how to construct the quotient of two mixed forms using the nomograph

III. Materials:

A. Student work sheet "Nomograph"

B. A twelve inch ruler

IV. Procedure:

A. Distribue the materials to each student.

B. Discuss the three scales A, B and C.

1. Scales A and B begin with one and end with ten. Each unit is divided into fourths.

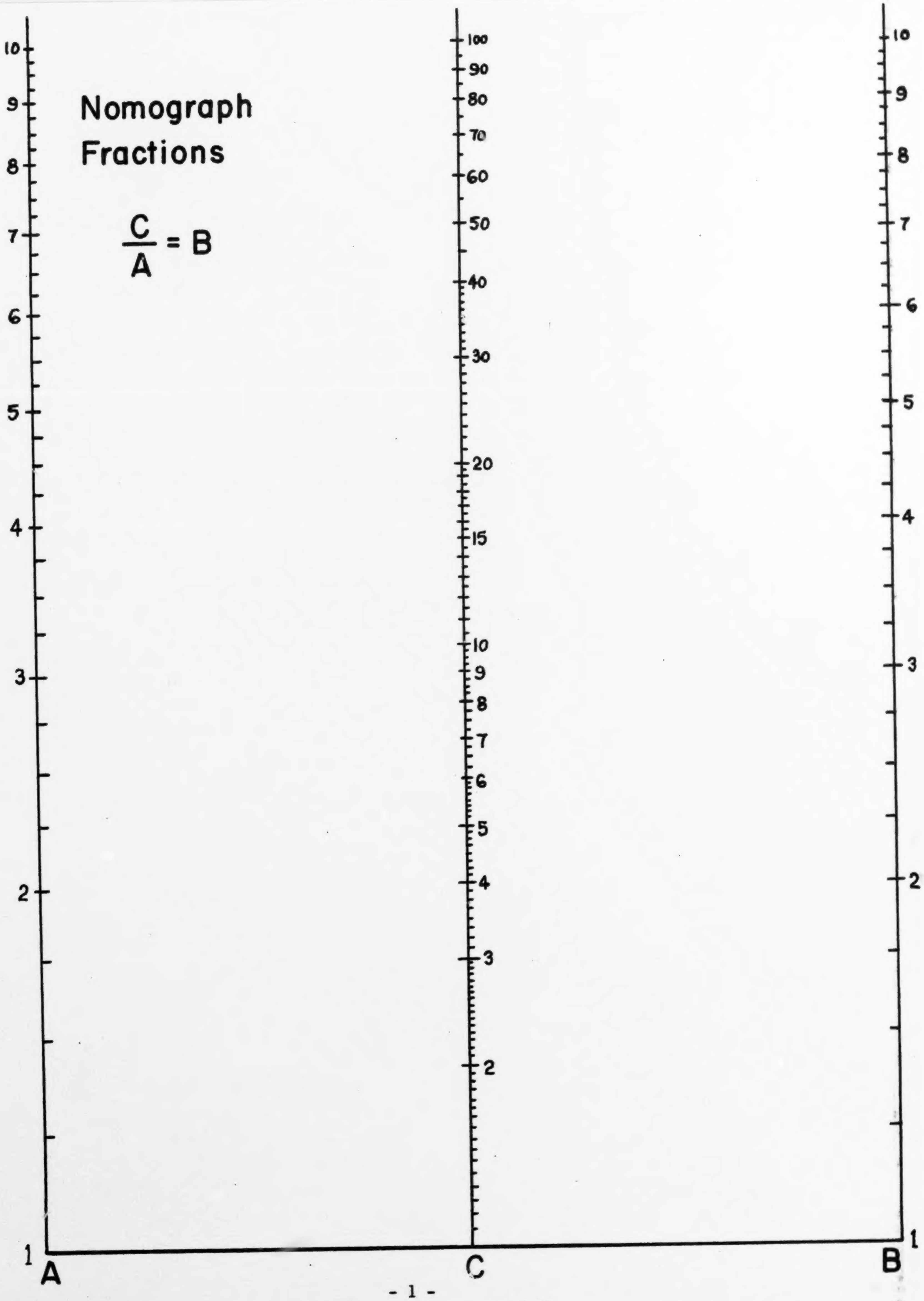
2. Scale C begins with one and ends with one hundred. The first two units are divided into sixteenths. Units six thru ten are divided into fourths. Units ten thru twenty are divided in half.

3. Locate some points on the scales and have the students identify them. Students will have to name points which lie between the increment marks.

4. Have students locate points on the scales. On scale C, make sure that the students can locate numbers like $20\frac{1}{2}$, 48 and $10\frac{1}{4}$.

C. In order to divide any two mixed forms $(6\frac{1}{4} \div 3\frac{1}{2})$ locate the dividend $(6\frac{1}{4})$ on scale C and the divisor $(3\frac{1}{2})$ on scale A. The line joining these two points will cross scale B at a point that represents the quotient $(1\frac{11}{14})$. The student will have to estimate the answer since the line will be between $1\frac{3}{4}$ and 2.

D. After the students have completed some written exercises, they could use the nomograph to check their results.



Nomograph
Fractions

$$\frac{C}{A} = B$$

A
1
2
3
4
5
6
7
8
9
10
C
2
3
4
5
6
7
8
9
10
15
20
30
40
50
60
70
80
90
100
B
1
2
3
4
5
6
7
8
9
10

NOMOGRAPH
SQUARE ROOT

Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

Demonstrate how to construct the square root of any square number by using the nomograph

III. Materials:

A. Student work sheet "Nomograph"

B. A twelve inch ruler

IV. Procedure:

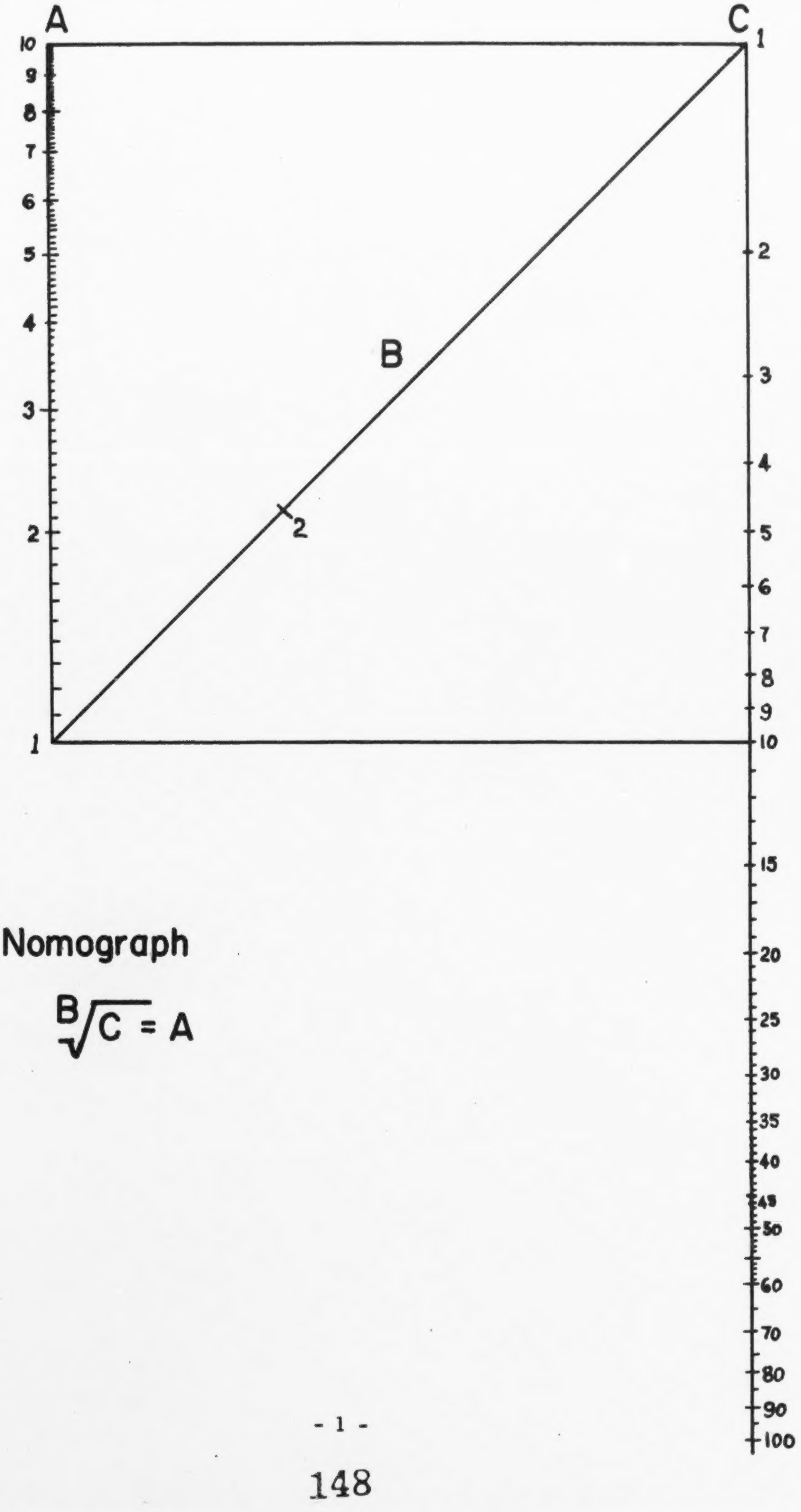
A. Distribute the materials to each student.

B. Discuss the three scales A, B and C.

1. Scale A begins with one and ends with ten. Each unit is divided into tenths.
2. Scale B, which is the diagonal of the square, has only a two indicated on it.
3. Scale C begins with one at the top of the page and ends with one hundred at the bottom.
4. Locate points on the scales and have students identify them.
5. Have student locate points on the scales.
6. Have students count one to one hundred on scale A.

C. In order to find $\sqrt{9}$ locate 9 on scale A and 2 on scale B. The line joining these two points will cross scale C at a point that represents the square root (3).

D. In order to find $\sqrt{49}$ locate 49 on scale A, and 2 on scale B. The line joining these two points will cross scale C at a point that represents the square root (7).


A
B
C
10
9
8
7
6
5
4
3
2
1
2
1
2
3
4
5
6
7
8
9
10
15
20
25
30
35
40
45
50
60
70
80
90
100

Nomograph

$\sqrt[B]{C} = A$

TILING A FLOOR

Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

Apply the principle for finding what per cent one number is of another number

III. Materials:

A. Graph paper

B. Colored pencils

C. Student work sheet entitled, "Tiling a Floor"

IV. Procedure:

A. The lesson can be motivated by discussing various tile patterns which the students have seen.

B. Distribute student work sheet entitled, "Tiling a Floor."

C. Have students do Exercises A and B.

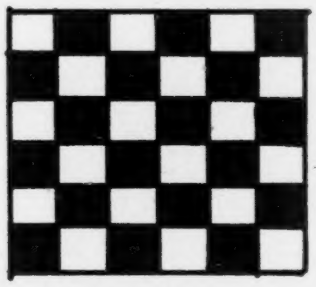
D. Have students do Exercise C. (Distribute graph paper.)

TILING A FLOOR

The modern tile floor is usually covered with square tiles. The tiles are arranged in colorful patterns. The man who designs the pattern must know the per cent of each color needed to cover the floor. He must also know the number of tiles needed.

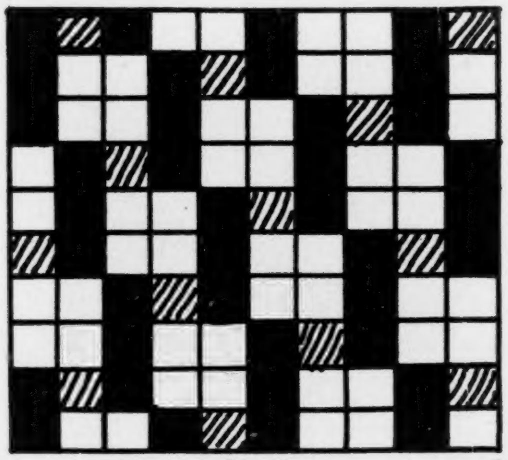
Consider the following tile patterns.

A.



1. How many black tiles are needed? ____________________
2. How many white tiles are needed? ____________________
3. What is the total number of tiles needed? ____________________
4. What per cent of the tiles are black? ____________________
5. What per cent of the tiles are white? ____________________

B.



1. How many striped tiles are needed? ____________________
2. How many black tiles are needed? ____________________
3. How many white tiles are needed? ____________________
4. What is the total number of tiles needed? ____________________
5. What per cent of the tiles are striped? ____________________
6. What per cent of the tiles are black? ____________________
7. What per cent of the tiles are white? ____________________

C. Design a pattern using the graph below. Find the percent of each color needed,

NOMOGRAPH
ALGEBRA

Teacher Commentary

I. Unit: Percent by Ratio and Proportion - Algebra

II. Objectives: The student should be able to:

Demonstrate how to construct a solution for a multiplication equation using the nomograph

III. Materials:

A. Student work sheet "Nomograph"

B. A twelve inch ruler

IV. Procedure:

A. Distribute the materials to each student.

B. Discuss the three scales A, X and C.

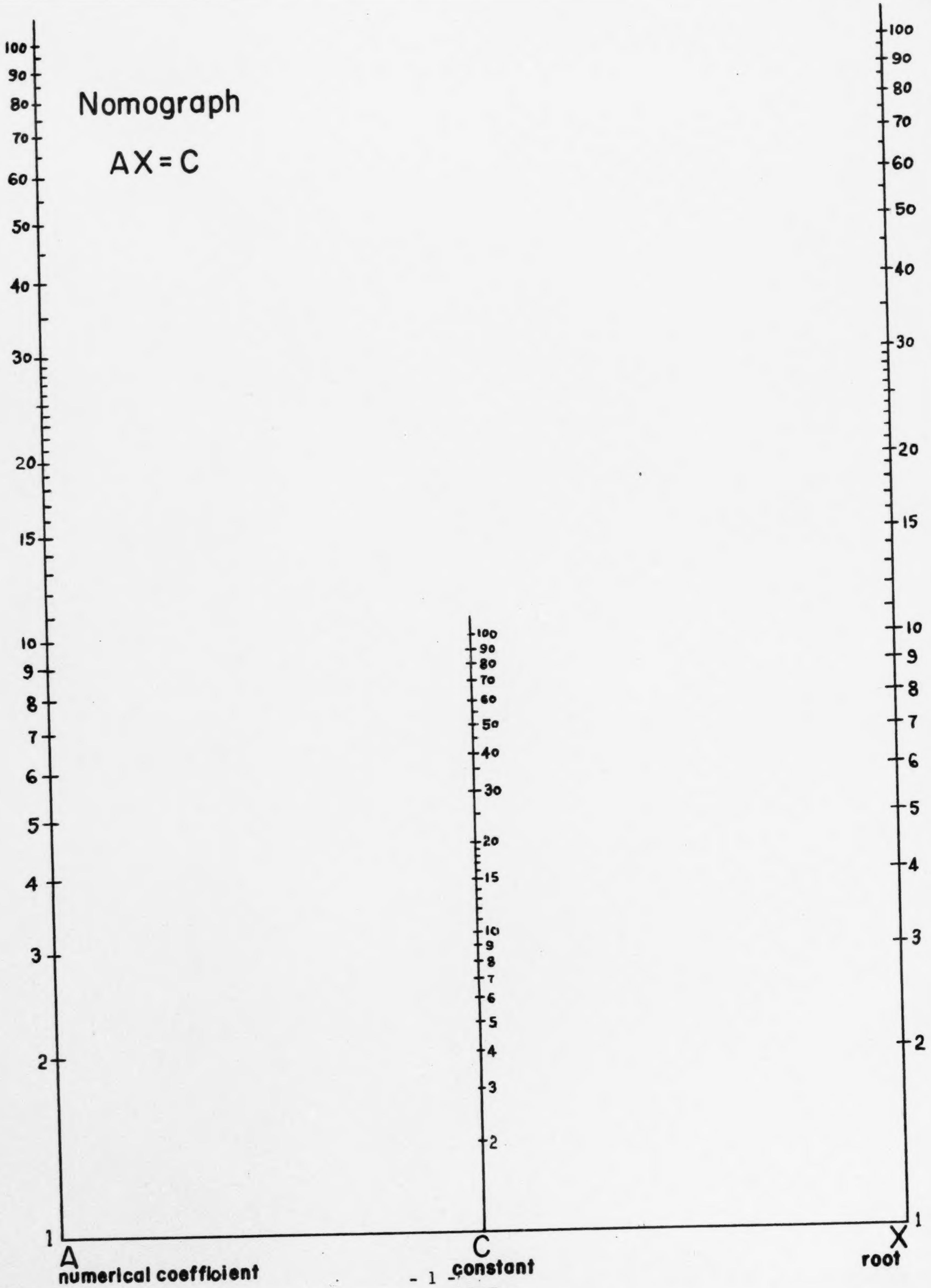
1. Scale A represents numerical coefficients from one to one hundred.
2. Scale X represents the roots from one to one hundred. Each unit is divided into fourths.
3. Scale C represents constants from one to one hundred.
4. Locate points on the scales and have students identify them.
5. Have students locate points on the scales.

C. In order to solve the equation $20x = 50$, locate 50 on scale C and 20 on scale A. The line joining these two points will cross scale X at a point that represents the root $(2\frac{1}{2})$.

D. After the students have completed some written examples, they could use the nomograph to check their results.

Nomograph

$AX = C$


100
90
80
70
60
50
40
30
20
15
10
9
8
7
6
5
4
3
2
1
A
numerical coefficient
100
90
80
70
60
50
40
30
20
15
10
9
8
7
6
5
4
3
2
C
constant
100
90
80
70
60
50
40
30
20
15
10
9
8
7
6
5
4
3
2
1
X
root

USING SHADOWS TO MEASURE HEIGHT

Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

- A. Identify a proportion with an unknown term
- B. Demonstrate a procedure for solving a proportion
- C. Apply the principle for solving a proportion by constructing its solution.

III. Materials:

- A. Yard stick
- B. Broom stick
- C. Tape measure

IV. Procedure:

- A. Ask the students to guess the height of the flagpole on the school grounds.
    1. Instruct each student to write his own estimate on a sheet of paper without conferring with another student.
    2. Collect and record estimates on the board in order of height.
    3. Students may want to find the average of these estimates.
- B. Discuss the way a surveyor might determine the height of something by using its shadow and the height of a stick placed upright nearby. The unknown height of an object and its shadow are related to the height of the stick and its shadow in the following way:

$$\frac{\text{unknown height of object}}{\text{object's shadow}} = \frac{\text{stick's height}}{\text{stick's shadow}}$$

- C. The students should relate this formula to the height of the flagpole:

$$\frac{\text{flagpole's height}}{\text{flagpole's shadow}} = \frac{\text{stick's height}}{\text{stick's shadow}}$$

D. Students should set up the following proportion to find the flagpole's height.

If

flagpole's shadow = 45 feet
height of stick = 2 feet
stick's shadow = 3 feet.

Then:

$$\frac{\square}{45} = \frac{2}{3}$$

$$3 \times \square = 2 \times 45$$

$$3 \times \square = 90$$

$$\square = 30$$

Thus, the height of the flagpole is 30 feet.

E. Students should be permitted to determine the height of the flagpole by setting up the broom stick, measuring the shadows of the flagpole and stick and following the formula set up. To avoid difficulties with multiplying and dividing fractions, the measurements should be recorded in inches to the nearest whole inch. This also will provide practice in converting inches to feet when the result is compared with the estimate made by the students.

F. Students should test the validity of the formula in various ways:

1. Choose one or two students in the class and determine their height by using the formula. The results should be checked by measuring.
2. Help the students to develop a plan for measuring the flagpole by using the rope of the flagpole. This can be done by attaching a string or rope to the hook for the flag and running this up the flagpole. The string may be marked when one end reaches the top. Then the string may be brought down and measured.
3. Obtain the elevation prints of the building on which the height of the flagpole is recorded.
4. Some classes may want to learn to use a simple transit to check their answers.

G. The objectives may be assessed by:

1. Selecting other objects such as the school building, a telephone pole, the backstop on the baseball diamond, or a tree and determining the height of each in the same manner.
2. Having students come to the board to demonstrate how they solved problems using proportions.

VERBAL PROBLEMS - 3

Teacher Commentary

A Tape Recording of Verbal Problems for Use in Grade 9

I. Materials:

- A. Tape recorder
- B. Eight-station listening post (optional)
- C. Tape entitled, "Verbal Problems - 3"
- D. Paper
- E. Pencil and eraser

II. Procedure:

- A. Preview the tape to determine if it will be used with a small group of students, or if it will be used as a class activity.
- B. The tape has 10 problems, and is 12 minutes long.
- C. Following the presentation of each problem, the answer is given after a suitable interval.
- D. Two additional tapes of similar design are also available - Part 1 presented in grade 7 and Part 2 presented in grade 8.

GEOMETRY

GEOMETRY

I. Master Chart - Grades Six through Eleven

II. Grade Nine Chart

III. Behavioral Objectives

IV. Activities

UNIT GEOMETRY GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Point	6	6		6						
Line	6	6		6	6					
Plane	6	6		7	6					
Closed Path	6	6		6	6					6
Segment	6,7	6,7	7	6,7	6					6
Congruent Segments	9	9	9		9					
Ray	6,7	6,7		6	6					6
Angles	6,7	6,7	7	6	6					
Vertex	7	7			7					
Right Angles	6	6	9	6	6					9
Acute Angles	9	9		9	9					9
Obtuse Angles	9	9		9	9					9
Straight Angles	9	9		9	9					9
Vertical Angles	9	9		9	9	9	9			
Supplementary Angles	9	9		9	9					9
Complementary Angles	9	9		9	9					9
Congruent Angles	9	9	9	9	9					
Triangles	6,7	6,7	10	6	6					

UNIT GEOMETRY GRADE(S) Seven through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Equilateral Triangle	8	8	8	8	8					8
Isosceles Triangle	8	8	8	8	8					8
Scalene Triangle	8	8	8	8	8					8
Right Triangle	9	9	9	9	9					9
Acute Triangle	9	9		9	9					9
Obtuse Triangle	9	9		9	9					9
Perpendicular Lines	9	9	9	9	9					
Parallel Lines	7	7	9	7	7					7
Transversal	10	10		10	10					
Corresponding Angles	10	10		10		10	10, 11			
Midpoint	7	7	7		7					
Partitioning a Segment			11							
Quadrilaterals	7	7		7	7					
Trapezoid	7	7		7	7					7
Parallelogram	7	7	10	7	7	10	10, 11			7
Rectangles	7	7	10	7	7	10	10, 11			7
Square	7	7	10	7	7	10	10, 11			7
Rhombus	7	7	10	7	7	10	10, 11			7
Polygon	8	8		8	8					

UNIT GEOMETRY GRADE(S) Eight through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Pentagon	8	8	9	8	8				8	8
Hexagon	8	8	8	8	8				8	8
Octagon	8	8	8, 9	8	8				8	8
Congruent Triangles	10	10			10	10	10			
Similar Triangles	10	10			10	10	10			
Corresponding Sides of Similar Triangles						10	10, 11	10, 11		
Circle	6	6	7	6	6					
Radius	6	6	7	6	6	7				
Diameter	6	6	7	6	6	7				
Chord	7	7	7	7	7					
Tangent	8	8	10	8	8	10				8
Secant	8	8		8	8					8
Central Angle	10	10	10	10	10					10
Inscribed Angle	10	10	10	10	10	10				10
Ellipse	10	10	10	10	10	10				
Angle Bisector	9	9	9	9	9	9				
Sum of Interior Angles of Triangles						9	9			
45°			9	9						
60°			9	9						

UNIT GEOMETRY GRADE(S) Seven through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
30°			9	9						
Median Triangle	7	7	7	7	7	7				
Altitude of Triangle	9	9	9	9	9					
Cube	7	7		7	7					7
Rectangular Solid	7	7		7	7					
Pyramid	8	8		8	8					
Cone	8	8		8	8					
Cylinder	8	8		8	8					
Sphere	8	8		8	8					8
Line of Symmetry	8	8		8	8					
Sum of Interior Angles of Quadrilaterals						9	9			
Sin	11	11				11	11	11		11
Cos	11	11				11	11	11		11
Tan	11	11				11	11	11		11
Trig Tables	11	11		11				11		
Other Polyhedrons	11	11		9, 10						
Pythagorean Theorem						10	10			
Region	7	7		7	7					
Sum of Interior Angles of a Polygon						8	8			

UNIT GEOMETRY GRADE(S) Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Congruent Segments	9	9	9		9					
Right ∠'s			9							9
Acute ∠'s	9	9		9	9					9
Obtuse ∠'s	9	9		9	9					9
Straight ∠'s	9	9		9	9					9
Vertical ∠'s	9	9		9	9	9	9			
Supplementary ∠'s	9	9		9	9					9
Complementary ∠'s	9	9		9	9					9
Congruent ∠'s	9	9	9	9	9					
Right △	9	9	9	9	9					9
Acute △	9	9		9	9					9
Obtuse △	9	9		9	9					9
Perpendicular Lines	9	9	9	9	9					
Parallel Lines			9							
Pentagon			9							
Octagon			9							
∠ Bisector	9	9	9		9	9				
Sum of Interior ∠'s of △						9	9			

UNIT GEOMETRY GRADE(S) Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
45°			9	9						
60°			9	9						
30°			9	9						
Altitude of △	9	9	9	9	9					
Sum of Interior ∠'s of Quadrilaterals						9	9			
Other Polyhedrons				9						

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Congruent Segments

The student should be able to:

1. Name and identify congruent segments and their symbols
2. Demonstrate the construction of congruent segments using compass and straightedge
3. Describe congruent segments

Right Angles

The student should be able to:

1. Demonstrate the construction of a right angle using compass and straightedge
2. Distinguish between right angles and other types of angles

Acute Angles, Obtuse Angles, Straight Angles

The student should be able to:

1. Name and identify these angles
2. Construct drawings of these angles using a straightedge or freehand sketch
3. Describe these angles in relationship to a right angle
4. Distinguish among these angles

Vertical angles

The student should be able to:

1. Name and identify vertical angles
2. Construct a drawing of a vertical angle using straightedge or freehand sketch
3. Describe vertical angles in terms of their surroundings
4. State the principle that the measures of vertical angles are equal
5. Apply the principle to name the measure of one of a pair of vertical angles, when given the measure of the other, by various illustrations

Supplementary Angles, Complementary Angles

Page

The student should be able to:

1. Name and identify supplementary and complementary angles
2. Construct a drawing of these angles using a protractor
3. Construct the measure of the complement or supplement of a given angle
4. Describe these angles by various illustrations
5. Distinguish between complementary and supplementary angles

Congruent Angles

The student should be able to:

1. Name and identify the figures and symbols for congruent angles
2. Construct a drawing of congruent angles using compass and straightedge, protractor or freehand sketch
3. Demonstrate a method for constructing an angle congruent to a given angle using compass and straightedge, protractor, or tracing
4. Describe a method of determining whether angles are congruent
5. Describe and demonstrate a method for testing whether angles are congruent

Right Triangles, Acute Triangles, Obtuse Triangles

The student should be able to:

1. Name and identify these triangles
2. Construct drawings of right, acute, and obtuse triangles using straightedge, freehand sketch or protractor
3. Construct models of these triangles with available materials
4. Demonstrate a method of constructing a right triangle using straightedge, compass, or protractor

Page

5. Describe by definitions and illustrations
6. Distinguish among these triangles

Perpendicular Lines

The student should be able to: GE-13

1. Name and identify the figure and symbol for perpendicular lines
2. Construct a drawing of perpendicular lines using straightedge, freehand sketch, or protractor
3. Demonstrate a method of constructing a line perpendicular to another line from a point on the line using a straightedge and a compass or protractor
4. Demonstrate a method of constructing a line perpendicular to another line from a point off the line using straightedge and compass
5. Describe by definition or in terms of surroundings

Parallel Lines

The student should be able to:

1. Demonstrate the construction of parallel lines using a straightedge and a compass or protractor

Pentagon, Octagon

The student should be able to:

1. Demonstrate the construction of the pentagon using a protractor
2. Demonstrate the construction of the octagon using a protractor

Angle Bisector

The student should be able to:

1. Name and identify an angle bisector
2. Demonstrate a method for constructing an angle bisector using a straightedge and a compass or protractor

3. Describe the angle bisector by definition
4. State the principle that the bisector of an angle divides the angles into two congruent angles

Sum of Interior Angles of Triangles

The student should be able to:

1. State the principle that the sum of the measures of the interior angles of a triangle is equal to 180°
2. Apply this principle to construct the measure of one angle when given the measures of the other two angles of a triangle

45°, 60°, 30°

The student should be able to:

1. Demonstrate the construction of these angles by using straightedge, compass, or protractor

Altitude of a Triangle

The student should be able to:

1. Name and identify the altitude of a triangle
2. Construct a drawing of the altitude of a triangle using straightedge and compass or freehand sketch
3. Demonstrate the construction of altitude of a triangle by straightedge and compass
4. Describe the altitude of a triangle by definition

Sum of Interior Angles of Quadrilaterals

The student should be able to:

1. State the principle that the sum of the interior angles of a quadrilateral is equal to 360°
2. Apply the principle to construct the measure of one angle given the measure of the other three angles

Other Polyhedrons

Page

The student should be able to:

1. Construct models of other polyhedrons using available materials

PERPENDICULAR LINES

Teacher Commentary

I. Unit: Geometry

II. Objectives: The student should be able to:

- A. Name and identify perpendicular lines
- B. Construct a drawing of perpendicular lines
- C. Describe perpendicular lines in terms of his surroundings

III. Materials:

- A. Rectangular sheets of unlined paper
- B. Straightedges

IV. Procedure:

- A. Tell the students that they are going to learn about perpendicular lines. Write this on the board and have the class pronounce it.
- B. Distribute the unlined paper. Have each student fold his sheet so that the side edges are together. Then have them fold again so that the creased edges are together. Unfold the paper.
- C. Point out that the pair of lines formed by the creases represent perpendicular lines.
- D. Locate other examples of perpendicular lines in the room; e.g. the corners of tiles on the floor, bricks in the wall, ceiling tiles, window frames, woodwork, etc.
- E. Ask the class to cite other examples of perpendicular lines in the room or from their environment.
- F. Distribute more unlined paper to make a drawing of perpendicular lines using a straightedge. Ask them to make a drawing of two lines which are not perpendicular. Have some drawings placed on the chalkboard and discussed.
- G. Suggested assessment procedure:
  - 1. Ask the students to indicate which of the following illustrate perpendicular lines:
    - a. A tic-tac-toe game
    - b. A times sign
    - c. A plus sign
    - d. A checker board

2. Show the class pictures from magazines. Have them identify the perpendicular lines in the pictures.

3. Distribute a work sheet which contains drawings of perpendicular and non-perpendicular lines. Have the students identify those which are perpendicular.

4. Ask students to describe perpendicular lines in terms of their surroundings and to make a drawing of a design which uses perpendicular lines.

MEASUREMENT

MEASUREMENT

I. Master Chart - Grades Six through Eleven

II. Grade Nine Chart

III. Behavioral Objectives

IV. Activities

UNIT MEASUREMENT GRADE(S) Seven through Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Area	7	7	7			7				
Square Inch	7	7		7	7	7				
Square Foot	7	7		7	7	7	7			
Square Yard	7	7		7	7	7	7			
Area of Square	7	7	7	7		7	7	7		7
Area of Rectangle	7	7	7	7		7	7	7		7
Area of Triangle	8	8	8		8	8	8			8
Area of Parallelogram	8	8	8		8	8	8			8
Area of Circle	8	8	8		8	8	8			8
Area of Other Polygons			9		9	9	9	9		
Acre	9	9	9		9	9				
Surface Area	9	9	9			9				
Total Surface Area of Cube	9	9			9	9	9			
Lateral Surface Area of Cube	9	9			9	9	9			
Total Surface Area of Box	9	9	9			9	9			
Lateral Surface Area of Box	9	9	9			9	9			
Total Surface Area of Cylinder	9	9	9			9	9			
Lateral Surface Area of Cylinder	9	9	9			9	9			

UNIT **MEASUREMENT** GRADE(S) **Six through Nine**

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Inch	6	6	6	6	6	6	6		6	
1/2 Inch	6	6	6	6	6		6		6	
1/4 Inch	7	7	7	7					7	
1/8 Inch	8	8	8							8
1/16 Inch	9	9	9							9
Foot	6	6	6	6	6	6	6		6	
Yard	6	6	6	6	6	6	6		6	8
Meter	8	8	8	8	8				8	8
Centimeter	8	8	8			8	8		8	9
Millimeter	9	9	9			9	9		9	9
Mile	7	7				7	7			
Perimeter	7	7	7			7	7			
Perimeter Square	7	7	7	7		7	7	7		
Perimeter Triangle	7	7	7	7		7	7	7		
Perimeter Rectangle	7	7	7	7		7	7	7		
Perimeter Polygons			8	8		8	8	8		
Circumference	8	8	8			8				
Pi	8	8			8	8	8			

UNIT MEASUREMENT GRADE(S) Six through Ten

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Micrometer	10	10	10							
Caliper	10	10	10							
Volume	10	10	10			10				
Cubic Inch	10	10		10	10					
Cubic Foot	10	10		10	10	10	10			
Cubic Yard	10	10		10	10	10	10			
Volume of Cube	10	10	10			10	10	10		
Volume of Rectangular Solid	10	10	10			10	10	10		
Volume of Cylinder	10	10	10			10	10	10		
Angular Measurement	9	9				9				
Degree	9	9			9	9	9	9		
Protractor	9	9	9	9	9	9	9	9		
Central Angle	10	10	10	10		10	10			10
Inscribed Angle	10	10	10	10		10	10			10
Corresponding Angles	10	10		10		10	10	10		
Ounce	6	6	6			6	6	6	6	
Pound	6	6	6			6	6	6	6	
Pint	6	6	6			6	6	6	6	

UNIT MEASUREMENT

GRADE(S) Six through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Quart	6	6	6			6	6	6	6	
Fluid Ounce	10	10	10			10	10	10	10	
Gallon	6	6	6			6	6	6	6	
Cup	10	10	10			10	10	10	10	
Degree Fahrinheit	6	6	6			6	6	6	6	
Degree Centigrade	7	7	7			7	7	7	7	
Similar Triangles (Ratio, Proportion)	10	10	10		10	10	10	10		
Scale Drawing			10	10	10	10		10		
Trig Tables	11	11		11	11					
Pythagorean	10	10	10			10	10	10		

UNIT MEASUREMENT GRADE(S) Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
$\frac{1}{16}$ Inch	9	9	9							9
Millimeter	9	9	9			9	9		9	9
Area of Other Polygons			9		9	9	9	9		
Acre	9	9			9					
Surface Area	9	9	9			9				
Total Surface Area of Cube	9	9			9	9	9			9
Lateral Surface Area of Cube	9	9			9	9	9			9
Total Surface Area of Rectangular Solid	9	9	9			9	9			9
Lateral Surface Area of Rectangular Solid	9	9	9			9	9			9
Total Surface Area of Cylinder	9	9	9			9	9			9
Lateral Surface Area of Cylinder	9	9	9			9	9			9
Angular Measurement	9	9				9				
Degree	9	9			9	9	9	9		
Protractor	9	9	9	9	9	9	9	9		

$\frac{1}{16}$ Inch

Page

The student should be able to:

1. Name and identify the $\frac{1}{16}$ inch unit
2. Demonstrate how to measure an object to the nearest $\frac{1}{16}$ inch
3. Distinguish the $\frac{1}{16}$ inch markings on a ruler from the $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ inch

Millimeter

The student should be able to:

1. Name and identify the millimeter unit of length on a meter stick
2. Demonstrate how to measure an object to the nearest millimeter
3. State the principle:

   1000 millimeters = 100 centimeters = 1 meter
4. Apply the principle by converting from meters to centimeters to millimeters and vice versa
5. Order millimeter, centimeter, and meter by arranging them according to size
6. Distinguish between a $\frac{1}{16}$ inch and a millimeter

Area of Other Polygons

The student should be able to:

1. Demonstrate how to measure the interior region of polygons by partitioning into familiar polygons
2. Describe the method of finding the area of polygons by partitioning into triangles and rectangles
3. State the principle that area of the interior region of a polygon is the sum of areas of the partial regions
4. Apply the principle by finding the areas of polygons
5. Interpret the principle by solving applied problems

Acre

Page

The student should be able to:

1. Name and identify an acre as a unit of land measure
2. Demonstrate an acre by measuring one on the school grounds
3. Describe an acre in terms of his surroundings
4. State the principle that an acre is an area that measures approximately 208' x 208'

Surface Area of a Polyhedron

The student should be able to:

1. Name and identify the surface area of a polyhedron
2. Demonstrate surface area by using models
3. State the principle that the surface area of a polyhedron is the sum of the area of the faces

Total and Lateral Surface Area of a Cube

The student should be able to:

1. Name and identify the total and lateral surface area of a cube
2. Describe the surface area of a cube by using models
3. State the principles:
    a. Total surface area = sum of the areas of the faces
    b. Lateral surface area = sum of the areas of the faces, excluding the ends
4. Apply the principles by finding the total and lateral surface areas of cubes when given their dimensions
5. Distinguish between lateral and total surface areas of a cube

Total and Lateral Surface Areas of a Rectangular Solid

M-17

The student should be able to:

1. Name and identify the total and lateral surface area of a box (rectangular solid)
2. Demonstrate tht total and lateral surface area by using physical models

Page

3. State the principles:
    a. Total surface area = sum of the areas of the faces
    b. Lateral surface area = sum of the areas of the faces excluding the ends
4. Apply the principles by finding total and lateral areas of boxes when given their dimensions
5. Distinguish between lateral and total surface areas of a box

Total and Lateral Surface Area of a Cylinder

The student should be able to:

1. Name and identify the total and lateral surface area of a cylinder
2. Demonstrate the total and lateral surface area of a cylinder by using models
3. State the principles:
    a. Total surface area = two times the area of the base and the area of the lateral surface
    b. Lateral surface area = height of cylinder and circumference of base
4. Apply the principles by finding the total and lateral surface areas of cylinders when given their dimensions
5. Distinguish between lateral and total surface areas of a cylinder

Angular Measurement

The student should be able to:

1. Name and identify an angle
2. State the principle that the measure of an angle is the number of congruent unit angles which can be placed in the interior of the angle

Degree and Protractor

The student should be able to:

1. Name and identify the degree and protractor
2. Demonstrate how to measure an angle with a protractor and how to draw an angle given its measure

	Page
3. Construct a model of a protractor using an original unit	M-11
4. Describe the protractor as an instrument to measure angles or in terms of its physical appearance	
5. State the principles:	
a. There are 360° around a point	M-13
b. The protractor is divided into 180 congruent angles, each called a degree	
6. Apply the principles of the protractor by measuring angles	
7. Interpret the principles of angle measure by measuring and then classifying angles as right, acute, obtuse, straight, complementary, and supplementary	

ANGULAR MEASUREMENT

Teacher Commentary

I. Unit: Measurement

II. Objectives: The student should be able to:

- A. Name and identify an angle
- B. State the principle that the measure of an angle is the number of congruent unit angles which can be placed in the interior of the angle

III. Materials:

- A. Scissors
- B. Work sheet entitled, "Angular Measurement"
- C. Overhead projector and overlays

IV. Procedure:

- A. Review the following principles through questioning.
  1. When we measure line segments we use a line segment as a unit of measure.
  2. When we measure the area of a plane figure we use a plane region as a unit of measure.
- B. Review the definition of an angle.
  1. Identify angles by choosing from a group of geometric figures drawn on the chalkboard.
  2. Draw some angles on the chalkboard. Have students name these angles.
- C. Pose the problem: What type of geometric figure should we use to measure an angle?
- D. Discuss the possibilities and explain that we must use an angle to measure an angle.
- E. Use the overhead projector and overlays to demonstrate how to find the measure of an angle using a unit angle. Do not use a protractor.

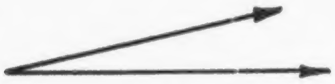
F. Distribute the work sheet entitled, "Angular Measurement." Distribute the scissors and have students complete problems 1-4. Discuss the results and explain the principle stated in Objective B.

G. Have students complete problem 5 which uses a different unit. Discuss their results and emphasize the principle of Objective B.

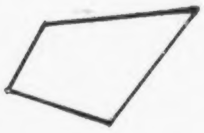
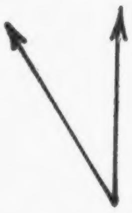
H. Suggested assessment procedures:

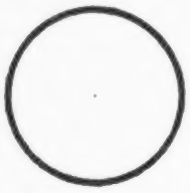
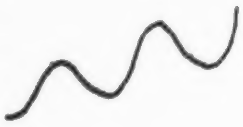
   1. Written exercises which could be used in a beginning drill.

      a. What is the name of this figure?



      b. Pick out the figures which are angles.

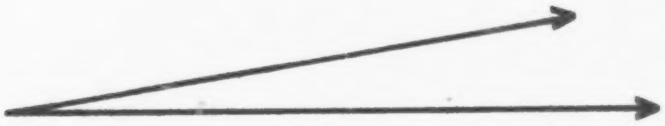



      c. What is the measure of an angle?

   2. An oral exercise could be employed with students explaining the principle in their own words.

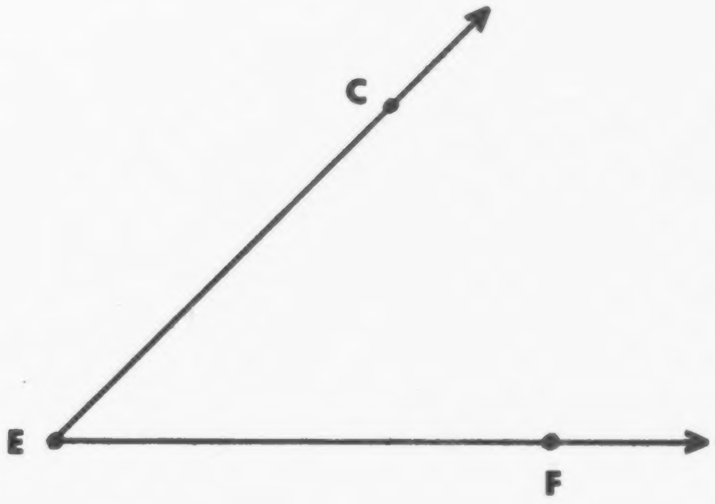
ANGULAR MEASUREMENT

1. Use this angle as a unit. Let's call this unit of measure a "zap."



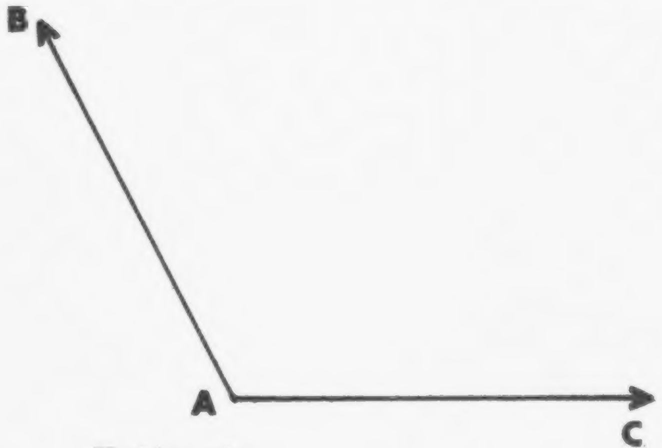
1 zap

2. Cut out this angle.
3. Now measure the following angle. Find out how many "zaps" you must place in the angle to fill it up. Remember that the vertices must meet and the zaps must be placed side by side.



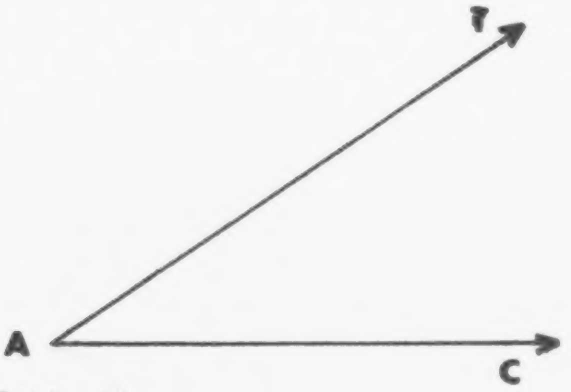
Measure ∠ CEF = _______ zaps

4. What do these angles measure to the nearest zap? First extimate your answer and then measure by using the "zap" as the unit of measure.



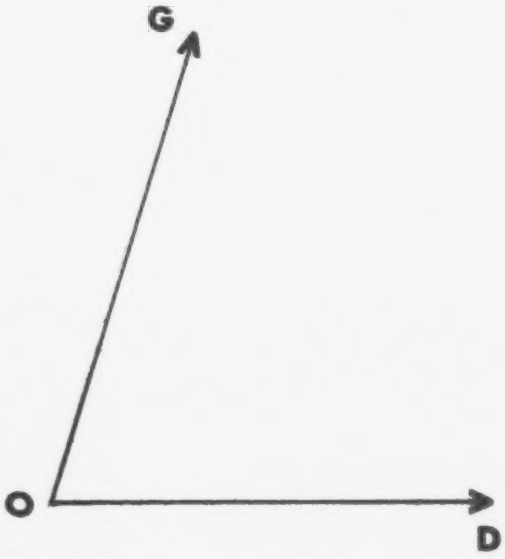
Estimate __________

Measure __________



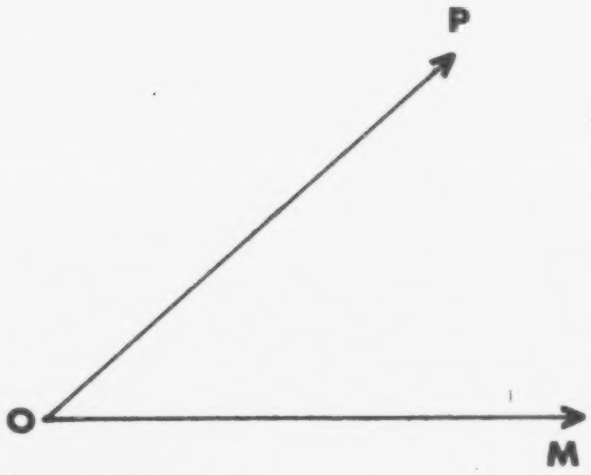
Estimate __________

Measure __________



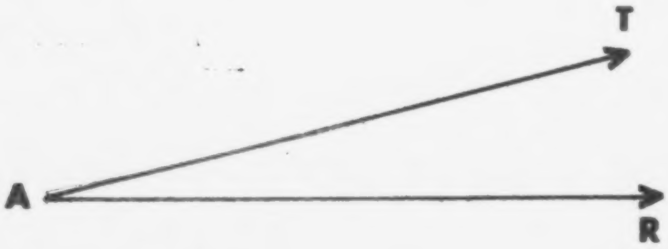
Estimate __________

Measure __________



Estimate __________

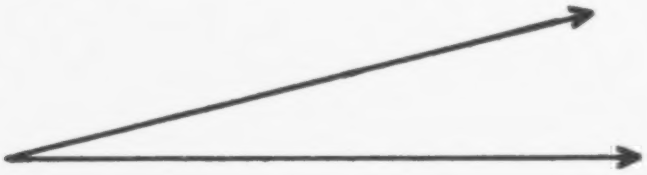
Measure __________



Estimate __________

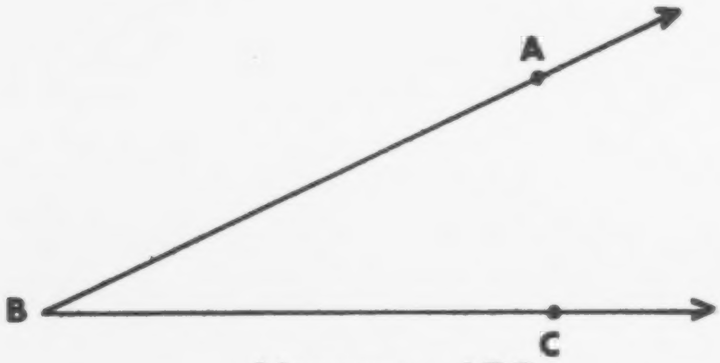
Measure __________

5. Now use this angle as a unit. Let's call this unit of measure a "dek." Cut out this angle and use it to measure the following angles to the nearest "dek."



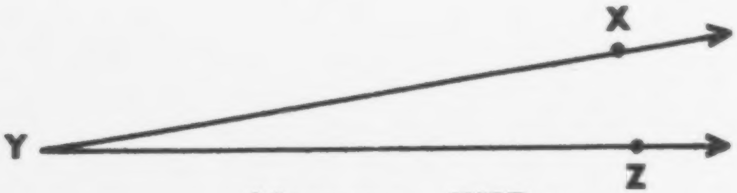
1 dek

a.



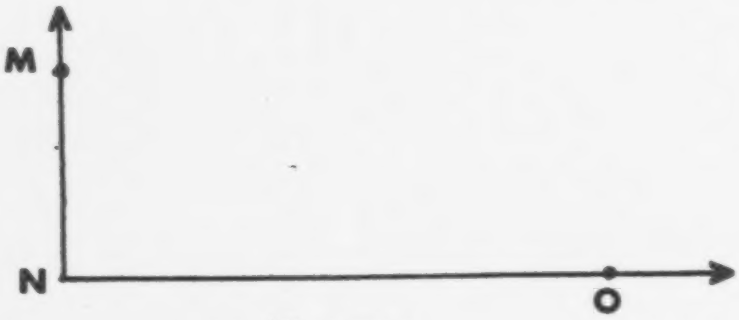
Measure ABC = ________

b.



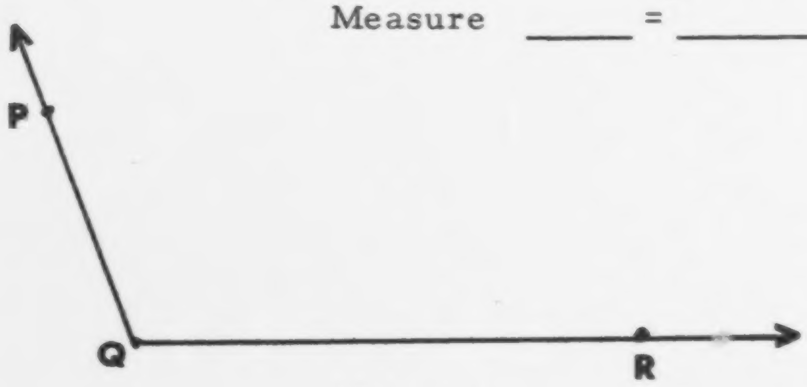
Measure XYZ = ________

c.



Measure _____ = ________

d.



Measure _____ = ________

COVERING THE PLANE

Teacher Commentary

I. Unit: Measurement

II. Objectives: The student should be able to:

- A. Apply the principle of the protractor by measuring angles
- B. State the principle that there are 360° around a point

III. Materials:

- A. Student work sheets entitled, "Covering the Plane"
- B. Scissors

IV. Procedure:

- A. Review regular polygons.
- B. Review the plane.
- C. Review the term "vertices."
- D. Distribute student work sheet.
- E. Discuss the problem on the work sheet.
- F. Do the part on equilateral triangles with the students. The teacher should have a large model for demonstration purposes.
- G. Have the students do parts B, C, D, and E. If necessary, the teacher should give help to the students.
- H. Assessment:
  1. Written Exercises

     Part F numbers 1 through 6.
  2. Oral Exercises
     - a. Give students models of regular monagons and ask them to check to see if they can be used to cover the floor.
     - b. Ask the students to use the equilateral triangles from Part A to try to cover the plane without putting all six vertices together.

I. Summary:

1. The only regular polygons which can be used to cover the plane are equilateral triangles, squares and regular hexagons.
2. There are 360° around a point.
3. The measure of each interior angle must be a factor of 360°.

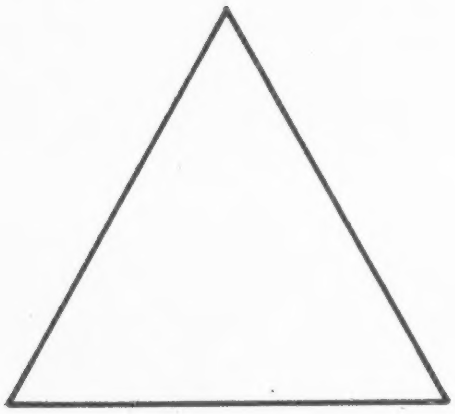
COVERING THE PLANE

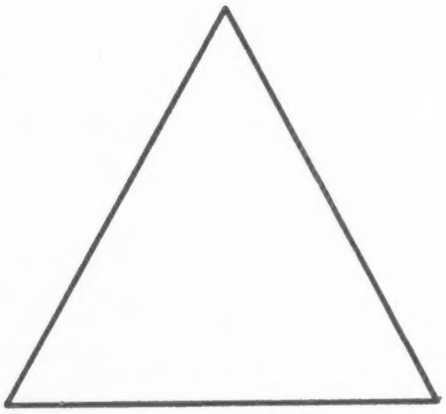
Joe wanted to cover the bathroom floor with tile. He decided to use tile which is in the shape of regular polygons. In a regular polygon all the sides have the same length and all the angles have the same measure. Joe also decided that he would place the vertices together.

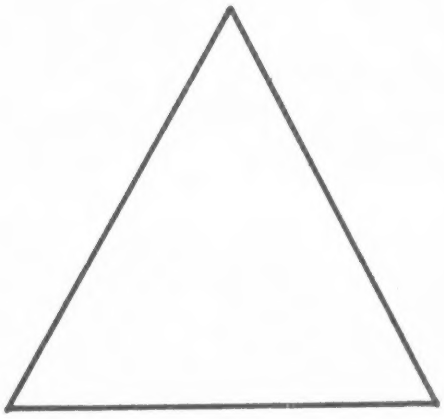
Which of the regular polygons can be used to completely cover the floor when the vertices are placed together? To answer this question, let's examine the following regular polygons.

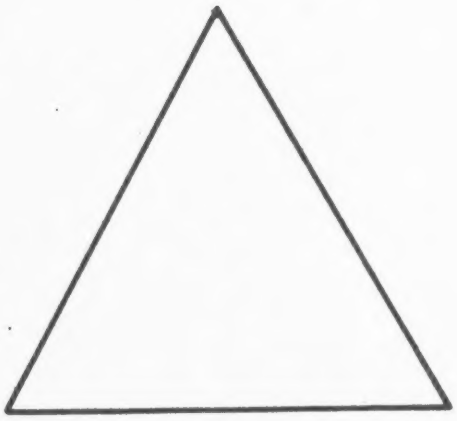
A. Equilateral Triangles

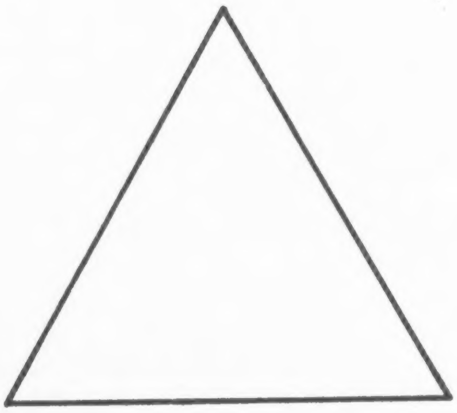
1. Cut out the triangles.
2. Place the vertices together.
3. Is the area completely covered where the vertices meet? ________
4. What is the measure of each angle? ________________
5. How many angles meet at a common vertex? ________________
6. What is the sum of the measures of all the angles which meet at the common vertex? ________________
7. Can Joe use tiles in the shape of equilateral triangles to cover the floor? ____________

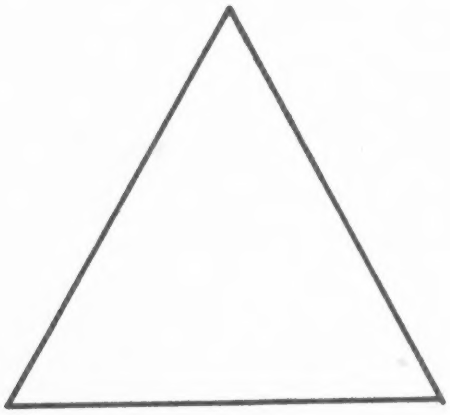






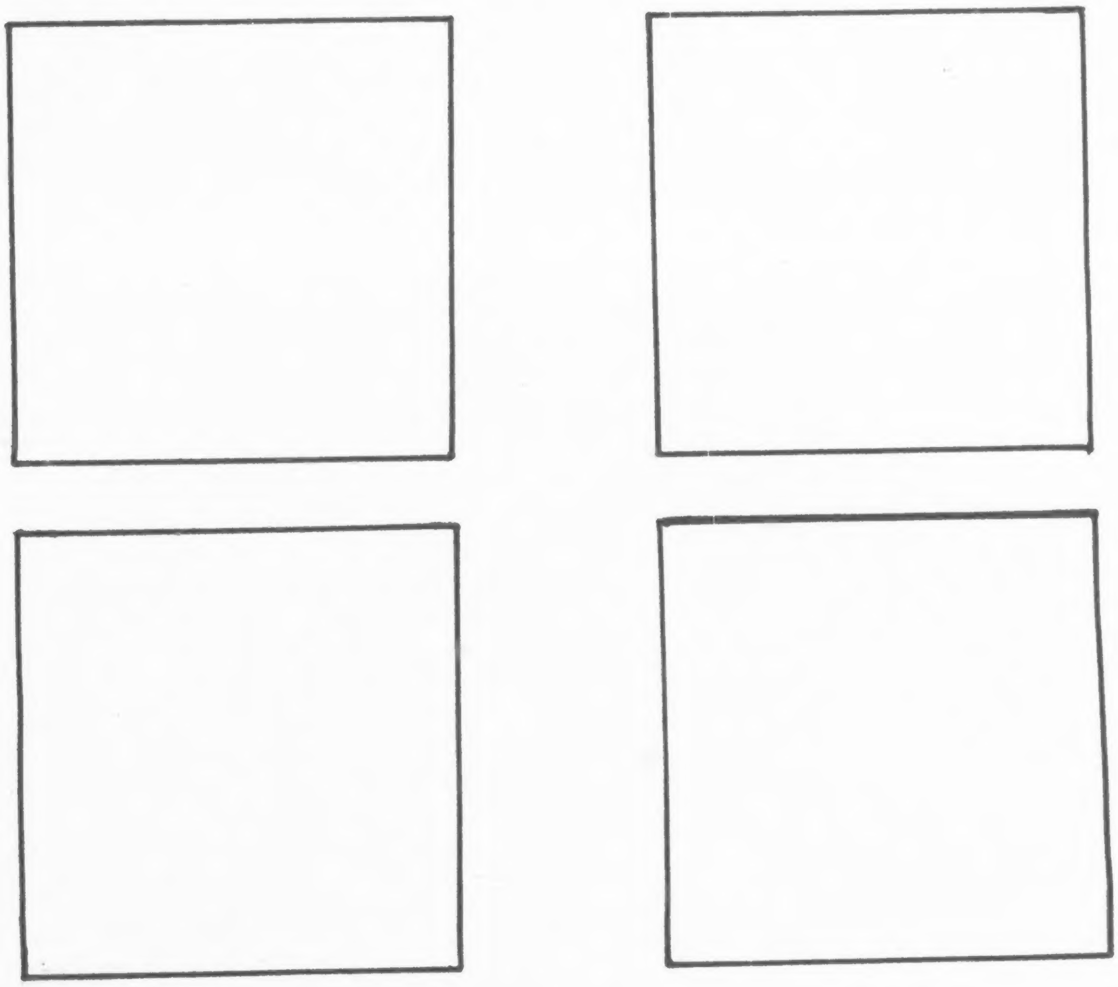






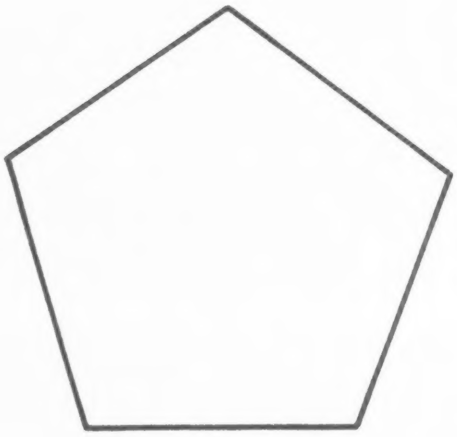
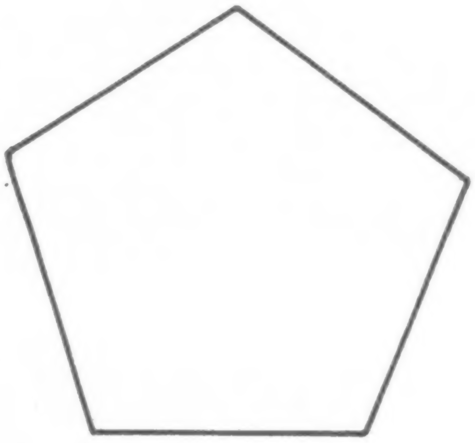
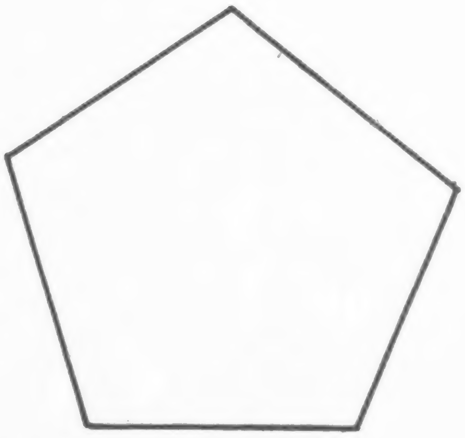
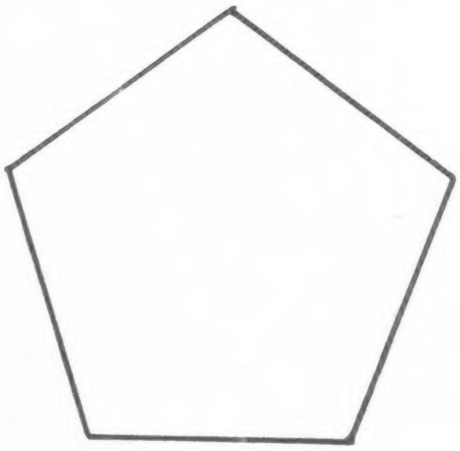
B. Squares

1. Cut out the squares.
2. Place the vertices together.
3. Is the area completely covered where the vertices meet? ____________
4. What is the measure of each angle? ______________________________
5. How many angles meet at a common vertex? ______________________
6. What is the sum of the measures of all the angles which meet at a common vertex? _______________________
7. Can Joe use square tiles to cover the floor? _______________



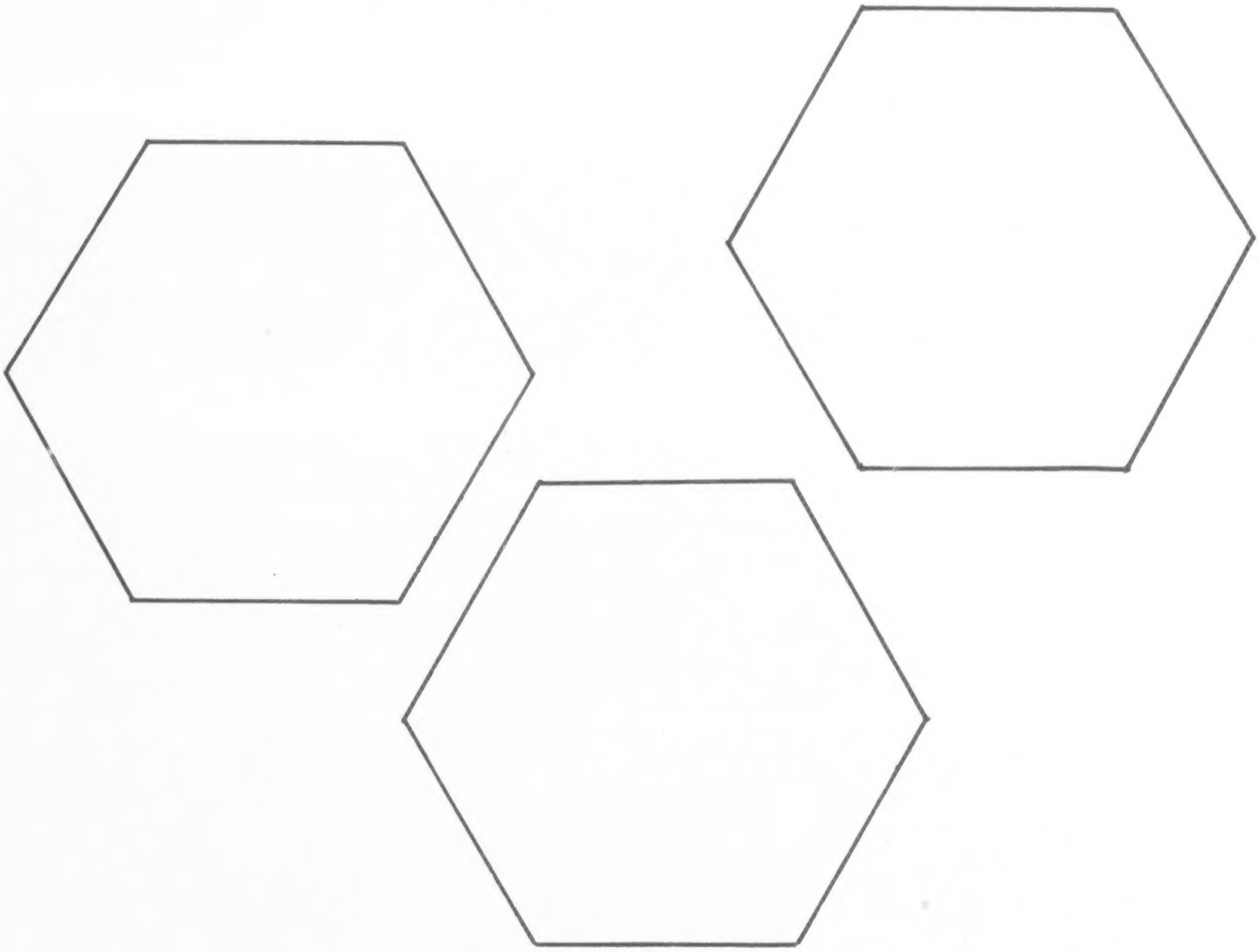
C. Regular Pentagons

1. Cut out the pentagons.
2. Place the vertices together.
3. Is the area completely covered where the vertices meet? ____________
4. What is the measure of each angle? ____________
5. What is the sum of the measures of three angles? ____________
6. What is the sum of the measures of four angles? ____________
7. How many degrees are around a point in a plane? ____________
8. Can Joe use tiles in the shape of regular pentagons to cover the floor? ____________

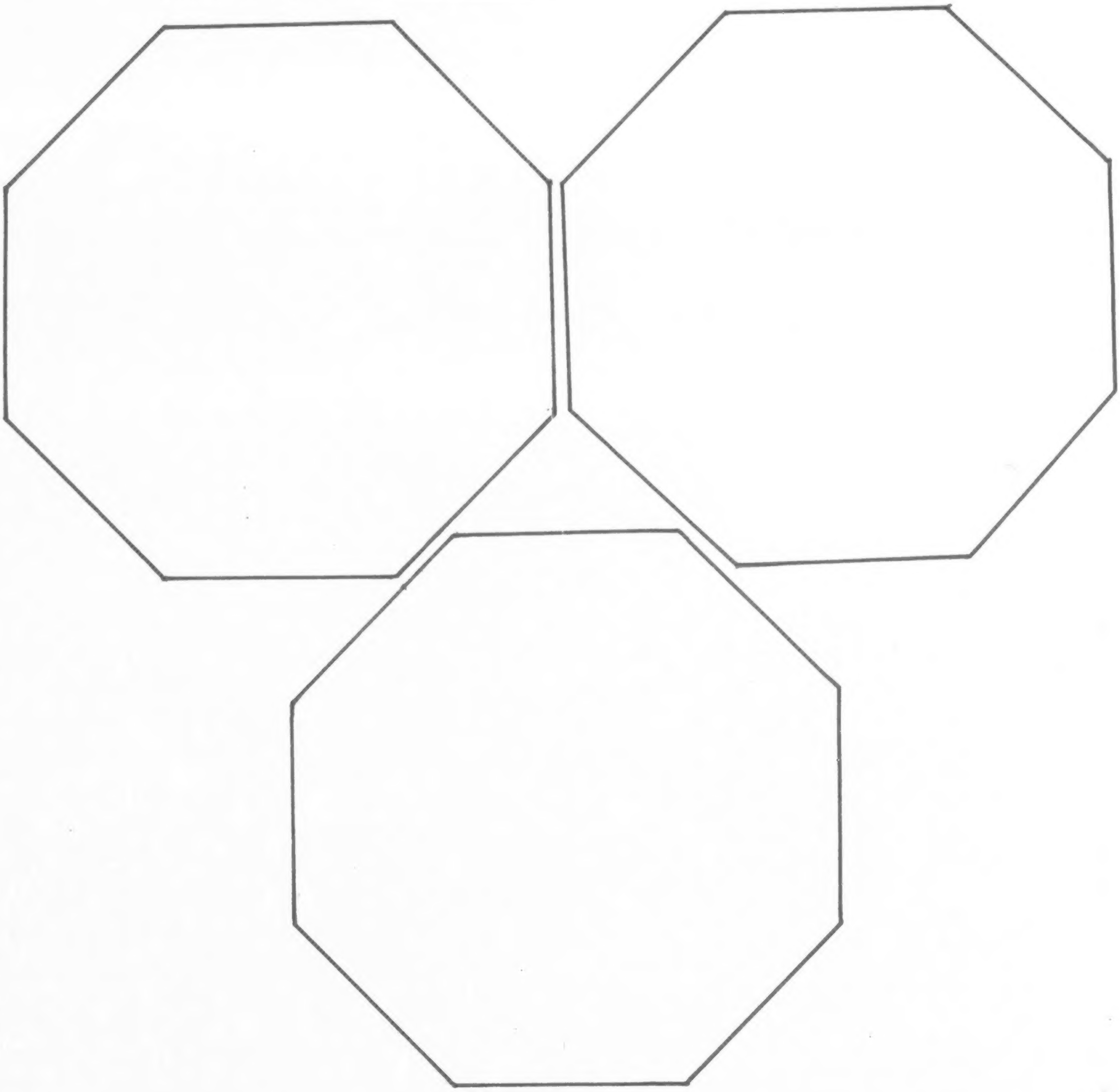
D. Regular Hexagons

1. Cut out the hexagons.
2. Place the vertices together.
3. Is the area completely covered where the vertices meet? __________
4. What is the measure of each angle? __________
5. How many angles meet at a common vertex? __________
6. What is the sum of the measures of three angles? __________
7. Can Joe use tiles in the shape of regular hexagons to cover the floor? __________



E. Regular Octagons

1. Cut out the octagons.
2. Place the vertices together.
3. Is the area completely covered where the vertices meet? ______________
4. What is the measure of each angle? ______________
5. What is the sum of the measures of three angles? ______________
6. What is the sum of the measures of four angles? ______________
7. Can Joe use tiles in the shape of regular octagons to cover the floor? ______________



F. 1. Which of the regular polygons cover the floor? ____________________

2. Do you think there might be other regular polygons that can be used? ________________

3. Explain your answer to problem 2.

____

4. Construct four rectangles which have dimensions of 1" x 2". Place the four vertices together. What is the sum of the measures of the four angles? ________________ Can rectangles be used to cover the floor? ________________

5. Use the four rectangles which you constructed in problem 4 and try to cover the floor without placing four vertices together. Can rectangles be used for covering the floor without putting four vertices together? ______________________ (Hint: Examine some brick work.)

6. Each angle of a regular decagon (10 sides) has a measure of 144°. Can regular decagons be used to cover a floor? ________________

TOTAL AND LATERAL SURFACE AREA OF A CUBE

Teacher Commentary

I. Unit: Measurement

II. Objectives: The student should be able to:

- A. Name and identify the total and lateral surface area of a cube
- B. Describe the surface area of a cube by using models
- C. State the principles:
  - 1. Total surface area = sum of the areas of the faces
  - 2. Lateral surface area = sum of the areas of the faces, excluding the ends
- D. Apply these principles by finding the total and lateral surface areas of cubes when given their dimensions
- E. Distinguish between lateral and total surface areas of a cube

III. Materials:

- A. Models of 6 cubes of the following dimensions. Each of the surfaces should be marked off in one inch squares.
  - 1. 2 x 2 x 2
  - 2. 3 x 3 x 3
  - 3. 4 x 4 x 4
  - 4. 5 x 5 x 5
  - 5. 6 x 6 x 6
  - 6. 7 x 7 x 7
- B. Student work sheet entitled, "Total and Lateral Surface Area of a Cube."

IV. Procedure:

- A. Review the definition of a cube.
- B. Have students construct cubes of given dimensions. (See the topic "Construction of the Five Regular Polyhedrons.")
- C. Review the definition of area and square units and how to find the area of a square.

D. 1. Use several of the models to demonstrate the meaning of total and lateral area of a cube. Have several students perform a similar demonstration with the remaining models.

   2. Pose the problem: Can you figure out a method for finding the total and lateral area of a cube?

   3. Divide the class into teams of 2 and give them sufficient time to solve the problem. If they seem to flounder, provide additional clues such as:

      a. Can you find the area of one face?

      b. Can you find the area of two faces?

   4. Discuss the suggested solutions to the problem and agree on one that is acceptable. Do not force the students to state their solutions in your terms.

E. Summarize the solutions on the board.

   1. Total Surface Area = sum of the areas of all the faces.

   2. Lateral Surface Area = sum of the areas of the faces, excluding the ends.

F. Divide the class into six groups.

G. Distribute student work sheet entitled, "Total and Lateral Surface Area of a Cube."

H. Give one cube to each group.

I. Have students pass the cubes to the next group when they have completed the information for the chart.

J. After each group has worked with each cube, discuss the results of the chart.

K. Assessment:

   1. Written exercises

      a. Do problems 2 through 7 on the work sheet entitled, "Total and Lateral Surface Area of a Cube."

   2. Oral exercises

      a. Give the students models and ask them to describe the surface area of a cube.

      b. With models ask the students to distinguish between the total and lateral surface area.

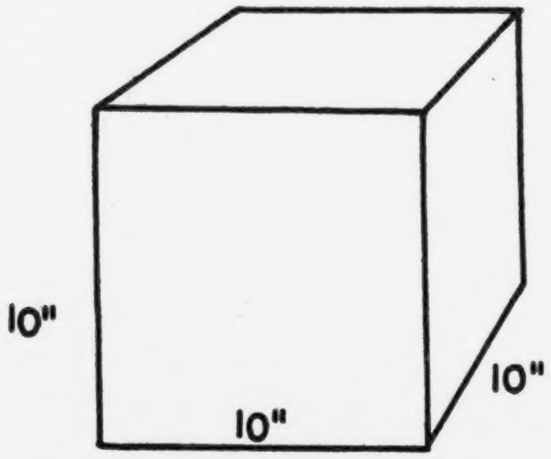
      c. Ask the students to state the principles in their own words.

TOTAL AND LATERAL SURFACE AREA OF A CUBE

1.

Size of Cube	2·2·2	3·3·3	4·4·4	5·5·5	6·6·6	7·7·7	r·r·r
No. of square units painted (Total Surface Area)							
Lateral Surface Area							

2. Find the total and lateral surface area of the cube drawn below.



Total Surface Area = ___________

Lateral Surface Area = _________

3. Suppose each side of a cube is doubled in length. Does the total surface area double? Hint: Look in the chart at the columns 2·2·2 and 4·4·4.

4. Suppose each side of a cube is doubled in length. Does the lateral surface area double? Use the hint from exercise 3.

5. Find the total surface area of a cube which measures 8" on each side.

6. Find the lateral surface area of a cube which measures 8" on each side.

7. Bill is building a stall for his horse. The stall will be in the shape of a cube. It measures 12' on each edge. How many square feet will Bill have to paint to cover the sides of the stall?

CAN YOU SEE AROUND CORNERS?

Teacher Commentary

I. Unit: Measurement

II. Objectives: The student should be able to:

- A. Name and identify the total surface area of a rectangular solid
- B. Demonstrate how to find the total surface area by using models
- C. State the principle:

  Total surface area equals the sum of the areas of the faces

- D. Apply the principle by finding the total surface area of boxes when given their dimensions

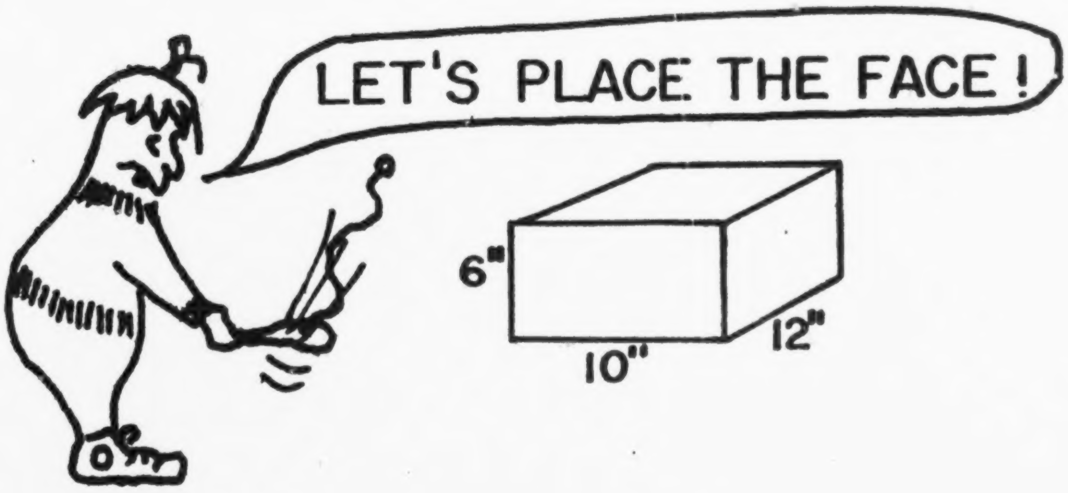
III. Materials:

- A. Work sheet entitled, "Can You See Around Corners?"
- B. Models of rectangular solids

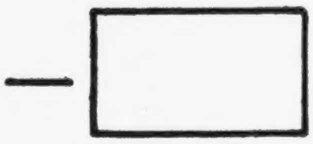
IV. Procedure:

- A. The lesson can be motivated by using the title page of the work sheet.
- B. Using the models emphasize the properties of rectangular solids. (The shapes of the visible faces will often provide clues to the shapes of the hidden faces.)
- C. Distribute student work sheets.
- D. Page 1 of the work sheets should be completed as a class activity. (Use models for verification of the size of the faces.)
- E. Students should complete pages 2 and 3 on their own.
- F. Check and discuss the results of pages 2 and 3.
- G. Assessment - Page 4 of the work sheets.
- H. The teacher may find that some students were not able to do page 4 while others were. The teacher should then divide the class into two groups. For those students who encountered difficulty in completing page 4, additional problems of the same type should be given. Those students who completed page 4, should be encouraged to do the enrichment topics included on pages 5, 6, and 7.

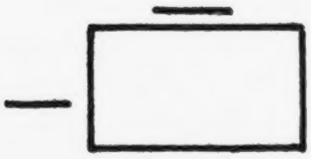
Can you see around CORNERS?



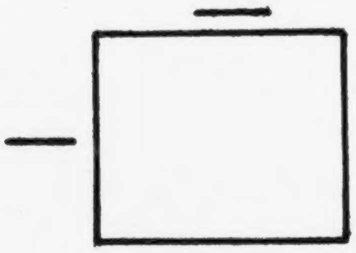
Area



Look at the front face.
Label its dimensions.
Find the area. ____________

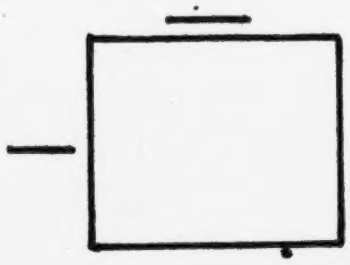


This is the back face.
Label the dimensions.
Find the area. ____________



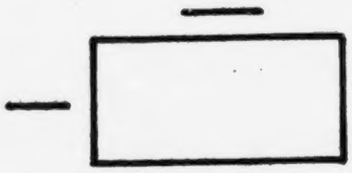
Look at the top face.
Label its dimensions.

Find the area. ____________

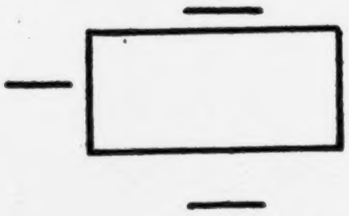


This is the bottom face.
Label its dimensions.

Find the area. ____________

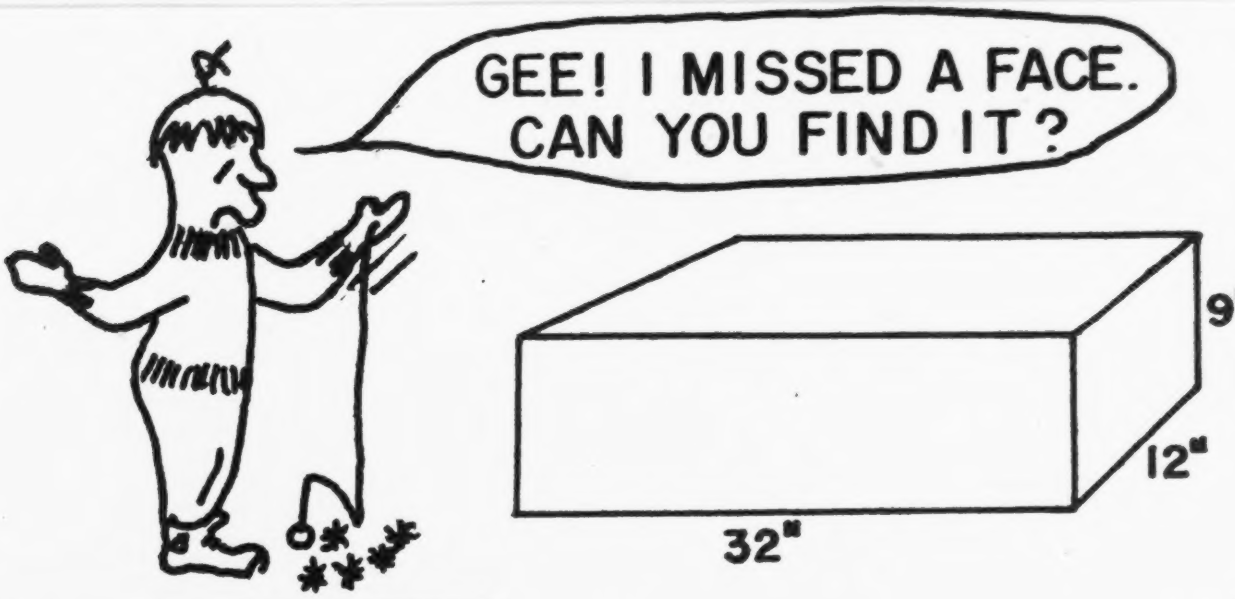


Look at the right face.
Label its dimensions.
Find the area. ____________



This is the left face.
Label its dimensions.
Find the area. ____________

Total Surface Area ____________

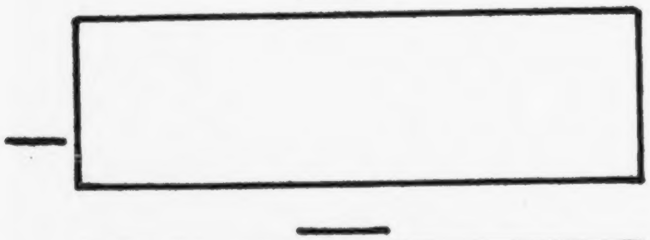

GEE! I MISSED A FACE. CAN YOU FIND IT?
9"
12"
32"

Area

This is the front face.

Label its dimensions.

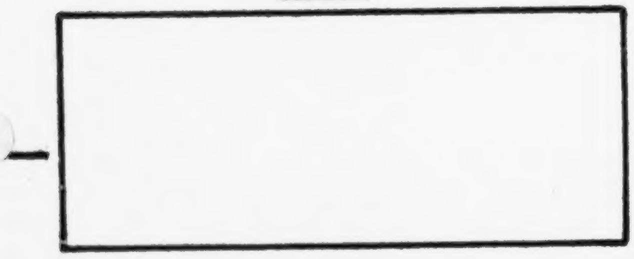
Area ______________ ______________



This is the ________ face.

Label its dimensions.

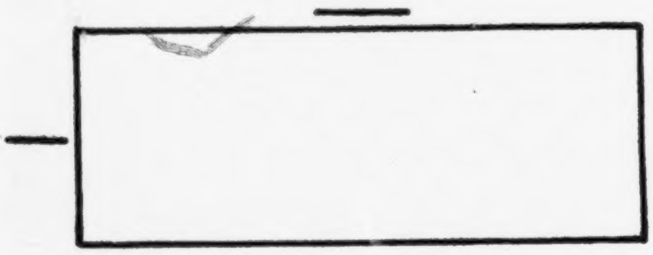
Area ______________ ______________



This is the top face.

Label its dimensions.

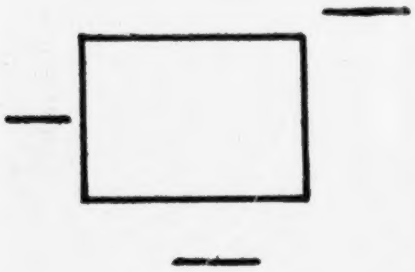
Area ______________ ______________



This is the ________ face.

Label its dimensions.

Area ______________ ______________



This is the right face.

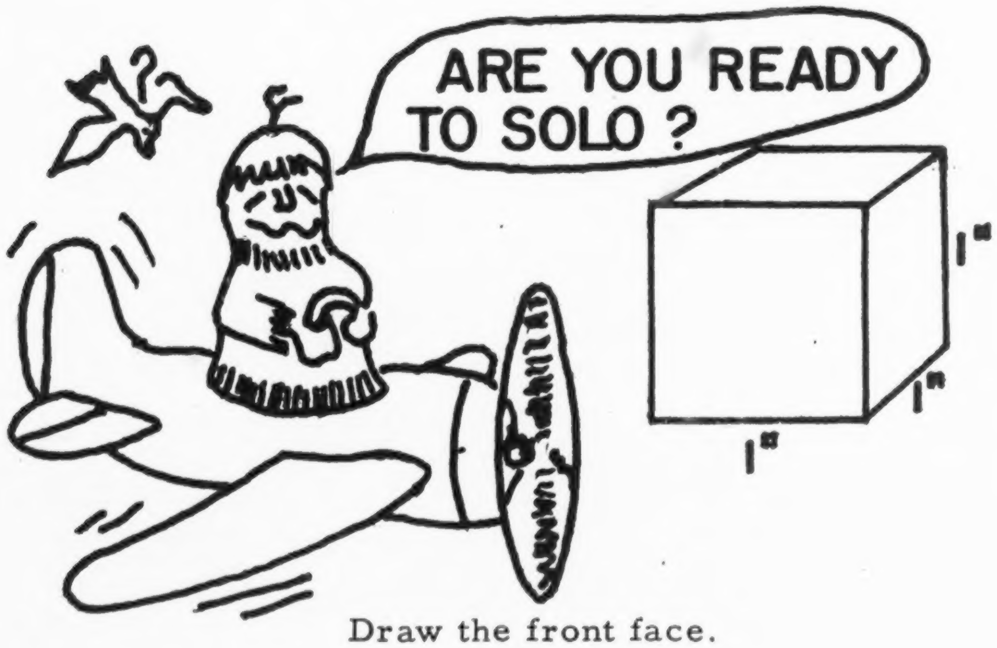
Label its dimensions.

Area ______________ ______________

PUT THE MISSING ONE HERE.

This is the ________ face.
Label its dimensions.
Area ______________ ______________

Total Surface Area ______________


ARE YOU READY TO SOLO ?
1"
1"
1"

Area

Draw the front face.

Label its dimensions.

Area ________________ ______________

Draw the rear face.

Label its dimensions.

Area ________________ ______________

Draw the right face.

Label its dimensions.

Area ________________ ______________

Draw the left face.

Label its dimensions.

Area ________________ ______________

Draw the top face.

Label its dimensions.

Area ________________ ______________

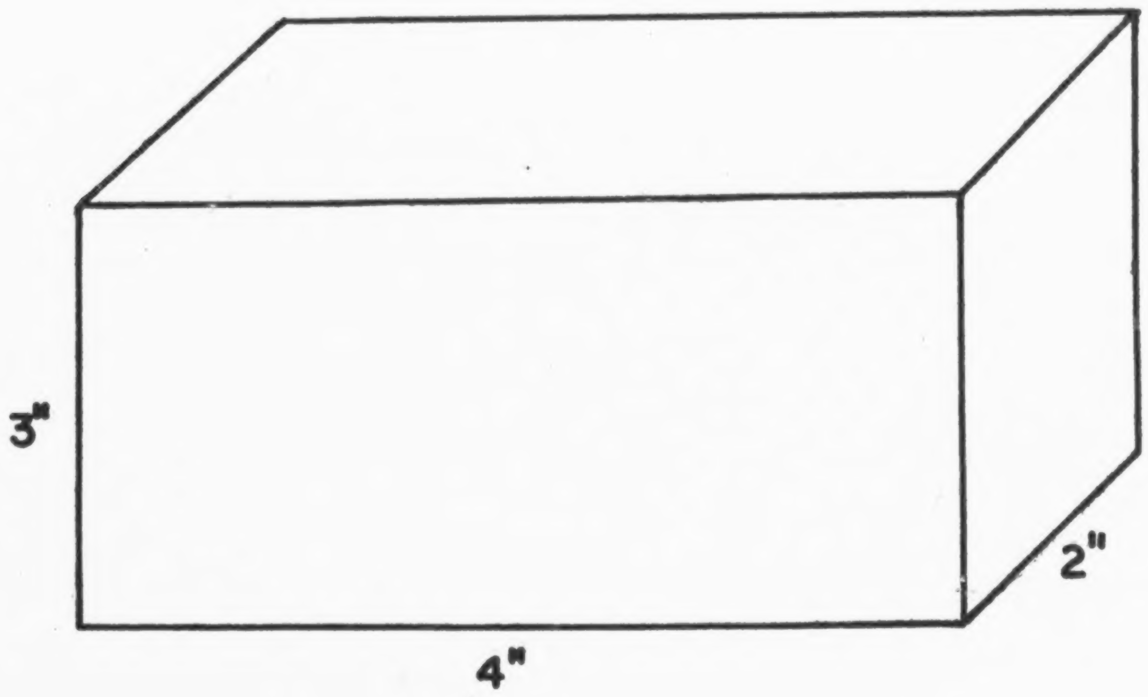
Draw the bottom face.

Label its dimensions.

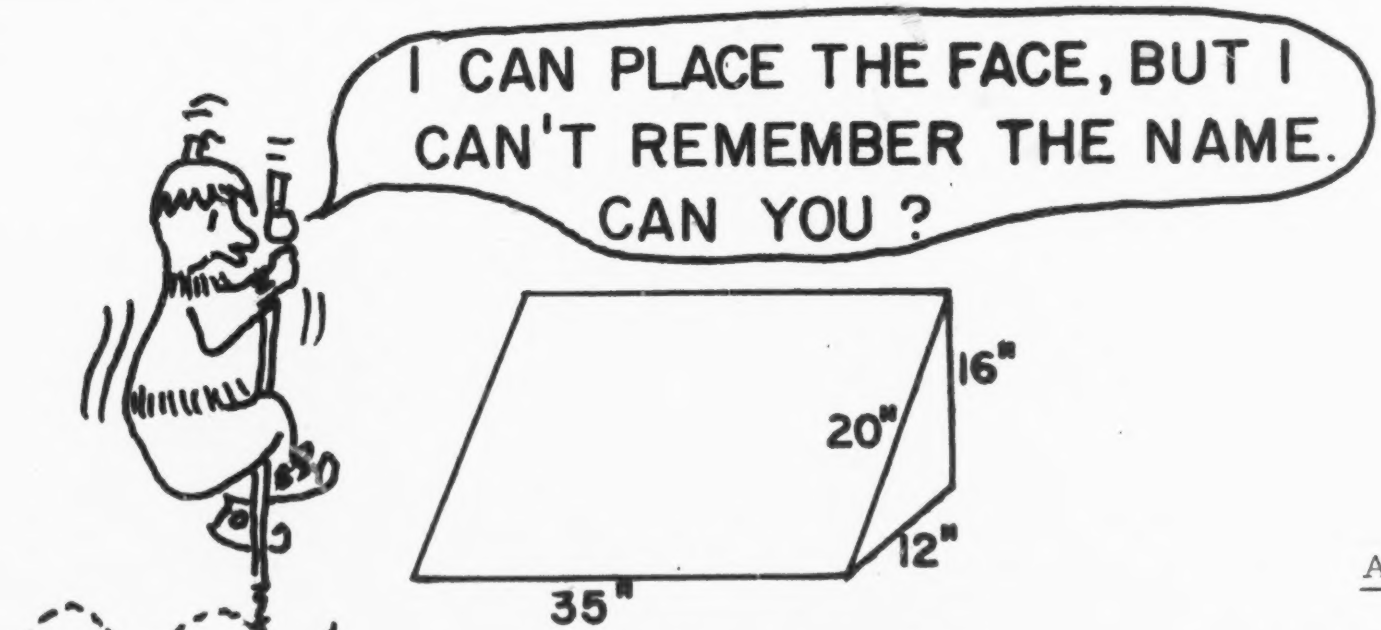
Area ________________ ______________

Total Surface Area ______________

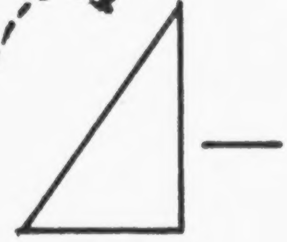
A. Find the total surface area of the following figure.



B. Explain the method you used in finding the total surface area.


I CAN PLACE THE FACE, BUT I CAN'T REMEMBER THE NAME. CAN YOU ?
16"
20"
12"
35"

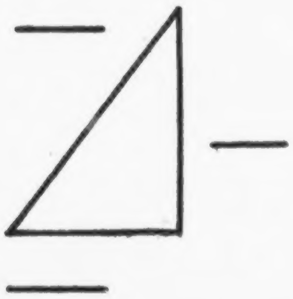
Area



This is the _______ face.

Label its dimensions.

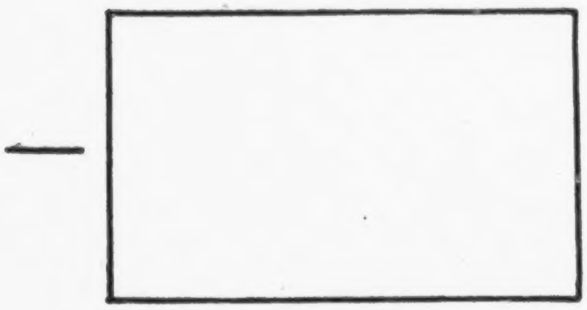
Area _______________ _______________



This is the _______ face.

Label its dimensions.

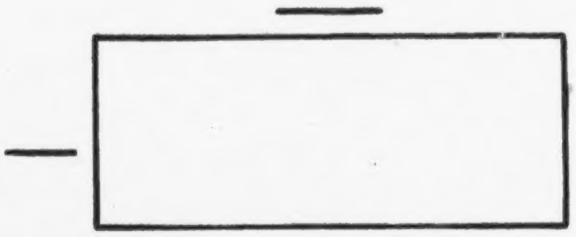
Area _______________ _______________



This is the _______ face.

Label its dimensions.

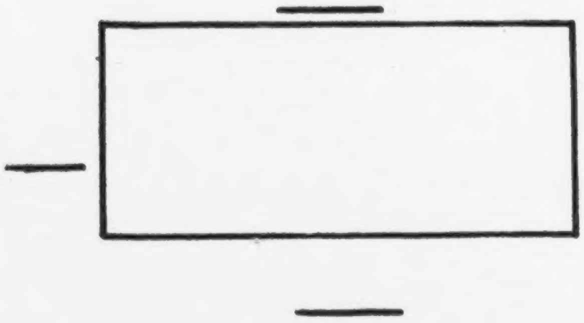
Area _______________ _______________



This is the _______ face.

Label its dimensions.

Area _______________ _______________

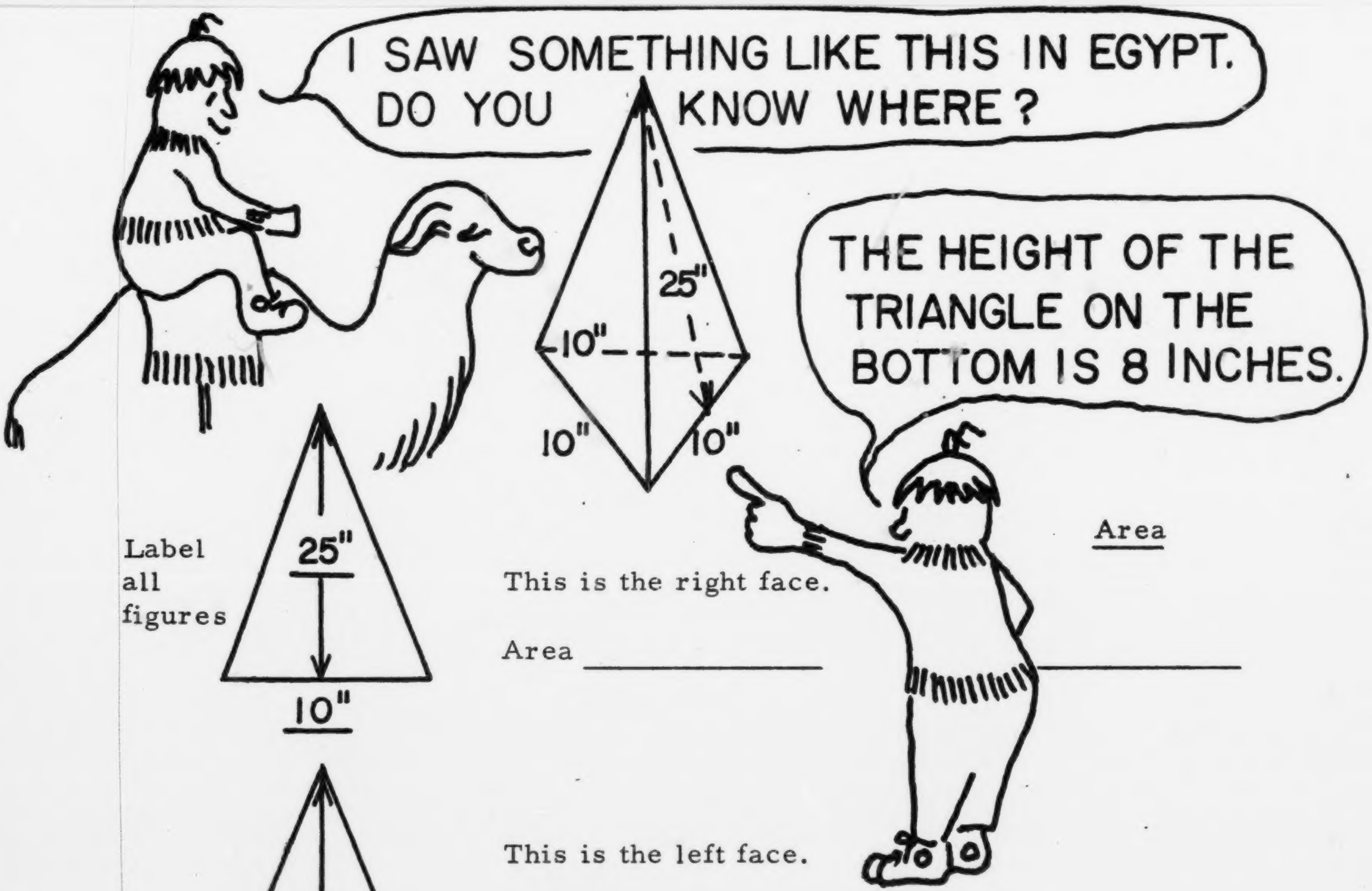


This is the _______ face.

Label its dimensions.

Area _______________ _______________

Total Surface Area _______________

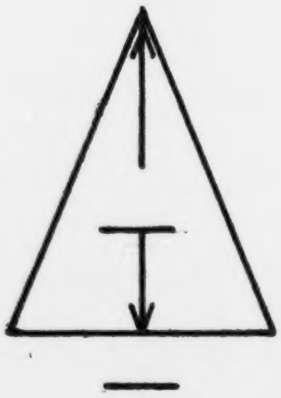

I SAW SOMETHING LIKE THIS IN EGYPT. DO YOU KNOW WHERE?
25"
10"
10"
10"
THE HEIGHT OF THE TRIANGLE ON THE BOTTOM IS 8 INCHES.
25"
10"

Label all figures

Area

This is the right face.

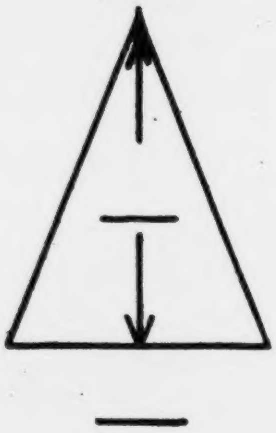
Area ____________ ____________



This is the left face.

Label its dimensions.

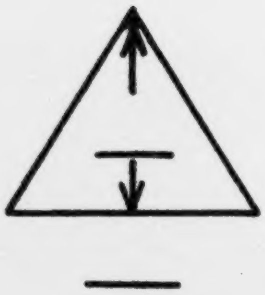
Area ____________ ____________



This is the back face.

Label its dimensions.

Area ____________ ____________

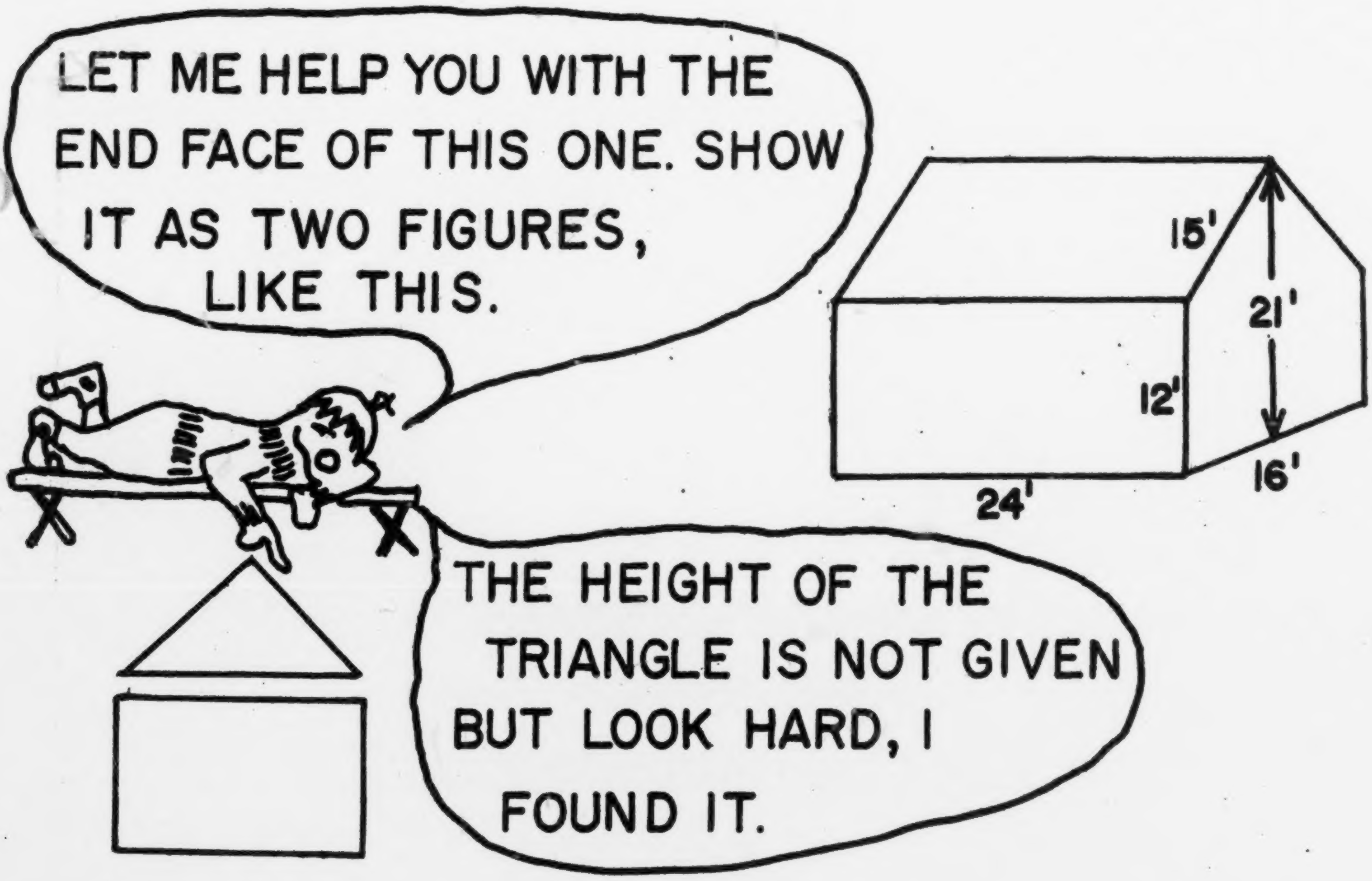


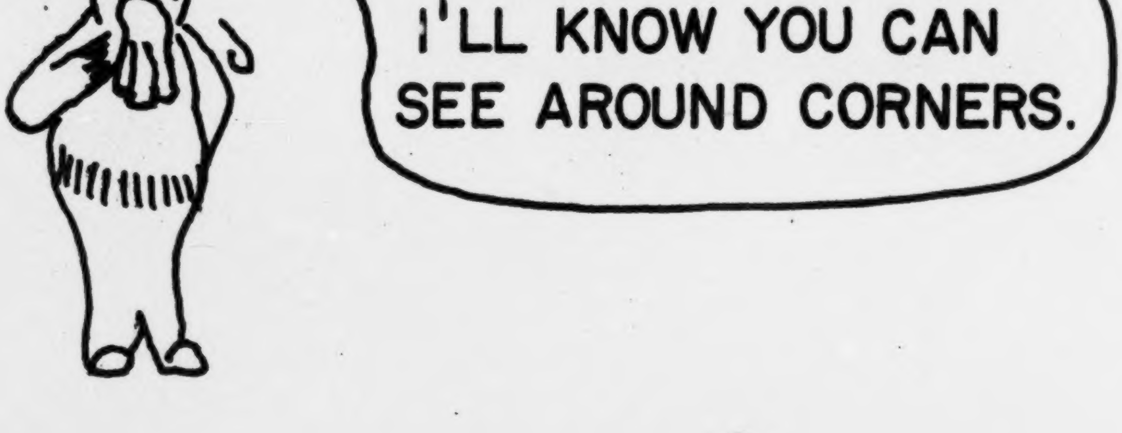
This is the bottom face.

Label its dimensions.

Area ____________ ____________

Total Surface Area ____________


LET ME HELP YOU WITH THE END FACE OF THIS ONE. SHOW IT AS TWO FIGURES, LIKE THIS.
15'
21'
12'
16'
24'
THE HEIGHT OF THE TRIANGLE IS NOT GIVEN BUT LOOK HARD, I FOUND IT.


FINISH THIS ONE AND I'LL KNOW YOU CAN SEE AROUND CORNERS.

GRAPHING

GRAPHING

I. Master Chart - Grades Six through Eleven

II. Grade Nine Chart

III. Behavioral Objectives

IV. Activities

UNIT **GRAPHING** GRADE(S) Seven through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Pictograph	7	7		7	7		7	7		
Bar Graph	7	7		7	7		7	7		
Line Graph	7	7		7	7		7	7		
Circle Graph	7	7			7		7	7		
Number Line	8	8		8	8	8	8			
Graphing Equalities	8	8		8		8				
Cartesian Products	8	8		8						
Graphing Ordered Pairs in First Quadrant	8	8	8	8	8					8
Coordinate Axes	9	9	9	9	9					9
Graphing Ordered Pairs	9	9	9	9	9	9	9			
Quadrants	9	9			9	9	9			
Table of Values	9	9	9	9	9			10		
Graphing Formulas	9	9		9	9			10		
Graphing Linear Equations and Inequations	10	10		10	10					10
Slope	10	10			10	10	10			
Graphing Simultaneous Equations	11	11		11	11	11	11			
Graphing Quadratic Equations and Inequations	11	11		11	11					11

UNIT GRAPHING GRADE(S) Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Coordinate Axes	9	9	9	9	9					9
Graphing Ordered Pairs	9	9	9	9		9	9			
Quadrants	9	9			9	9	9			
Table of Values	9	9	9	9	9					
Graphing Formulas	9	9		9	9					

GRAPHING - Grade 9

Page

Coordinate Axes

The student should be able to:

1. Name and identify coordinate axes
2. Demonstrate how to draw and label coordinate axes
3. Construct a pair of coordinate axes using a straightedge or freehand sketch with labeling
4. Describe coordinate axes as a vertical line and a horizontal line which intersects at a point called the origin or by a specific example
5. Distinguish coordinate axes from a number line

Graphing Ordered Pairs

The student should be able to:

1. Name and identify ordered pairs on a graph
2. Demonstrate how to graph ordered pairs with reference to the coordinate axes
3. Construct a graph from ordered pairs GR-13
4. Describe the procedure for plotting ordered pairs
5. State the principle of ordered pairs with reference to coordinate axes as:
    a. The first number in the pair tells the number of units to the right or left or the origin
    b. The second number in the pair tells the number or units up or down from the origin
6. Apply the principle by naming the ordered pairs associated with the given points

Quadrants

The student should be able to:

1. Name and identify the four quadrants
2. Describe quadrants as the four regions formed by the coordinate axes excluding the coordinate axes

Page

3. State the principles that:
    a. Quadrant one contains points which represent ordered pairs in which both numbers are positive
    b. Quadrant two contains points which represent ordered pairs in which the first number is negative and the second number is positive
    c. Quadrant three contains points which represent ordered pairs in which both numbers are negative
    d. Quadrant four contains points which represent ordered pairs in which the first number is positive and the second number is negative
4. Apply the principle by determining the quadrant in which given ordered pairs are located

Table of Values

The student should be able to: GR- 6

1. Name and identify a table of values GR-10
2. Demonstrate how to construct a table of values
3. Construct a table of values from a given mathematical sentence
4. Construct a graph using a table of values
5. Describe a table of values as an organized way to represent ordered pairs in a chart

Graphing Formulas

The student should be able to:

1. Name and identify the graph of a formula GR-17
2. Construct a graph given a table of values and formulas
3. Describe a graph as a pictorial representation of a formula
4. Apply the principle by solving related problems

THE SWINGING BALL

Teacher Commentary

I. Unit: Graphing

II. Objectives: The student should be able to:

- A. Demonstrate how to construct a table of values
- B. Construct a graph given a table of values

III. Materials:

- A. Four blocks of wood
- B. A ping pong ball
- C. Four super balls
- D. Heavy sewing thread
- E. Student work sheet entitled, "The Super Ball"

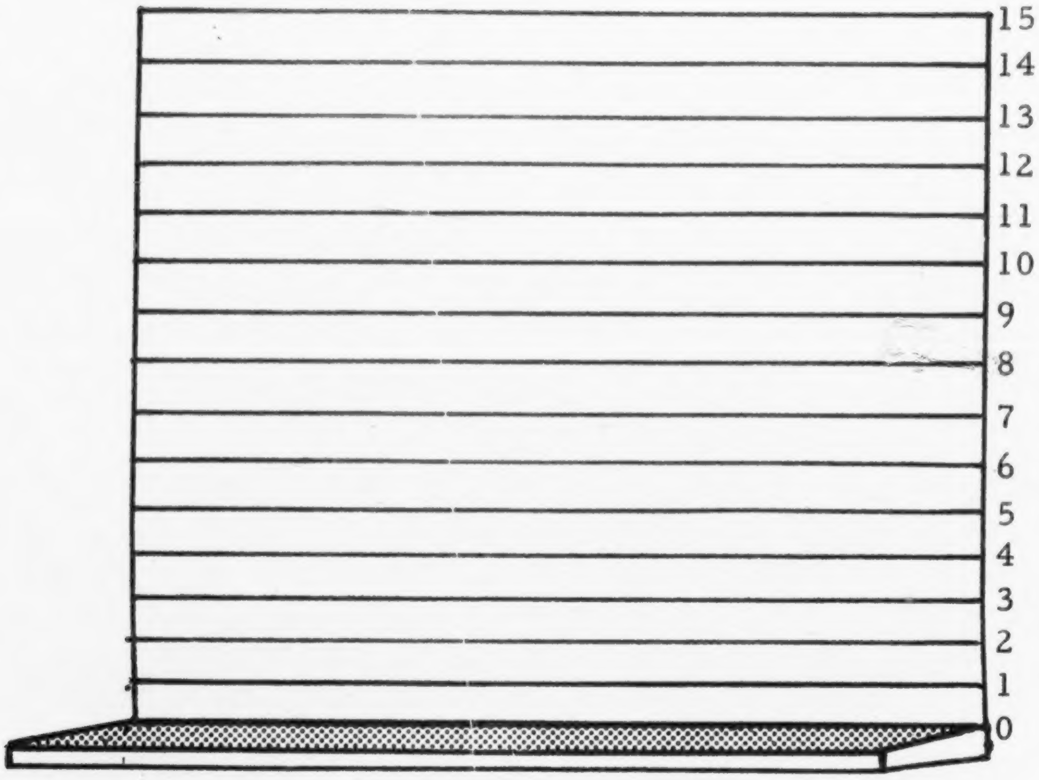
IV. Procedure:

A. This lesson provides for performing an experiment, collecting data and constructing a graph. In the experiment, we will swing a ping pong ball to collect data for a table of values and a graph. Observe:

1. The height from which the ball is dropped.
2. The height to which the ball rebounds.

B. Preparation for this experiment follows:

1. To prepare the board for measurement:
    - a. Use a staff marker to mark the chalkboard with horizontal lines at 3 inch intervals. You should be able to mark off 15 of these 3 inch intervals.
    - b. Number each line, beginning with "1" placed 3 inches from the bottom of the board.



2. To prepare the swinging ball:

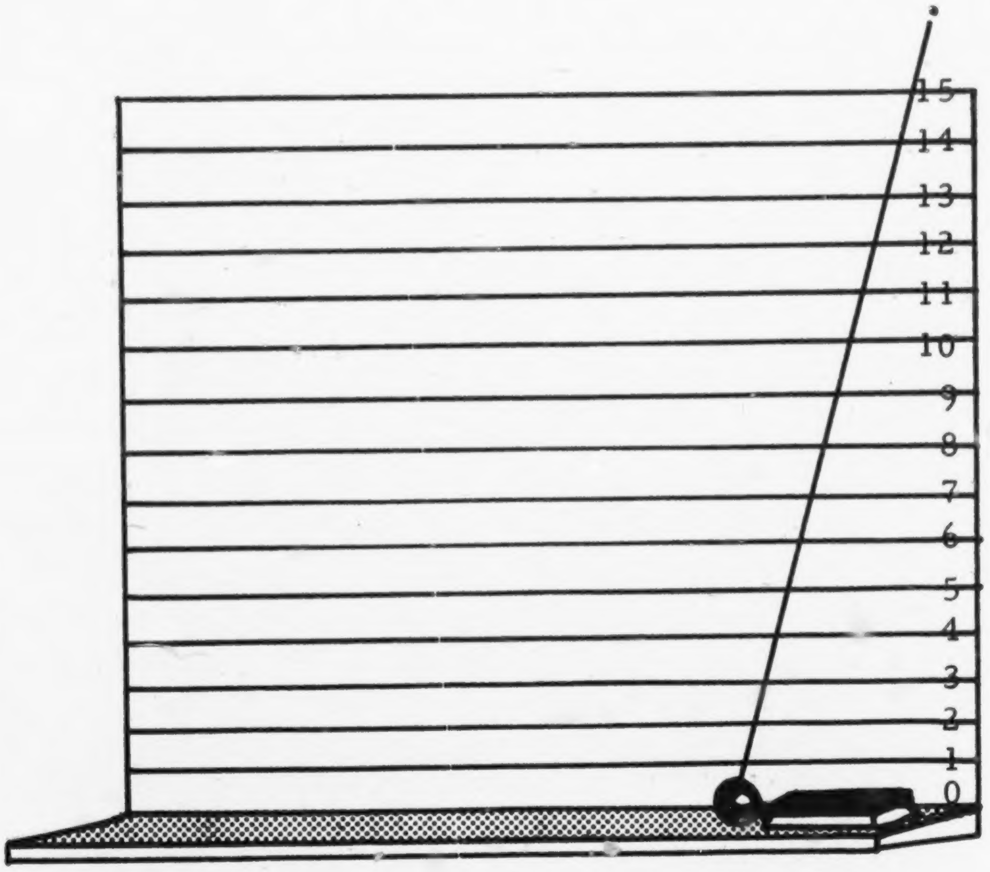
   a. Attach one end of the thread to a map hook just above the chalkboard.

   b. The thread should be long enough to extend to the chalkledge.

   c. Use a piece of masking tape to attach the ping pong ball at the other end of the thread. The center of the ball should be at the bottom of the chalkboard, at a point corresponding to a zero point.

   d. A block of wood should be placed on the chalkledge directly below the map hook. Fasten the block with masking tape to keep it stationary.

C. The set-up for the experiment should look similar to the figure below:



D. The student is to raise the ball to a certain height, say 13, being sure to keep the string taunt and release the ball. The students are then to observe how high the ball rebounds after striking the wooden block.

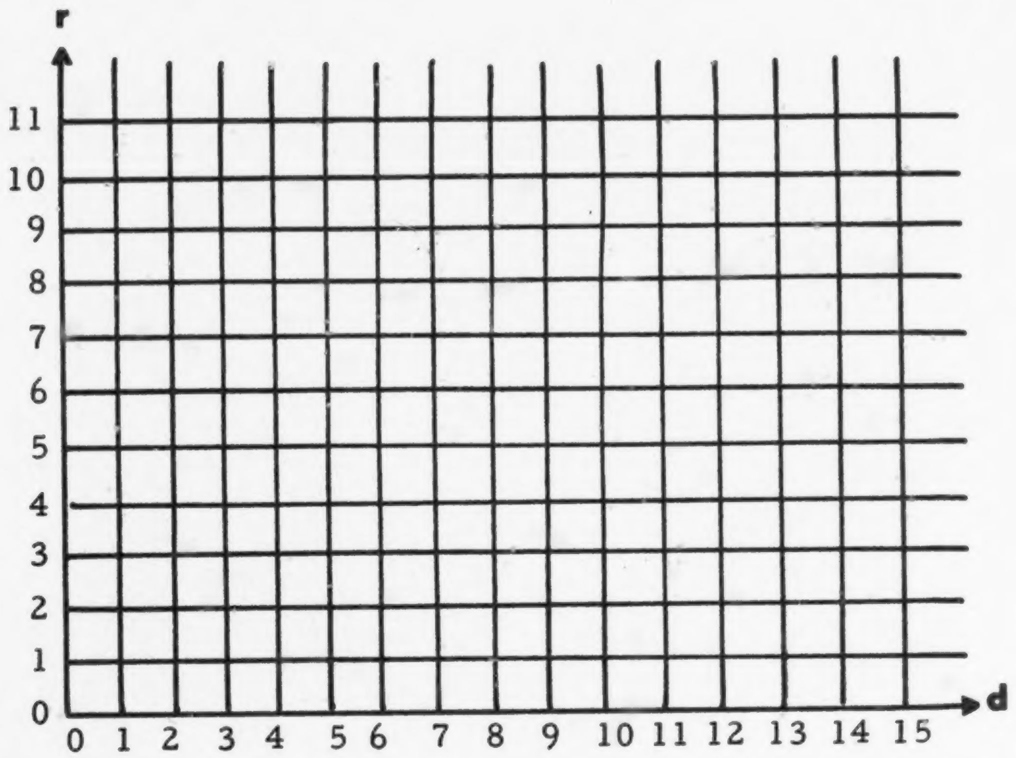
As students swing the ball, they collect the data in a table similar to the one on the right. Have the table on the board. The columns are headed d (height dropped) and r (height of rebound).

d	r
15	
14	
13	
12	
11	
10	
9	
8	
7	
6	
5	
4	
3	
2	
1	

E. Have different students drop the ball while others observe the height of the rebound. Have a student record the results in the table.

F. Construct a set of coordinates using the table on the preceding page. The coordinates should be of the form (d, r).

G. Have a graph prepared on the chalkboard. Plot the points which correspond to the set of coordinates.



H. Assessment: Super Ball

1. Divide the class into four groups. Have them perform the experiment to collect data using the super ball. Have four stations for this experiment prepared before class begins. Use the same procedure as used for the ping pong ball.

2. Give each student a work sheet to record his data from the experiment.

   After each group has finished collecting the data, have them construct a set of coordinates of the form (d, r), then graph the coordinates.

THE SUPER BALL

1. Swing the ball a number of times. For each swing, record the height from which the ball is dropped in column d, and the height of rebound in column r.

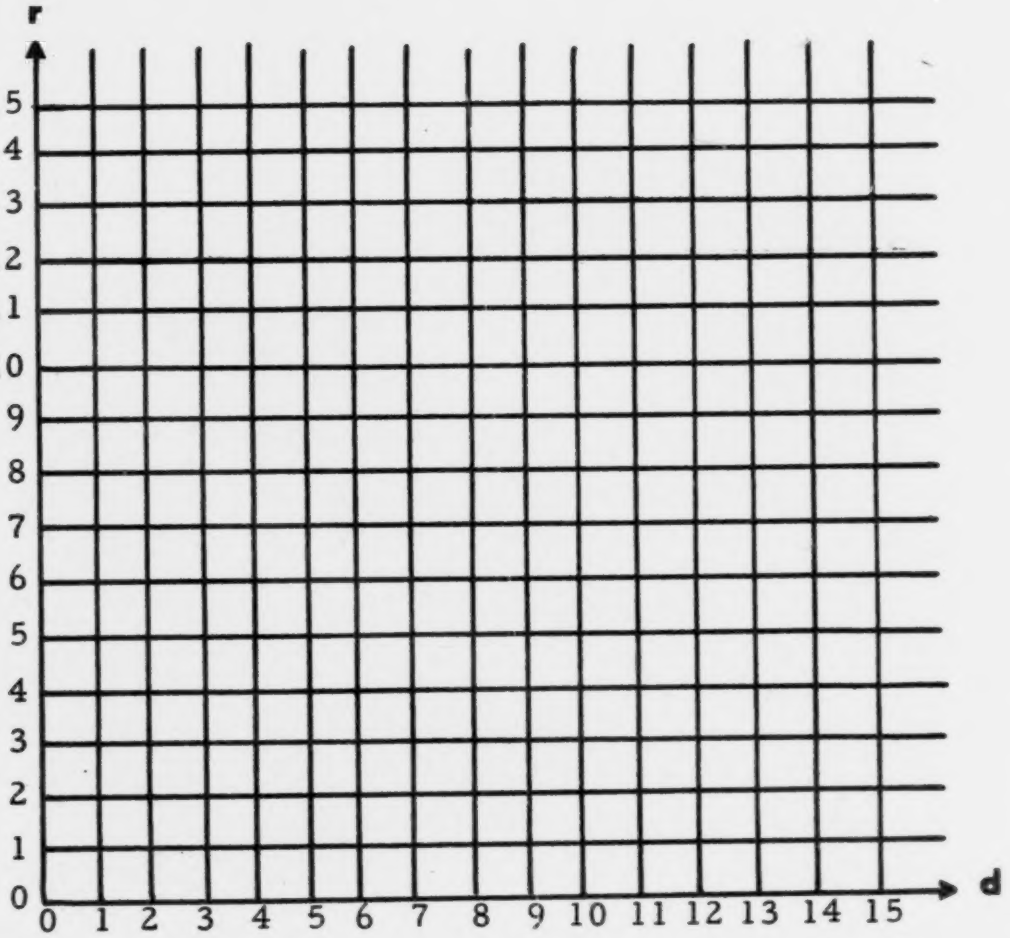
d	r

2. Use the table to construct a set of ordered pairs of the form (d, r). Write your ordered pairs below.

____

____

3. Use the set of coordinates to construct a graph of the data from the swinging ball experiment.


r
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
d

A DOWNHILL EXPERIMENT

Teacher Commentary

I. Unit: Graphing

II. Objectives: The student should be able to:

- A. Construct a graph from experimental data
- B. Interpret a graph of experimental data

III. Materials:

- A. Board (approximately 2' to 4' long by 1' wide)
- B. Model car
- C. Protractor
- D. Ruler
- E. Grid paper or something on which to lay off parallel lines

IV. Procedure:

- A. A grid must be placed on the top of a demonstration table. This grid consists of a series of parallel lines approximately 6" apart. This can be accomplished by placing a long sheet of paper on the table top and making parallel lines, perhaps with a magic marker or with masking tape. The demonstration could also be performed on the floor by using the tile blocks as a grid.
- B. Begin the experiment by placing a board on the table at an angle of 0°.
- C. Place the model car immediately behind a fixed starting line which has been drawn across the board (perhaps with a magic marker). The car, of course, will not roll in this position.
- D. Next, set the board at an angle of 5°. (Refer to figure 1.) Again place the car at the fixed line on the board and let the car roll down the inclined plane across the grid. Record the distance the car has traveled across the grid at this angle.
- E. Repeat this process for angles of 10°, 15°, etc. Record this data, which students have gathered in chart form. (Refer to figure 2.)
- F. Next, have students record the charted information as "ordered pairs." (Refer to figure 3.)
- G. Finally, have students plot these ordered pairs on the graph. (Refer to figure 4.)

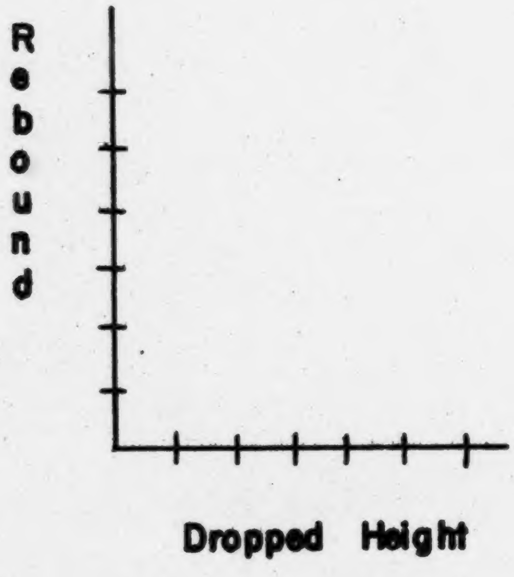
H. The students should now be led in a discussion to interpret their findings. Students should connect the points on the graph and as a result be able to estimate distances of angles which have not been plotted. Additional questions of the following type should be asked here.

1. What distance did the car travel at an angle of 30°, 32°, 45°, 57°, 60°?
2. What happens at 90°? Why?
3. As the angle increases what happens to distance? Is this always true or only to a certain point?

I. To assess the students ability in gathering and graphing data, an experiment of the following type may be performed:

The class could be divided into 3 or 4 groups. Each group could use a super ball, ping pong ball, tennis ball, golf ball, or handball. After marking equal intervals on the wall with parallel lines (by using masking tape), the students could drop the ball from different heights and record the rebounds.

The process would be the same in that the students would first record the data in chart form, second, convert data to ordered pairs in the form of (dropped height, rebound) and third, graph the results.



The following illustrations are provided for purposes of explanation.

Figure 1

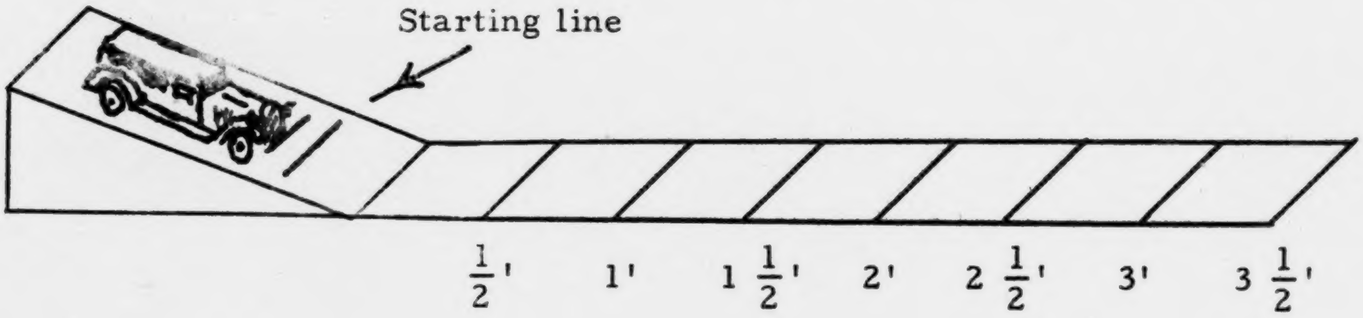

Starting line
$\frac{1}{2}$'
1'
1 $\frac{1}{2}$'
2'
2 $\frac{1}{2}$'
3'
3 $\frac{1}{2}$'

Figure 2

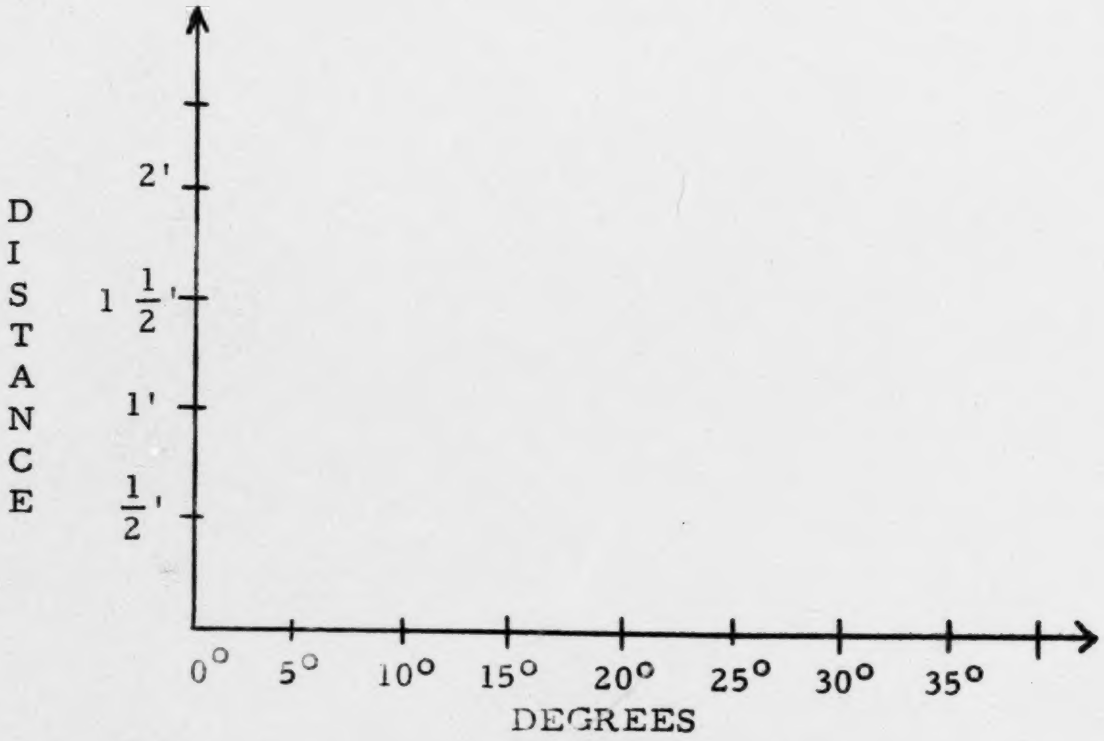
Degrees	5°	10°	15°	20°	25°	30°	
Distance in feet	2'	2 $\frac{1}{2}$'	2 $\frac{3}{4}$'				

Figure 3

Ordered pairs

(5°, 2'), (10°, 2 $\frac{1}{2}$'), (15°, 2 $\frac{3}{4}$'), etc.

Figure 4



GIVE YOUR CAR A BRAKE

Teacher Commentary

I. Unit: Graphing

II. Objectives: The student should be able to:

   A. Demonstrate how to graph ordered pairs with reference to the coordinate axes

   B. Apply the principle by naming the ordered pairs associated with the given points.

   C. Construct a graph given ordered pairs

III. Materials:

   A. Student work sheet "Give Your Car a Brake"

   B. Student work sheet "Evaluation Questions"

IV. Procedure:

   A. Introduce the topic through a class discussion on automobile safety. Students may speculate on stopping distances at various speeds.

   B. Pass out the student work sheet "Give Your Car a Brake."

   C. The class should complete the discussion questions as a group. Each section of the discussion questions should be discussed. The teacher may want to have the students do some of the questions on their own, and then discuss their work.

   D. The student work sheet "Evaluation Questions" may be completed at the end of this lesson, or on the following day.

GIVE YOUR CAR A BRAKE

Bob Johnson is an automobile test driver. One of his jobs is to test stopping distance. He has to find the distance that cars, traveling at different speeds, will go after the brakes have been applied. The table below shows the results of several tests. The first column lists the speed and the second column lists the total stopping distance.

Test Number	Speed MPH	Stopping Distance
1.	10	20
2.	20	43
3.	30	82
4.	40	126
5.	50	183
6.	60	251
7.	70	328

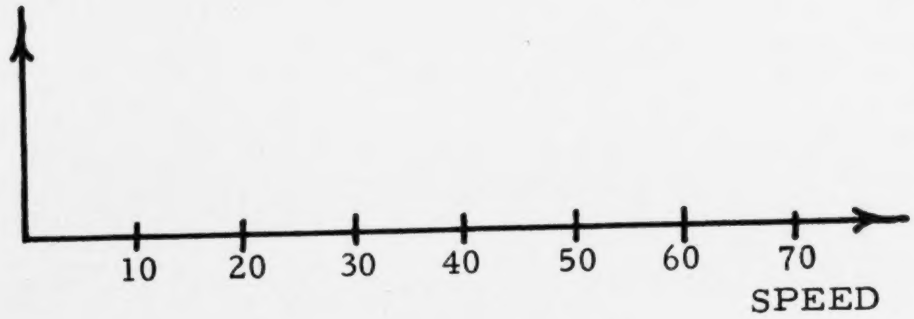
Use the table and answer the following:

1. In each test the speed was increased by ________mph.
2. If the speed is increased from 10 mph to 20 mph, the stopping distance increases ________ ft.
3. If the speed increases from 20 mph to 30 mph the stopping distance increases _________ ft.
4. If the speed is increased from 30 mph to 40 mph, the stopping distance increases _________ ft.
5. If the speed is increased from 40 mph to 50 mph, the stopping distance increases _________ ft.
6. If the speed is increased from 50 mph to 60 mph, the stopping distance increases _________ ft.

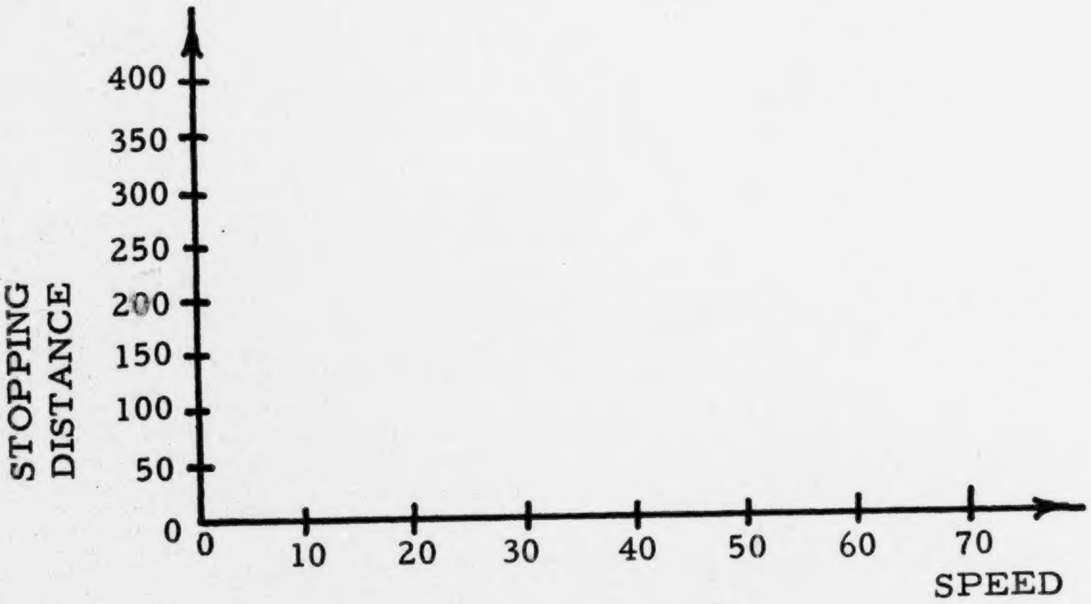
7. If the speed increases from 60 mph to 70 mph, the stopping distance increases ________ ft.

8. Does the stopping distance increase by the same number of feet each time? ___________

The testing company decided to make a graph of the results. On the horizontal axis they made a scale of speed.


10
20
30
40
50
60
70
SPEED

On the vertical axis they made a scale of distance.


400
350
300
250
200
150
100
50
0
STOPPING DISTANCE
0
10
20
30
40
50
60
70
SPEED

9. Next they wrote the table of values as ordered pairs. Below is the table you saw on page 1, but a third column has been added for the ordered pairs. The first three have already been completed. You complete the rest.

Test Numbers	Speed MPH	Stopping Distance	Ordered Pairs
1.	10	20	(10, 20)
2.	20	43	(20, 43)
3.	30	82	(30, 82)
4.	40	126	
5.	50	183	
6.	60	251	
7.	70	328	

10. The first number in each ordered pair represents ____________.
The speed increases by 10 mph each time.

11. The second number in each ordered pair represents ___________.
The distance increases by a greater number of feet each time.

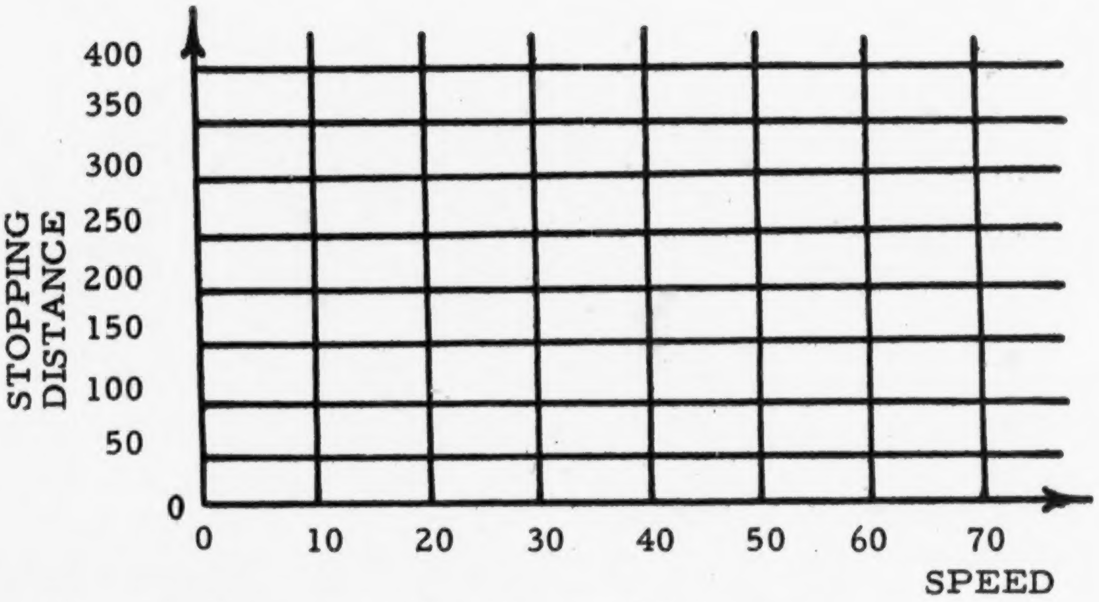
The company considered the two parts of each ordered pair:

a. speed - each increase is the same 10 mph.

b. distance - each increase is different.

They wondered if the pattern of the points would be a straight line.

12. Plot the points named by the list of ordered pairs in the last section to see if a straight line is formed.



13. Does the graph form a straight line? ____________

14. Connect the points with a smooth curve.

EVALUATION QUESTIONS

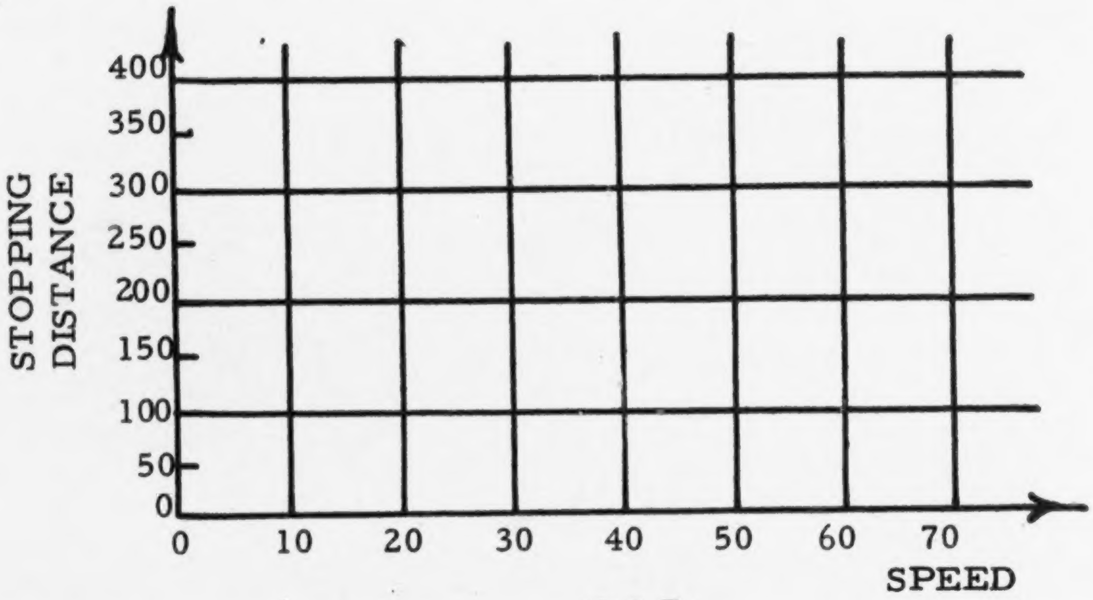
A second test drive made the same tests. Part of his results are shown in the table below.

Test	Speed	Stopping Distance	Ordered Pairs
1.	30	75	(30, 75)
2.	40	125	(40, 125)
3.	50	200	
4.	60	260	
5.	70	325	

1. Use the table and answer the following:
   a. In each test the speed was increased by ____________ mph.
   b. If the speed is increased from 30 mph to 40 mph, the stopping distance is increased by ________ ft.
   c. If the speed is increased from 40 mph to 50 mph, the stopping distance is increased __________ ft.
   d. Is the increase in stopping distance the same each time? _________

2. In the third column of the table, write the speed and stopping distance as ordered pairs. The first two have already been done.

3. Plot the points named by the ordered pairs. Connect the points with a smooth curved line.



TRIANGLES

Teacher Commentary

I. Unit: Graphing

II. Objectives: The student should be able to:

- A. Demonstrate how to graph ordered pairs with reference to the coordinate axes
- B. Construct a graph consisting of ordered pairs
- C. Demonstrate how to construct a table of values

III. Materials:

- A. Student work sheets: "Triangles," "Discovery Exercises," "Evaluation Exercises"
- B. Board diagrams of triangles #1 and #2
- C. Chart from "Discovery Exercises" on the board
- D. Ruler for each student
- E. Graph paper

IV. Procedure:

- A. Distribute work sheets "Triangles" and "Discovery Exercises."
- B. Discuss "Triangles" with the students.
  1. x represents the length of the base of the triangle.
  2. y represents the length of the dotted line connecting the midpoints of the other two sides.
- C. Discuss the directions to #1 on "Discovery Exercises."
- D. As the teacher measures segments x and y, the students should do so on their work sheets. Record the results on the individual and board charts. Repeat for triangle #2.
- E. Discuss the directions for the remaining exercises. Make sure the terms coordinates and graphing coordinates are familiar to all the students.
- F. The exercise may be completed independently or as a group.
- G. Ask the students to examine the results of their measurements for a pattern. Some students may be able to make this generalization in their own words: If a line joins the midpoints of two sides of a triangle, then it is parallel to the third side and equal to one-half of it.

H. The work sheet "Evaluation Exercises" is an assessment item for this lesson.

DISCOVERY EXERCISES

1. Complete the following chart by measuring the segments x and y for each triangle. (nearest $\frac{1}{4}$")

Triangle	x	y
1		
2		
3		
4		
5		
6		
7		

2. What seems to be the relationship between the numbers in column x and column y? ________________________________

3. Write the values in columns x and y as coordinates. The first two are done for you.

   $(3, 1\frac{1}{2})$, $(4, 2)$ ________________________________

4. Graph the set of coordinates in exercise 3.

5. What pattern is suggested by the points on the graph? Draw a line to show this pattern, and extend the line above and below the points that you plotted. ________________

6. If x represents the measure of the base of a triangle, and y represents the measure of the segment joining the midpoints of the other two sides, then ________________.

   (a) $y = x$ (b) $y = \frac{1}{2}x$ (c) $y = 2x$ (d) $y = x + 2$

7. Use the graph to answer the following questions:

   (a) Locate the point (0, 0). This point is called the origin. Does your graph go through the origin?

   (b) Suppose the measure of x is 7. What would be the measure of y?

   ____

   (c) Suppose the measure of y is 4. What would be the measure of x?

   ____

   (d) As x increases, what happens to y? ____________________

EVALUATION EXERCISES

1. Below are 5 segments. Measure from the point 0 to the point x. This distance is called a. Place your results in the table at the right. Complete the table by measuring from the point 0 to the point y. Call this distance b.

Seg. 1

Seg. 2

Seg. 3

Seg. 4

Seg. 5

Segment	a	b
1		
2		
3		
4		
5		

2. Construct a graph using the ordered pairs (x, y) from the table in number 1.

3. Which of the following shows the relationship between a and b?
   (a) $a = b$ (b) $b = 2a$ (c) $b = 3a$ (d) $b = \frac{1}{2}a$

4. Use your graph to answer the following questions.
   (a) Does your graph go through the origin? ____________
   (b) If segment a is $1\frac{1}{2}$, what is the measure of b? ____________
   (c) If segment a is 5, what is the measure of b? ____________
   (d) As a increases, what happens to b? ____________

PROBABILITY STATISTICS

PROBABILITY AND STATISTICS

I. Master Chart - Grades Six through Eleven

II. Grade Nine Chart

III. Behavioral Objectives

IV. Activities

UNIT PROBABILITY GRADE(S) Eight through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Event	8	8	8							
Certainty - Uncertainty	8	8		8	8					
Sample Spaces - Ordered Arrangements	8	8		8	8					8
Equally and Unequally Likely Outcomes	8	8	8			8	8			8
Probability	8	8				10				
Ordered Arrangements (Permutations)	9	9		9	9	9				
Factorial	9	9		9	9		9			
Permutations of N things taken N at a time	9	9		9		9	9			
Permutations of N things taken R at a time	9	9		9		9	9			9
Tree Diagram	9	9		9	9		9			
Box Diagram	9	9		9	9		9			
Decreasing and Increasing Probability	10	10		10		10	10			
Probability of 0 and 1	10	10		10		10	10			
Independent and Dependent Events	10	10		10	10	10	10	10		10
Complementary Events	10	10		10		10	10			
Experimental Probability	10	10	10		10					
Theoretical Probability	10	10			10					10
Sample	11	11	11		11		11	11		11

UNIT PROBABILITY GRADE(S) Eight through Eleven

TOPIC	NAME	IDENTIFY	DEMON- STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN- GUISHING
Circular Permutation	11	11		11	11	11	11			
Combination	11	11		11	11					11
Combinations of N things taken N at a time	11	11		11		11				
Combinations of N things taken R at a time	11	11		11	11	11	11			
Preduction	11	11		11	11	11	11			
Chances (Odds)	11	11		11		11	11			11

UNIT STATISTICS GRADE(S) Nine through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Data	9				9				9	
Range	9	9	9		9	9	9			9
Rank	9	9	9		9		9	9		9
Interval	9	9	9			9	9			9
Tally	9	9	9		9		9			
Frequency	9	9	9	9	9					9
Frequency Table	9	9		9	9	9		9		9
Mean	9	9				9	9	9		9
Median	9	9				9	9	9		9
Mode	9	9				9	9	9		9
Pictograph	9	9						9		
Circle Graph	9	9		9				9		
Bar Graph	9	9		9				9		
Line Graph	9	9		9				9		
Normal Curve	10	10		10	10			10		
Pascal's Triangle	10	10	10	10			10	10		
Percentile	10	10		10	10	10	10	10		10
Correlation	11	11			11		11	11		
Scattergram	11	11		11	11	11	11			11

UNIT PROBABILITY GRADE(S) Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
The Fundamental Principle of Counting		9					9			
Ordered Arrangements (Permutations)	9	9		9	9	9				
Factorial	9	9		9	9		9			
Permutations of n Things Taken n at a Time	9	9		9		9	9			
Permutations of n Things Taken r at a Time	9	9		9			9			9
Tree Diagram	9	9		9	9		9			
Box Diagram	9	9		9	9		9			

UNIT STATISTICS GRADE(S) Nine

TOPIC	NAME	IDENTIFY	DEMON- STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN- GUISHING
Data	9				9				9	
Range	9	9	9		9	9	9			9
Rank	9	9	9		9		9	9		9
Interval	9	9	9			9	9			9
Tally	9	9	9		9		9			
Frequency	9	9	9	9	9					9
Frequency Table	9	9		9	9	9		9		9
Mean	9	9				9	9	9		9
Median	9	9				9	9	9		9
Mode	9	9				9	9	9		9
Pictograph	9	9						9		
Circle Graph	9	9		9				9		
Bar Graph	9	9		9				9		
Line Graph	9	9		9				9		

PROBABILITY - Grade 9

Page

The Fundamental Principle of Counting

The student should be able to:

1. Describe the fundamental principle by using specific examples
2. Apply the fundamental principle of counting to solve related problems

Ordered Arrangements (Permutations)

The student should be able to:

1. Name and identify ordered arrangements
2. Construct all the possible arrangements from given events with appropriate restrictions
3. Describe ordered arrangements as all the possible ways objects can be arranged
4. State the principle that the number of possible arrangements or orders increases if the number of things to be arranged increases

Factorial

The student should be able to:

1. Name and identify the factorial symbol
2. Construct products indicated by factorial notation
3. Describe a factorial as a short way of indicating products of the first in counting numbers
4. Apply the principle of factorial notation to solve problems concerned with arrangements of n things taken n at a time

Permutations of n Things Taken n at a Time

The student should be able to:

1. Construct the number of n things taken n at a time by applying the principle of the box diagram
2. State the principle that the permutations of n thing taken n at a time is n!
3. Apply the principle by calculating the total number of permutations from a given number of objects

Permutations of n Things Taken r at a Time

Page

The student should be able to:

1. Construct permutations by listing all possible arrangements
2. Apply the principle by calculating the number of permutations of n things taken r at a time
3. Distinguish between permutations of n things taken n at a time and permutations of n things taken r at a time

Tree Diagram

The student should be able to:

1. Name and identify tree diagrams
2. Construct a tree diagram given appropriate data
3. Describe a tree diagram
4. Apply the principle of the tree diagram to solve related problems

Box Diagram

The student should be able to:

1. Name and identify box diagrams
2. Construct a box diagram given appropriate data
3. Describe a box diagram
4. Apply the principle of the box diagram to solve related problems

STATISTICS - Grade 9

Page

Data

The student should be able to:

1. Name data
2. Describe data as a collection of information
3. Order data according to a given rule

Range

The student should be able to:

1. Name and identify range
2. Demonstrate a procedure for finding the range using data
3. Describe the range as a measure of extremes, and not necessarily an indication of the variability in the data
4. State the principle: range = high score - low score
5. Apply the principle by computing the range from a set of data

Rank

The student should be able to:

1. Name and identify rank
2. Demonstrate a procedure for finding the rank using a set of data
3. Describe rank as a position of a score within a set of data which have been arranged from highest to lowest
4. Apply the principle by finding the rank of specific scores given a set of data
5. Interpret the principle by using related problems
6. Distinguish between rank and range

Interval

The student should be able to:

1. Name and identify interval
2. Demonstrate how to construct appropriate intervals using a set of data

Page

3. State the principle that intervals are used to group data in useful order
4. Apply the principle by arranging data into appropriate intervals
5. Distinguish between range and interval

Tally

The student should be able to:

1. Name and identify the standard tally
2. Demonstrate how to use a tally
3. Describe a tally as a mark used to represent the occurrence of an event
4. Apply the principle for representing data by tallies

Frequency

The student should be able to:

1. Name and identify frequency
2. Demonstrate how to find the frequency of events in an interval
3. Construct the frequency of events from the tallies
4. Describe frequency as the number of times a particular event occurs
5. Distinguish between tally and frequency

Frequency Table

The student should be able to:

1. Name and identify a frequency table
2. Construct a frequency table when data and appropriate intervals are given
3. Construct a frequency table from experimental data
4. Describe the various parts of a frequency table

Page

5. State the principle that a frequency table is a chart for organizing data
6. Interpret a frequency table to solve related problems
7. Distinguish a frequency table from a graph

Mean, Median, Mode

The student should be able to:

1. Name and identify mean, median, and mode P-12
2. State the principles:
    a. The mean is the sum of the scores divided by the number of scores P-17
    b. The median is the middle score after the scores have been ranked from highest to lowest
    c. The mode is the score that occurs most frequently
3. Apply the principles by computing the mean, median, and mode of a given body of data
4. Interpret the mean, median, and mode in related problems
5. Distinguish between mean, median, and mode

Graphs

The student should be able to:

1. Name and identify line graphs, bar graphs, pictographs,and circle graphs
2. Construct bar graphs, line graphs, and pictograms of statistical data P-12 P-14
3. Construct a circle graph given appropriate data
4. Interpret graphs of statistical data to answer related questions

CLIPS PLEASE

I. Unit: Probability and Statistics

II. Objectives: The student should be able to:

   A. Order data according to a given rule

   B. Apply the principle for representing data by tallies

   C. Apply the principle of mean by computing the mean for a given body of data

   D. Construct a bar graph from statistical data

III. Materials:

   A. 10 paper clips for each pair of students

   B. Work sheet entitled, "Clips Please!"

   C. Grid entitled, "Clips Please!"

   D. Square

   E. Work sheets entitled, "Number Keys!"

IV. Procedure:

   A. Distribute the following materials to each pair of students

      1. 10 paper clips

      2. Work sheet entitled, "Clips Please!"

      3. Grid entitled, "Clips Please!"

      4. Square

   B. Ask the students to make guesses as to what they think would be the average number of paper clips they could drop with their eyes closed into the square.

   C. Help students make plans as to how they could collect data and organize it to prove or disprove their guesses.

      1. One student in each pair should hold his hand with the 10 paper clips in it about 1 foot to $1\frac{1}{2}$ feet above the square, close his eyes, and drop the clips.

      2. The number of clips inside the square should be counted. If a clip is on the line it is considered outside the square.

3. This number should be recorded by a tally mark in the correct row on the tally sheet.
4. The above procedure should then be done 25 times by each pair.
5. Total the tally marks in the third column of the tally sheet.

D. Students should then observe their tally marks in the third column of the tally sheets and answer the following questions:

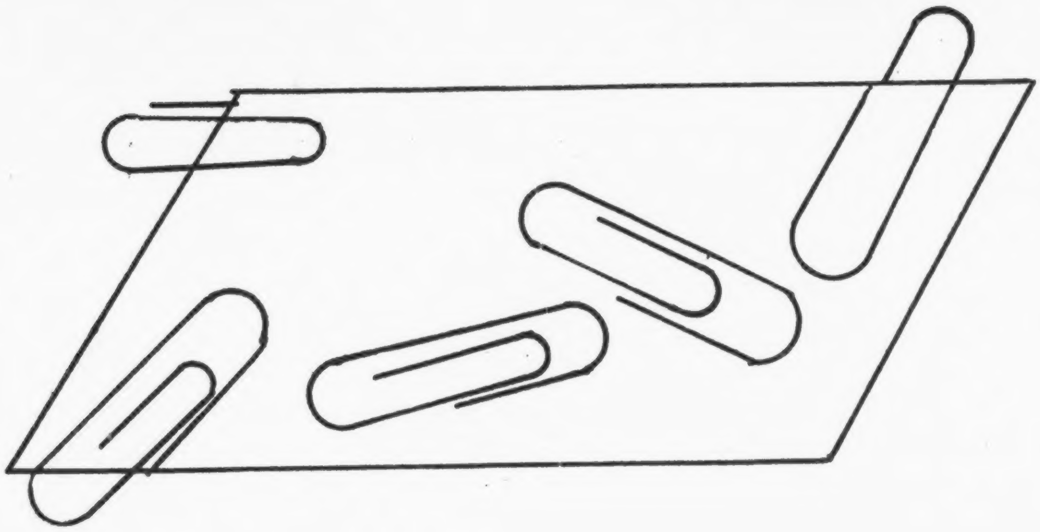
1. What was the greatest number of clips which landed in the square?
2. What was the smallest number of clips in the square?
3. Which row has the greatest number of tally marks?
4. If you dropped the clips one more time, how many clips would you expect to land in the square?
5. What was the average number of clips to land in the square? Emphasize that this average is called the mean.

$$\frac{\text{Total Clips to Land in Square}}{\text{Total Times Dropped}} = \text{Mean}$$

E. Using the data collected, students may then construct a bar graph.

F. The objectives may be assessed by having the student do the work sheet entitled, "Number Keys!"

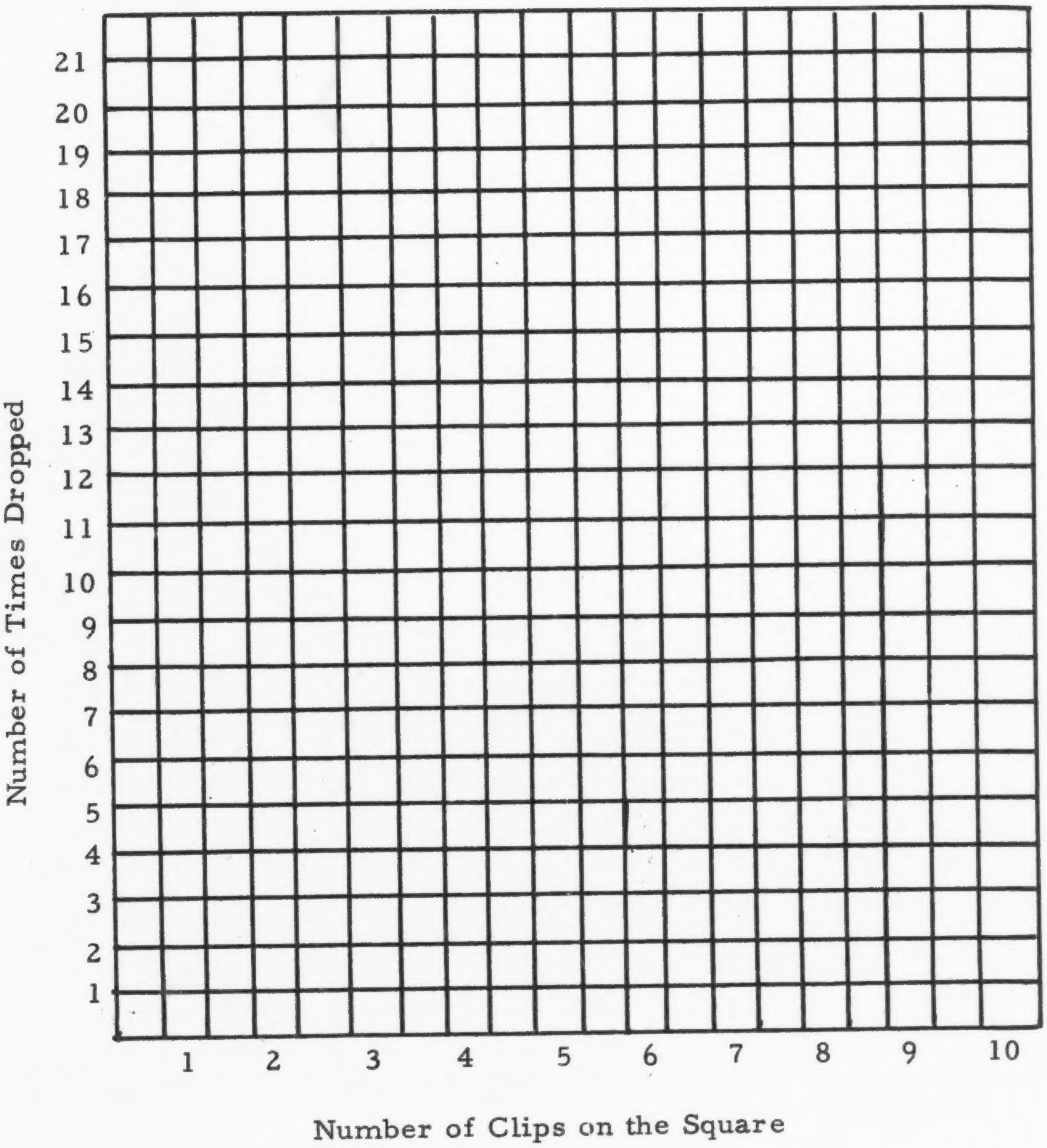
CLIPS PLEASE!



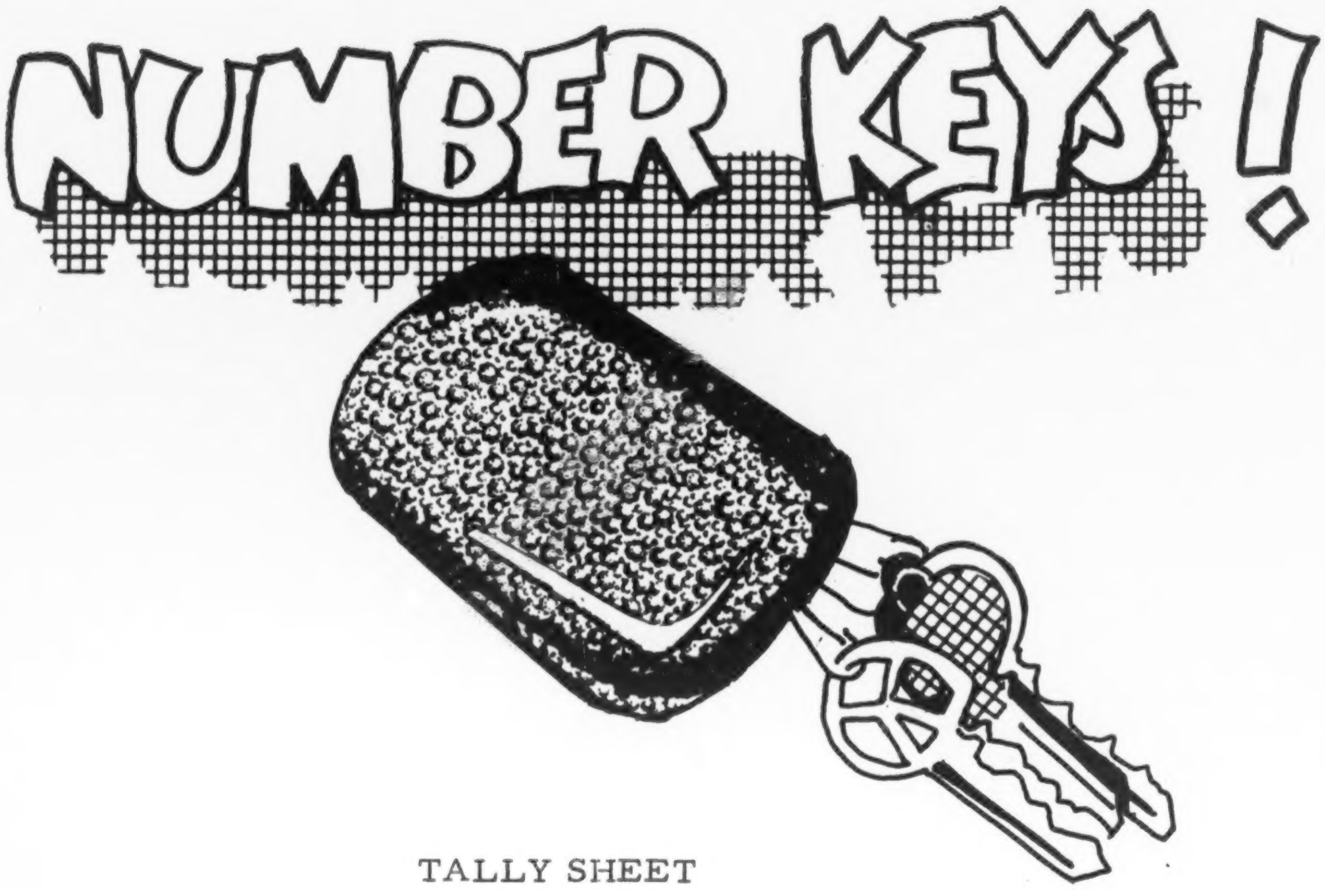
TALLY SHEET

Number of Clips in the Square	Tallies	Total Number of Tallies
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
	Total	

CLIPS PLEASE!



NUMBER KEYS!



TALLY SHEET

Number of Keys on a Key Ring	Tallies	Total Number of Tallies
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
	Total	

I. 1. Below is a list of the number of keys found on 25 key rings.

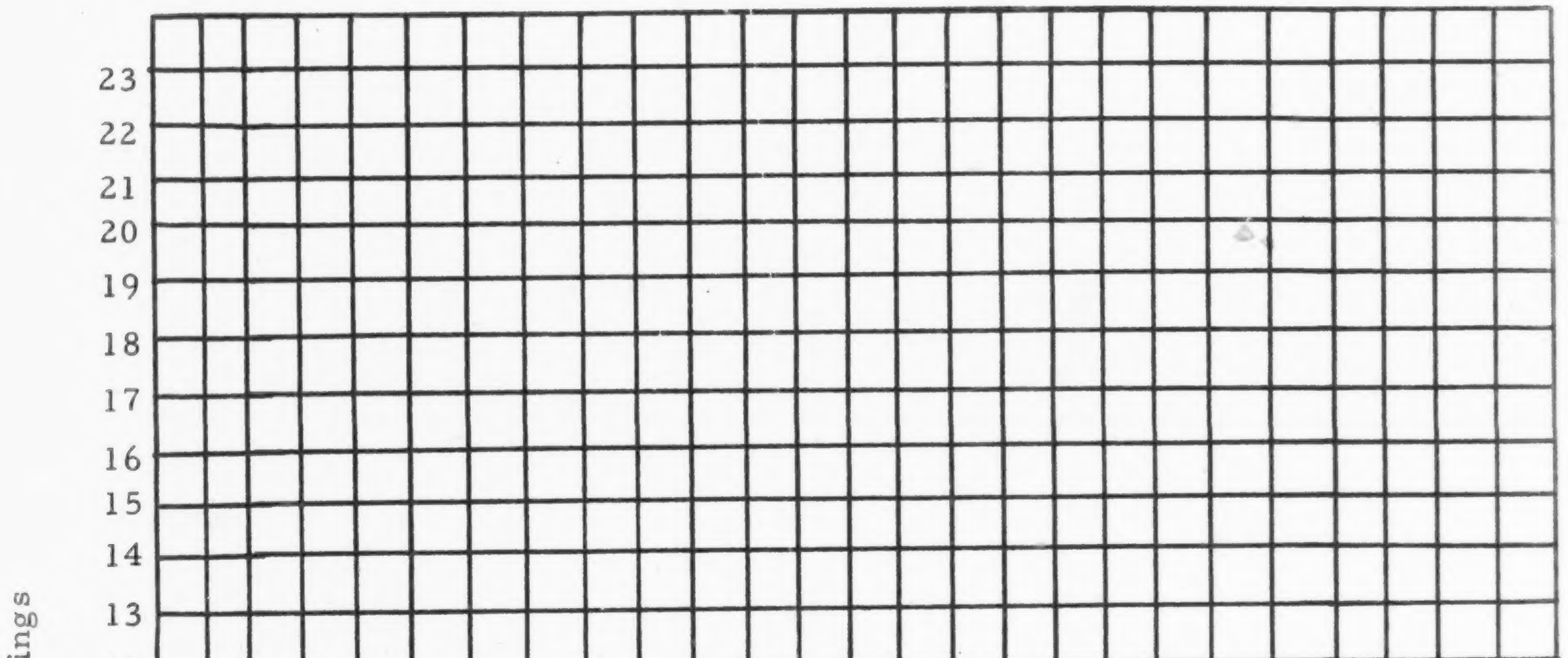
2. Record the information on your tally sheet.

6	4	7	6	6
8	5	5	13	12
4	3	5	8	6
12	9	8	2	1
6	3	6	2	2

II. Use your tally sheet to answer the following questions.

1. Which row has the most number of tallies?
2. Which row has the least number of tallies?
3. If you ask someone how many keys he has, what number of keys might you expect to find?
4. How many keys are there all together?
5. What is the average number of keys? $\frac{\text{Total No. of Keys}}{\text{Total No. of Key Rings}}$

III. Use the data you collected and organized to complete the bar graph below.



A HIT RECORD?

Teacher Commentary

I. Unit: Probability and Statistics

II. Objectives: The student should be able to:

   A. Order data according to a given rule

   B. Construct a line graph of statistical data

   C. Interpret a line graph of statistical data to solve related problems

III. Materials:

   A. Bulletin board or classroom chart.

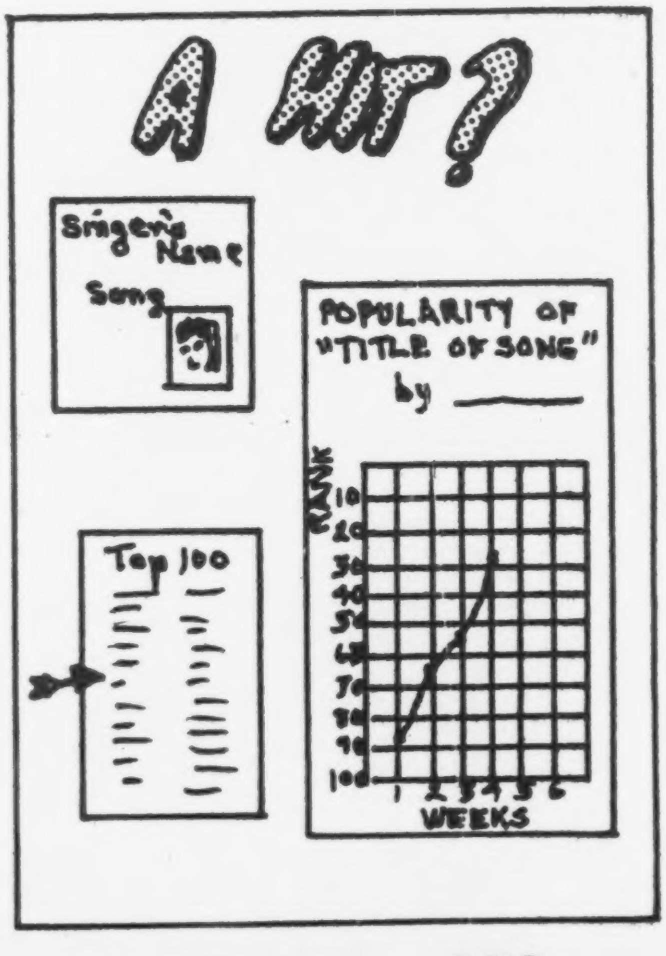
   B. Hit record listings for several weeks obtained from radio stations, record stores and departments, and magazines such as Billboard, Cashbox, or Variety

   C. Student work sheet

IV. Procedure:

   A. Select a new record release by a popular artist or group.

   B. Prepare a bulletin board or classroom chart similar to the one below:


A HIT?
Singer's Name
Song
Top 100
POPULARITY OF "TITLE OF SONG" by ______
RANK
10
20
30
40
50
60
70
80
90
100
1 2 3 4 5 6
WEEKS

The display should include:

1. Information for motivation purposes such as ads, pictures, and record charts obtained from Billboard or other weekly magazines.
2. The line graph completed for the first four weeks
    a. The scale along the bottom of the graph should indicate the date.
    b. The scale along the left side of the graph should indicate how the record ranks in terms of sales.

C. Discuss the bulletin board with the class. Include the following questions:

1. How many have heard the selected recording?
2. Is this a "hit" record? A definition of "hit" must be formed. Probably the class will decide that a record must be in the top ten to be a "hit".
3. What are some of the qualities that make a record a "hit"?
4. What was the position of the record the first week? second week? third week? fourth week?
5. Can you tell from the graph what position the record will hold on the fifth week?
6. What determines the position of a record during a given week? Billboard listings are compiled from national retail sales and radio station airplay by the Music Popularity Department of Record Market Research, Billboard Magazine. Most local listings are compiled in a similar way from local sources.
7. In which week can we see the greatest increase in popularity?
8. How many positions did the record increase during the first week?
9. How does the scale along the left side of the graph differ from a regular scale? The smallest number is at the top of the graph and the largest at the bottom.
10. Why would the graph be unsatisfactory if this scale were written in the regular way? As the popularity of the record increased, the points on the graph would be plotted lower on the scale.
11. What does this graph tell you about this record? This will depend on the record, but these general points should be brought out:
    a. The week of the greatest increase in popularity, or

the least increase in popularity

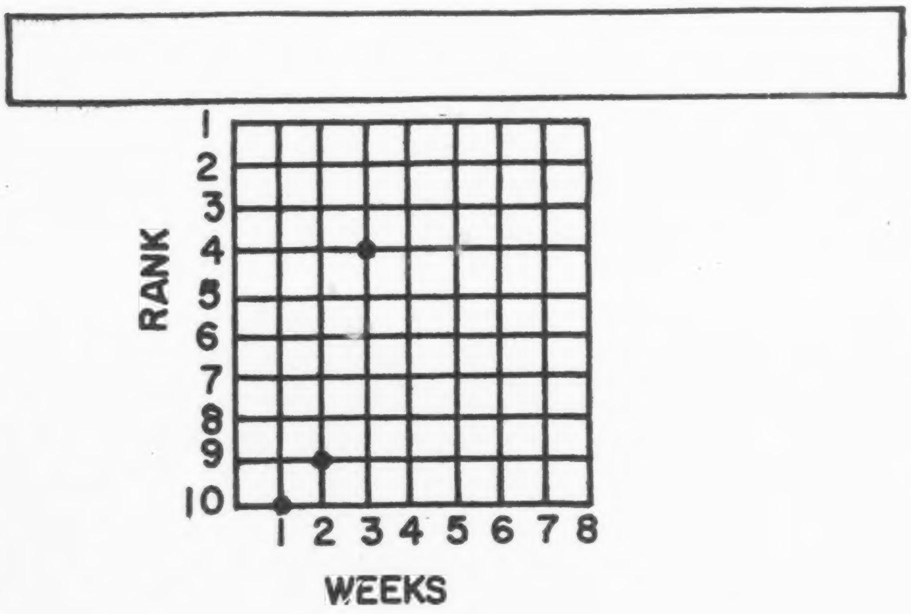
   b. Rate of increase - average weekly increase

D. Continue to plot the success of the record for several weeks.

E. The student work sheet entitled, "Graphing A Hit" may be used as assessment or the students may collect data on a record that has been a hit for a series of weeks and organize the data to construct another line graph.

GRAPHING A HIT

A new combo, "The Millionaires," recorded a song called "Rockin Taxes." The graph below shows the position of the record in the top 10 for the first three weeks.



I'm a $$ $airo
yea yea yea

1. Plot on the graph where the record would be if it became number 2 on the fourth week, was number 1 for the next two weeks, and then was number 5 the seventh week.
2. Give the graph a title in the space provided.
3. How many weeks are shown on this graph? ____________________
4. How many weeks did it take "Rockin Taxes" to get to the top of the chart? ____________________
5. How long did "Rockin Taxes" remain in the number 1 position? ____________________
6. Can you tell what the position of "Rockin Taxes" would be on the eighth week? ____________________
7. Does the graph tell the position of the record after $3\frac{1}{2}$ weeks? __________
8. Between what two weeks was the increase in popularity the greatest? ____________________
9. When did the popularity of the record start to go down? ______________
10. Was this record a "hit"? ____________________

MEAN, MEDIAN, MODE

Teacher Commentary for Film

I. Unit: Statistics

II. Objectives: The student should be able to:

- A. Name and identify the mean, median, and mode
- B. State the principles:
    - 1. The mean is the sum of the scores divided by the number of scores
    - 2. The median is the middle score after the scores have been ranked from highest to lowest
- C. Apply these principles by computation
- D. Interpret the mean, median, and mode in related problems
- E. Distinguish among the mean, median, and mode

III. Materials: Motion picture film - Mean, Median, Mode. McGraw Hill Book Co.; Central Film Library

IV. Procedure:

- A. This film should be used immediately after teaching the measures of central tendency. Emphasize objectives C, D, and E.
- B. Motivate the students immediately before showing the film.
- C. List guide questions on the board in order to insure that students know the reason for viewing this film and what to look for.

Examples:

1. How do you compute the mean? the median? the mode?
2. Given these scores:

85
49
72
51
72
85

What is the mean?
What is the mode?
What is the median?

What would be the best average to use?

D. Check the answers to the guide questions to evaluate what the students learned from the film.

E. This film is about 15 minutes long. Therefore, it may be used as one band in a lesson.

ALGEBRA

ALGEBRA

I. Master Chart - Grades Six through Eleven

II. Grade Nine Chart

III. Behavioral Objectives

IV. Activities

UNIT ALGEBRA GRADE(S) Six through Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Set	6, 8	6, 8		6	6					6
Member	6, 7	6, 7			6					6
Types of Sets	6, 7	6, 7		6	6, 7					6, 7
Relationship Between Sets	6, 8, 9	6, 8, 9		6, 8	6, 8					6, 8
Methods of Describing Sets	6	6			6					6
Operations With Sets	6	6			6					6
Language of Algebra	6, 7, 8 9, 10	6, 7, 8, 9 10		9	7, 8, 9	7			9	6, 7, 8, 9
Symbols for Operations	6, 7, 8	6, 7, 8, 9	6	8, 9	9					6, 7, 8
Symbols for Grouping	6, 10	6, 10	6, 7	10		6, 7, 9, 10	6, 7, 9, 10		10	6, 10
Evaluating Algebraic Expressions			9, 10, 11	10, 11	9, 10, 11	9, 10, 11	9, 10, 11			
Number Line	8, 9	8, 9	9	8, 9		9	9		8, 9	9
Operations With Rationals	9, 10	9, 10	10	9	9	9, 10	9, 10			9
Similar Terms	10, 11	10, 11	10, 11		10, 11	10, 11	10, 11			10, 11
Open Sentences With One Operation			9	9		9	9			
Open Sentences With Combined Operations	9, 11	9, 11	9, 11		9, 11					
Inequalities With One Operation	9	9	9		9					9
Inequalities With Combined Operations	11	11			11	11	11			11
Rationals	9	9		9	9				9	9

UNIT ALGEBRA GRADE(S) Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Subset	9	9								
Words Implying Operations	9	9								9
Words Implying Grouping	9	9								
Constant	9	9			9					9
Ordered Pair	9	9			9					
Symbols of Multiplication		9		9	9					
Exponent in Order of Operations						9	9			
Evaluating Algebraic Expressions			9			9				
Solution of Formulas for Dependent Variable			9							
Formula Derivation			9		9					
Rationals	9	9		9	9				9	
Correspondence of Pts. on Number Line to Rationals	9	9		9					9	
Addition of Rationals on Number Line			9	9			9			
Multiplication of Rationals on Number Line			9	9		9	9			
Addition of Rationals	9	9		9	9	9	9			
Properties of Addition	9	9		9		9	9			9
Multiplication of Rationals	9	9	9			9	9			
Properties of Multiplication	9	9	9			9	9			9

UNIT ALGEBRA GRADE(S) Nine

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Subtraction of Rationals (As Inverse of Addition)	9	9		9			9			9
Division of Rationals (As Inverse of Multiplication)	9	9		9			9			9
Distributive Property	9	9		9			9			9
Multiplication Property of Zero				9		9	9			9
Solution of Equations Involving One Operation	9	9	9	9		9	9			
Solution of Equations Involving Combined Operation	9	9	9		9					
Solution of Inequalities	9	9	9		9					9

ALGEBRA - Grade 9

Page

Subset

The student should be able to:

1. Name and identify the symbol for subset

Words Implying Operations

The student should be able to:

1. Name and identify words which denote operations; e.g. "increased by," "diminished by," "product of," etc.
2. Distinguish among these words

Words Implying Grouping

The student should be able to:

1. Name and identify words which denote grouping; e.g. "the quantity," "the sum of," "the difference of"

Constant

The student should be able to:

1. Name and identify a constant
2. Describe a constant as a symbol which always represents the same number in a situation
3. Distinguish between constant and variable

Ordered Pair

The student should be able to:

1. Name and identify an ordered pair
2. Describe an ordered pair using examples

Symbols of Multiplication

The student should be able to:

1. Identify the operation of multiplication when no symbol is given between two variables or between a variable and a constant

Page

2. Describe operation of multiplication when no symbol separates two variables
3. Construct examples which illustrate the operation of multiplication when no symbol separates two variables

Exponent in Order of Operations

The student should be able to:

1. State the principle that a quantity should be raised to a power before performing any other operations
2. Apply the above principle to solve related problems

Evaluation of Algebraic Expressions

The student should be able to:

1. State and apply the principle that the value of an algebraic expression can be found if each variable is replaced by a constant
2. Demonstrate how to use the above principle

Evaluation of Formulas for Dependent Variable

The student should be able to:

1. Demonstrate a procedure for evaluating the dependent variable in a formula

Formula Derivation

The student should be able to:

1. Describe a method of deriving a formula from a simple relationship written in tabular form
2. Demonstrate how to use the above method

Rational Numbers

The student should be able to:

1. Name and identify the symbols for the positive and negative rational numbers; e. g.

   $+3, \quad \frac{+3}{4}, \quad -4, \quad \frac{-2}{4}$

2. Construct the negative rational numbers as a "mirror image" of the positive rational numbers

Page

3. Describe a rational number as any number which can be expressed in fractional form where the denominator is not zero

4. Order subsets of the rational numbers

Correspondence of Points on the Number Line to Rational Numbers

The student should be able to:

1. Name and identify points representing rational numbers on the number line

2. Construct a number line representing the rational numbers

3. Order subsets of the rational numbers on the number line

Addition of Rational Numbers on Number Line

The student should be able to:

1. Demonstrate how to add rational numbers using the number line

2. Apply the principle for adding rational numbers using the number line A-14 A-18

3. Construct the addends when given a graph of an addition problem

Addition of Two Positive Rational Numbers

The student should be able to:

1. Name and identify the addition of two positive rational numbers A-14

2. State the principle that when two positive rational numbers are added the sum is a positive rational

3. Apply the above principle in various problems

4. Demonstrate the above principle

Addition of Two Negative Rational Numbers

The student should be able to:

1. Name and identify the addition of two negative rational numbers A-14

2. State the principle that the sum of two negative rational numbers is a negative rational number

Page

3. Apply the principle to solve related problems
4. Demonstrate how to use the above principle

Addition of a Positive Number and a Negative Rational Number

The student should be able to:

1. Name and identify the addition of a positive rational number and a negative rational number
2. Describe that the sum of two rational numbers having unlike signs may be found by subtracting the "smaller" from the "larger" and giving the answer the sign of the "larger," or through specific examples A-14
3. Apply the principle to solve related problems
4. Demonstrate how to use the principle

Properties of Addition of Rational Numbers

(The statements below will apply to the following properties: closure, associativity, commutativity, and identity element)

The student should be able to:

1. Name and identify each property
2. Construct an example of each property
3. State and apply the principle of each property
4. Distinguish among the properties

Additive Inverse of a Rational Number

The student should be able to:

1. Name and identify the additive inverse of a rational number
2. State the principle that the sum of a rational number and its additive inverse is zero
3. Apply the principle to solve related problems
4. Construct an example which illustrates the principle

Page

Multiplication of Rational Numbers on Number Line

The student should be able to:

1. Demonstrate how to multiply rational numbers (positive times positive and negative times positive) on the number line
2. State and apply the principle that a multiplication may be expressed as repeated addition
3. Construct an example of multiplication using the number line
4. Construct the factors when given the graph of a multiplication problem

Multiplication of Rational Numbers

The student should be able to:

1. Name and identify the operation of multiplication involving rational numbers

Multiplication of Two Positive Rational Numbers

The student should be able to:

A-13

1. State and apply the principle that the product of two positive rational numbers is a positive rational number
2. Demonstrate how to use the principle to solve related problems

Multiplication of a Positive Rational Number and a Negative Rational Number

A-13

The student should be able to:

1. State and apply the principle that the product of a positive rational number and a negative rational number is a negative rational number
2. Demonstrate how to use the principle to solve related problems

Multiplication of Two Negative Rational Numbers

The student should be able to:

A-13

1. State and apply the principle that the product of two negative rational numbers is a positive rational number
2. Demonstrate how to use the principle to solve related problems

Properties of Multiplication of Rational Numbers

Page

(The statements below apply to the following properties: closure, associativity, commutativity, and identity element)

The student should be able to:

1. Name and identify each property
2. Construct an example of each property
3. State and apply the principle of each property
4. Distinguish among the properties

Multiplicative Inverses of Rational Numbers

The student should be able to:

1. Name and identify the multiplicative inverse for every rational number except zero
2. State and apply the principle that the product of a rational number (except zero) and its multiplicative inverse is one
3. Construct an example illustrating the principle
4. Distinguish between inverse element and identity element

Distributive Property of Multiplication over Addition of Rational Numbers

The student should be able to:

1. Name and identify the distributive property
2. Construct an example of the distributive property
3. Apply the principle to solve related problems
4. Distinguish among the distributive property and the associative properties for addition and multiplication

Multiplicative Property of Zero for Rational Numbers

The student should be able to:

1. State and apply the principle that the product of any rational number and zero is zero
2. Construct an example of the principle
3. Distinguish between the product of a rational number and zero and the product of a rational number and one

Page

Subtraction as the Inverse Operation of Addition for Rational Numbers

The student should be able to:

1. Name and identify the operation of subtraction of rational numbers
2. Apply the principle that $a - b = a + (-b)$ when a and b are rational numbers — A-18
3. Construct an example of subtraction of rational numbers
4. Distinguish subtraction from addition

Division as the Inverse Operation of Multiplication for Rational Numbers

The student should be able to:

1. Name and identify the operation of division using rational numbers
2. Apply the principle that $a \div b = a \cdot \frac{1}{b}$ when a, b are rational numbers, $b \neq 0$
3. Construct an example using the principle
4. Distinguish between the operations of multiplication and division

Open Sentences Involving Addition and Subtraction

(Use examples of the form $x + \frac{3}{2} = \frac{7}{6}$ and $x - \frac{1}{2} = \frac{5}{4}$. The latter example should first be rewritten as $x + (-\frac{1}{2}) = \frac{5}{4}$

The student should be able to:

1. State and apply the principle that the solution for $x + a = b$ may be found by adding the additive inverse of a
2. Construct examples illustrating this rule
3. Demonstrate the procedure for solving equations

Page

Open Sentences Involving Multiplication and Division

(Sentences of the type $\frac{x}{a} = 1$ should be first expressed as $\frac{1}{a} \cdot x = 0$)

The student should be able to:

1. State and apply the principle that the solution may be found by multiplying both members of the equation by the multiplicative inverse of the coefficient of the variable
2. Construct examples to illustrate this principle
3. Demonstrate the procedure for solving equations

Open Sentences Involving Combined Operations

(The following operations will be considered with respect to open sentences involving multiplication-addition, multiplication-subtraction, multiplication-division; division-addition, division-subtraction; addition-division, subtraction-division)

The student should be able to:

1. Name and identify the operations involved
2. Demonstrate a procedure for determining the solution for various equations
3. Describe the method used in the solution of an equation

Solution of Inequalities Involving One Operation

The student should be able to:

1. Name and identify open sentences involving inequality
2. Demonstrate a procedure for solving inequalities
3. Describe the procedure for solving equations involving these operations
4. Distinguish inequalities from equalities

NOMOGRAPH
MULTIPLY-RATIONAL NUMBERS
Teacher Commentary

I. Unit: Algebra

II. Objectives: The student should be able to:

Demonstrate how to construct the product of two rational numbers using the nomograph

III. Materials:

A. Student work sheet "Nomograph"

B. A twelve inch ruler

IV. Procedure:

A. Distribute the materials to each student.

B. Discuss the three scales A, B and C.

1. Scales A and B begin with one in the middle of the nomograph. The points on these scales represent either positive or negative numbers above the line. Scale A is divided into positive numbers below the line and Scale B is divided into negative numbers below the line.

2. Scale C begins with one in the middle of the nomograph. Positive numbers up to one hundred are found above the line and negative numbers up to one hundred are found below the line.

3. Locate points on the scales and have the students identify them.

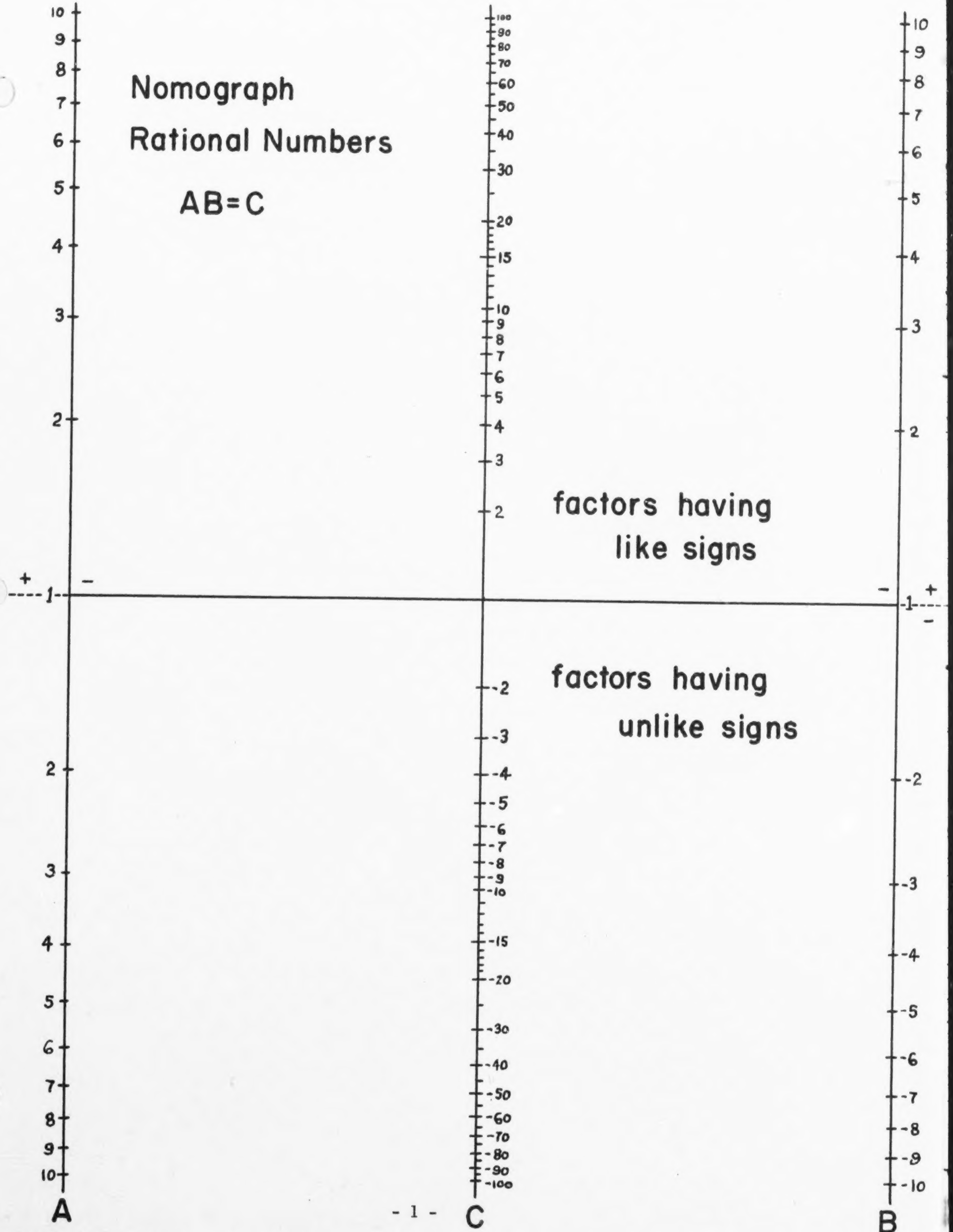
4. Have students locate points on the scales.

C. The top portion of the nomograph will be used when the two factors have like signs. The bottom portion of the nomograph will be used when the two factors have unlike signs.

D. In order to multiply (-2) (-3), locate -2 on scale A and (-3) on scale B. The line joining these two points will cross scale C at a point that represents the product (+6). Follow the same procedure when multiplying (2) (3).

E. In order to multiply (3) (-2) use the lower portion of the nomograph. Locate +3 on scale A and -2 on scale B. The line joining these two points will cross scale C at a point that represents the product (-6).

F. After the students have completed some written exercises, they could use the nomograph to check their results.



Nomograph

Rational Numbers

AB=C

10
9
8
7
6
5
4
3
2
+
−
1
2
3
4
5
6
7
8
9
10
A
100
90
80
70
60
50
40
30
20
15
10
9
8
7
6
5
4
3
2
factors having
like signs
factors having
unlike signs
-2
-3
-4
-5
-6
-7
-8
-9
-10
-15
-20
-30
-40
-50
-60
-70
-80
-90
-100
C
10
9
8
7
6
5
4
3
2
−
+
1
−
-2
-3
-4
-5
-6
-7
-8
-9
-10
B

THE MEANING OF SIGNED NUMBERS AND HOW TO ADD THEM

Teacher Commentary

I. Unit: Fundamental Operations

II. Objectives: The student should be able to:

- A. Name and identify a signed number
- B. Demonstrate how to add signed numbers using the number line
- C. Demonstrate how to name the opposite (additive inverse) of a number
- D. Demonstrate how to rename a signed number using the operation of addition

III. Materials:

- A. Filmstrip-of-the-month entitled "The Meaning of Signed Numbers and How to Add Them"
- B. Tape recorded presentation entitled "Adding Signed Numbers," parts I, II, and III
- C. Parts I, II, and III of the work sheet entitled "The Meaning of Signed Numbers and How to Add Them"
- D. Pencil, eraser, and ruler
- E. Tape recorder
- F. Filmstrip projector

IV. Procedure:

- A. Preview the tape, filmstrip, and work sheet.
- B. The tape has been recorded at a speed of $7\frac{1}{2}$.
- C. The tape and filmstrip have been divided into three lessons.

  Do not give more than one part during any one class period.

  The title of each part is as follows:

  1. Part I. The Meaning of Signed Numbers - Frames 6 - 22 - Tape time approximately 14 minutes
  2. Part II. Adding Two Positive Numbers and Adding Two Negative Numbers - Frames 24 - 29 - Tape time approximately 8 minutes

3. Part III. Adding a Positive and a Negative Number - Frames 30 - 41 - Tape time approximately 15 minutes

D. Do each part of the work sheet following each part of the tape. Because some parts of the work sheets might prove difficult, permit the students, with directed study, to do the work sheet in class.

E. Evaluation - If the students were able to satisfactorily complete the work sheets, the purposes for which this presentation were intended will have been achieved.

Solutions:

Part I

A. 1. $x + 8 = 7$

2. $x = 2$, $x = 1$, $x = 0$, $x = -1$

3. $\begin{array}{r} 10 \\ \underline{-11} \end{array}$

4. 3, 2, 1, 0, -1

B. 1. zero

2. right

3. left

12. +5, -5

14. +3, -3

15. $-4\frac{1}{2}$

16. +6

17. -25

18. +1000

Part II

A. 1. right

2. left

3. a. 0

b. 4

c. 6

d. +4, +6

e. +10

4. a. 0
   b. 6
   c. 3
   d. -6, -3
   e. -9

B. 1. 0, 5, right, 1, right, +6, +6
   2. 0, 3, left, 5, left, -8, -8
   3. 0, 8, right, 7, right, +15, +15
   4. 0, 6, left, 9, left, -15, -15
   5. 0, 35, right, 26, right, +61, +61
   6. 0, 17, left, 28, left, -45, -45

Part III

A. 1. 0
   2. that number
   3. 0
   4. -10
   5. +7
   6. -5
   7. -15
   8. +35
   9. a. 0
      b. 3
      c. 7
      d. +3, -7
      e. -4
   10. a. 0
       b. 5
       c. 10
       d. -5, +10
       e. +5

B. 1. 0, 4, right, 3, left, +1, +1
   2. 0, 5, left, 7, right, +2, +2

3. 0, 6, left, 4, right, -2, -2

4. 0, 4, left, 3, right, -1, -1

5. 0, 50, right, 35, left, +15, +15

6. 0, 43, left, 18, right, -25, -25

C. 1. a. (-4) + (-3)

   b. opposite

   c. 0

   d. -3

2. a. +3, (+3) + (+1), 0, +1

   b. +5, (+5) + (+2), 0, +2

   c. -4, (-4) + (-2), 0, -2

   d. -3, (-3) + (-1), 0, -1

3. a. -3

   b. +22

   c. +22

   d. +50

   e. -23

   f. +10

Name ______________________

Date ______________________

THE MEANING OF SIGNED NUMBERS
AND
HOW TO ADD THEM

PART I

THE MEANING OF SIGNED NUMBERS

A. Complete the following sentences by placing in the space provided a word that will make each sentence true.

1. The equation below, that has a negative number for its solution, is ________________.

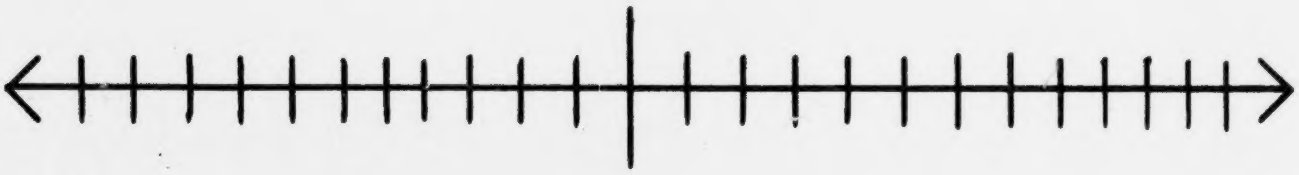
   $x + 8 = 10$ $x + 8 = 9$ $x + 8 = 8$ $x + 8 = 7$

2. Find the solution of all the equations in Problem 1.
3. The subtraction problem below whose difference is a negative number is ______________.

   $\begin{array}{r}14\\ \underline{-11}\end{array}\quad\begin{array}{r}13\\ \underline{-11}\end{array}\quad\begin{array}{r}12\\ \underline{-11}\end{array}\quad\begin{array}{r}11\\ \underline{-11}\end{array}\quad\begin{array}{r}10\\ \underline{-11}\end{array}$

4. Find the difference of all the subtractions in Problem 3.

B. Below is part of a number line. Answer the following questions.



1. The number in the middle of the number line is ____________.
2. Positive numbers are placed to the __________ of zero on the number line.
3. Negative numbers are placed to the __________ of zero on the number line.
4. On the number line, place the number 0 in the correct place.
5. On the number line, place the positive numbers on the correct side of zero.
6. On the number line, place the negative numbers on the correct side of zero.

7. Mark $+\frac{1}{2}$ on your number line.

8. Mark $-2\frac{1}{2}$ on your number line.

9. Mark $+4\frac{1}{2}$ on your number line.

10. Mark $-4\frac{1}{2}$ on your number line.

11. On your number line draw a circle around all the points that are 5 units from zero.

12. The points you found in Problem 11 were ______ and ______.

13. On your number line draw a square around all of the points that are 3 units from zero.

14. The points you found in Problem 13 were ______ and ______.

15. On your number line the opposite of $+4\frac{1}{2}$ is _______.

16. On your number line the opposite of -6 is _________.

17. The opposite of +25 is ________.

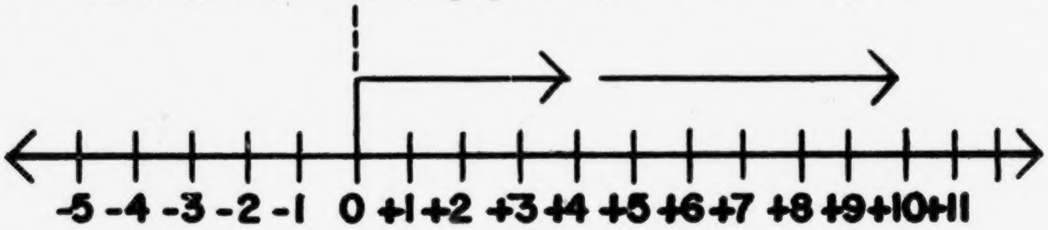
18. The opposite of -1000 is _______.

PART II

ADDING TWO POSITIVE NUMBERS AND TWO NEGATIVE NUMBERS

A. Complete the following sentences by placing in the space provided a word that will make the sentence true.

1. A positive movement on the number line is always to the ________.
2. A negative movement on the number line is always to the ________.
3. Answer the following questions concerning the number line.



   a. In the additions of these two numbers the starting point was ________.

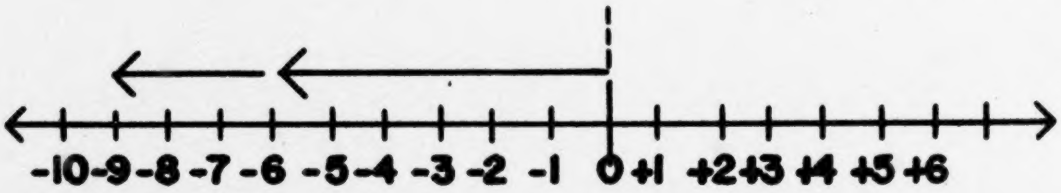
   b. The first movement was to the right __________ units.

   c. The second movement was to the right _________ units.

   d. This number line shows the addition of the signed numbers ________ and ________.

   e. The sum of these two numbers is _________.

4. Answer the following questions concerning the number line.



   a. In the addition of these two numbers, the starting point was __________.

   b. The first movement was to the left __________ units.

c. The second movement was to the left ________ units.

d. This number line shows the addition of the signed numbers _______ and ________.

e. The sum of these two number is __________.

B. The following problems are addition problems. For each problem make a number line with your ruler. By using arrows show how you would perform the addition. Answer the questions following each addition.

1. (+5) + (+1)

To add (+5) + (+1) I start at ______. I then move ______ units to the ______. I then move ______ unit to the ______. I am located at ______. The sum of (+5) + (+1) is ______.

2. (-3) + (-5)

To add (-3) + (-5) I start at ______. I then move ______ units to the ______. I then move ______ units to the ______. I am located at ______. The sum of (-3) + (-5) is ______.

3. (+8) + (+7)

To add (+8) + (+7) I start at ______. I then move ______ units to the ______. I then move ______ units to the ______. I am located at ______. The sum of (+8) + (+7) is ______.

4. (-6) + (-9)

To add (-6) + (-9) I start at ______. I then move ______ units to the ______. I then move ______ units to the ______. I am located at ______. The sum of (-6) + (-9) is ______.

5. Without using the number line explain how you would add (+35) + (+26).

To add (+35) + (+26) I start at ______. I then move ______ units to the ______. I then move ______ units to the ______. I am now located at ______. The sum of (+35) + (+26) is ______.

6. Without using the number line explain how you would add (-17) + (-28).

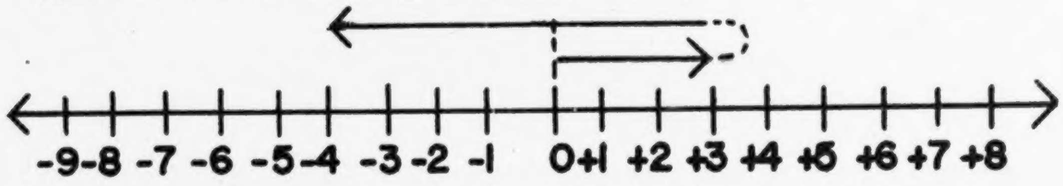
To add (-17) + (-28) I start at ______. I then move ______ units to the ______. I then move ______ units to the ______. I am now located at ______. The sum of (-17) + (-28) is ______.

PART III

ADDING A POSITIVE AND A NEGATIVE NUMBER

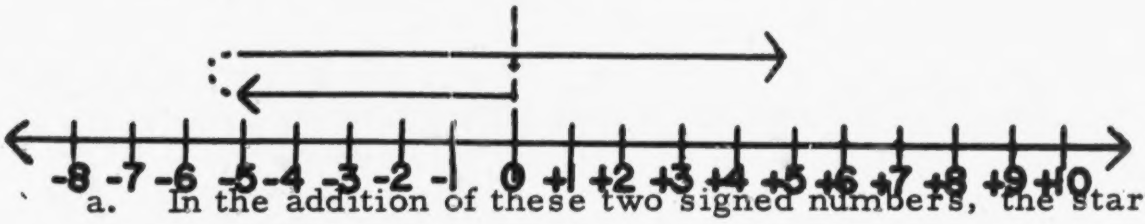
A. Complete the following sentences by placing in the space provided a word or number that will make the sentence true.

1. When adding on the number line, the number that has no movement is __________.
2. Whenever 0 is added to any signed number, the sum is always __________.
3. The sum of a signed number and its opposite is __________.
4. The number that makes the number sentence $(+10) + \square = 0$ true is __________.
5. The number that makes the number sentence $(-7) + \square = 0$ true is __________.
6. The number that makes the number sentence $(0) + (-5) = \square$ true is __________.
7. The opposite of +15 is __________.
8. The opposite of -35 is __________.
9. Answer the following questions concerning the number line.



   a. In the addition of these two signed number the starting point was __________.
   b. The first movement was to the right ________ units.
   c. The second movement was to the left ________ units.
   d. This number line shows the addition of the signed numbers __________ and __________.
   e. The sum of these two number is __________.

10. Answer the following questions concerning the number line.



a. In the addition of these two signed numbers, the starting point was ________.

b. The first movement was to the left ________ units.

c. The second movement was to the right ________ units.

d. The number line shows the addition of the signed numbers ________ and ________.

e. The sum of these two numbers is ________.

B. The following problems are addition problems. For each problem make a number line with your ruler. By using arrows show how you would perfrom the addition. Answer the questions following each addition.

1. (+4) + (-3)

To add (+4) + (-3) I start at ________.

I then move ________ units to the ________.

I then move ________ units to the ________.

I am located at ________. The sum of (+4) + (-3) is ________.

2. (-5) + (+7)

To add (-5) + (+7) I start at ________.

I then move ________ units to the ________.

I then move ________ units to the ________.

I am located at ________. The sum of (-5) + (+7) is ________.

3. (-6) + (+4)

To add (-6) + (+4) I first start at __________.
I then move _________ units to the _________.
I then move _________ units to the _________.
I am located at __________. The sum of (-6) + (+4) is ________.

4. (-4) + (+3)

To add (-4) + (+3) I first start at __________.
I then move _________ units to the ________.
I then move _________ units to the ________.
I am located at __________. The sum of (-4) + (+3) is ________.

5. Without using the number line explain how you would add (+50) + (-35).

To add (+50) + (-35) I start at ___________.
I then move _________ units to the _______.
I then move _________ units to the _______.
I am located at __________. The sum of (+50) + (-35) is _______.

6. Without using the number line explain how you would add (-43) + (+18).

To add (-43) + (+18) I start at __________.
I then move _________ units to the ________.
I then move _________ units to the ________.
I am located at __________. The sum of (-43) + (+18) is _______.

NOMOGRAPH
ADD-SUBTRACT - RATIONAL NUMBERS
Teacher Commentary

I. Unit: Algebra

II. Objectives: The student should be able to:

Demonstrate how to construct the sum and difference of two rational numbers using the nomograph

III. Materials:

A. Student work sheet "Nomograph"

B. A twelve inch ruler

IV. Procedure:

A. Distribute the materials to each student.

B. Discuss the three scales A, B and C.

1. Scales A and B begin with zero in the middle of the nomograph. Positive numbers up to ten are found above the line, and negative numbers up to negative ten are found below the line.

2. Scale C begins with zero in the middle of the nomograph. Positive numbers to twenty are found above the line and negative numbers to negative twenty are found below the line.

3. Locate some points on the scales and have the students identify them. Be sue to include fractions.

4. Have the students locate points on the scales.

C. In order to add 6 + (-5), locate six on scale A and -5 on scale B. The line joining these two points will cross scale C at a point that represents the sum (+1).

D. In order to subtract (-2) - (+6), locate negative two on scale C and six on scale A. The line joining these two points will cross scale B at a point that represents the difference (-8).

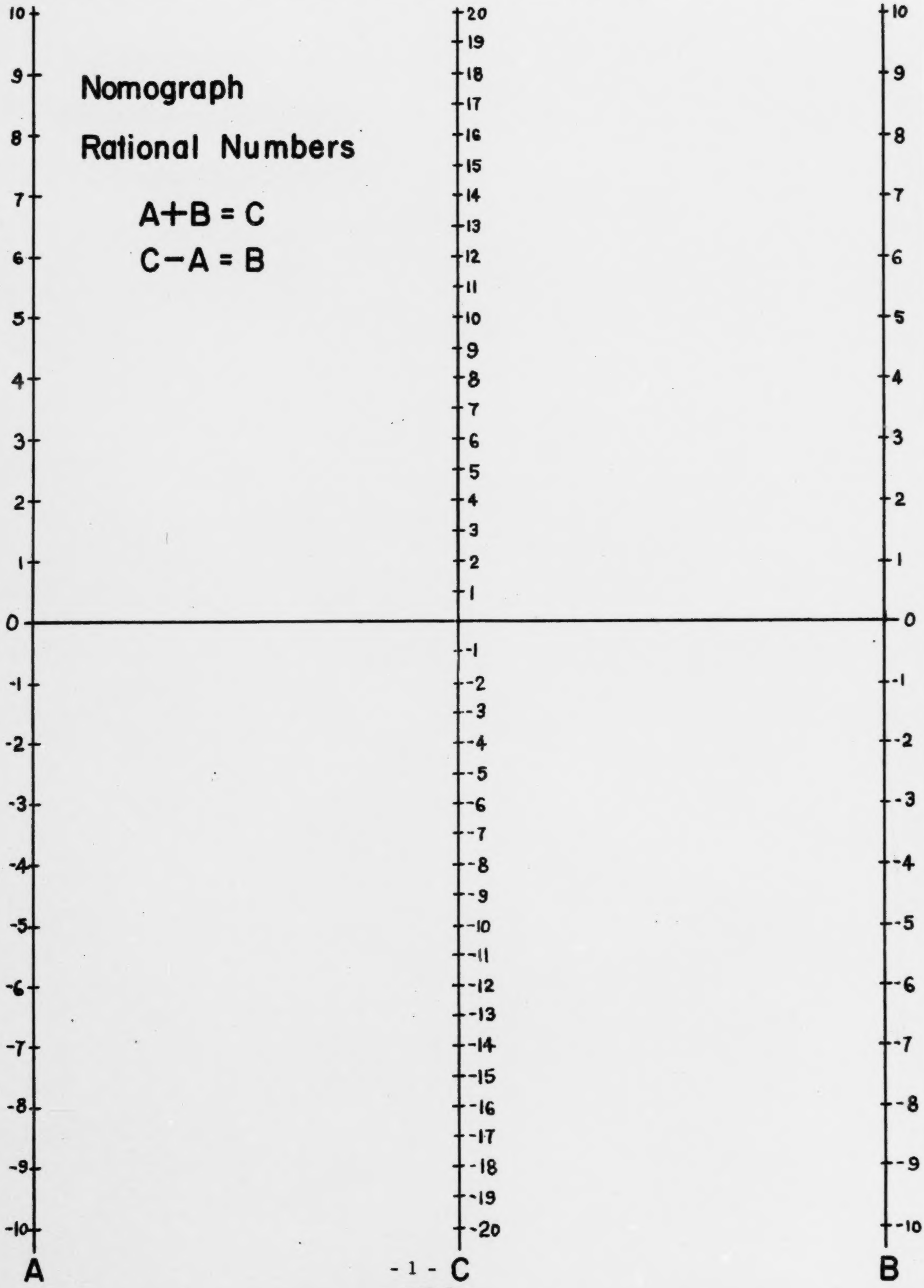
E. After the students have completed some written exercises, they could use the nomograph to check their results.

Nomograph

Rational Numbers

$A+B = C$

$C-A = B$


A
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
C
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
-13
-14
-15
-16
-17
-18
-19
-20
B
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10

LOGIC

LOGIC

I. Master Chart - Grades Six through Eleven

UNIT LOGIC GRADE(S) Ten and Eleven

TOPIC	NAME	IDENTIFY	DEMON-STRATE	CONSTRUCT	DESCRIBE	STATE THE PRINCIPLE	APPLY THE PRINCIPLE	INTERPRET	ORDER	DISTIN-GUISHING
Equivalent Phrases and Sentences	10	10		10						8, 10
Assumptions	10	10		11	10					10
"If - Then" Statements	10, 11	10, 11		10				10		10, 11
Converse	11	11		11	11	11		11		11
Inverse	11	11		11	11	11		11		11
Circular Reasoning (Doubletalk)										11
Valid Arguments					10, 11					10, 11
Non-Valid Arguments					10, 11					10, 11
Syllogism (Chain of Logic)										11
Indirect Proof						11	11			
Venn Diagrams	10	10						10		
Open Sentences	8, 10	8, 10		8, 10	8, 10					8, 10
Closed Sentences	8, 10	8, 10		8, 10	8, 10					8, 10
Deductive Reasoning					11					11
Inductive Reasoning					11					11
Counterexample				10	10	10	10			

RECREATIONAL ACTIVITIES

RECREATIONAL ACTIVITIES

The units contained in this section are supplementary activities for recreation. These may be used to provide variety throughout the year as well as in the daily lesson.

HIDDEN MESSAGE

Teacher Commentary

A Recreational Activity on Adding, Subtracting, and Rounding Fractions

I. **Materials:** **Student work sheets**

II. **Procedure:**

- **A. Distribute a work sheet to each student**
- **B. Each student is to decode the hidden message**
- **C. Solution: M A T H I S A U S E F U L T O O L**

A HIDDEN MESSAGE

Secret messages are sometimes in code. Often the code will have numbers or letters in it. Today you will figure out a message by solving examples.

I. Round each answer to the nearest whole number. Each solution stands for a letter.

1. $10\frac{1}{4} + 2\frac{1}{2}$ ____13____
2. $6 - 5\frac{1}{4}$ ________
3. $17\frac{1}{8} + 3\frac{1}{8}$ ________
4. $6\frac{1}{8}$ more than $1\frac{3}{4}$ ________
5. $\frac{7}{8}$ less than 10 ________
6. $4\frac{2}{3} + 14\frac{1}{4}$ ________
7. $20\frac{1}{8} - 18\frac{3}{4}$ ________
8. $3\frac{1}{3} + 10\frac{3}{4} + 6\frac{2}{3}$ ________
9. $6\frac{3}{4} + 5\frac{7}{8} + 6\frac{1}{2}$ ________
10. $8\frac{3}{8} - 3\frac{1}{4}$ ________
11. $14\frac{1}{4} - 8\frac{3}{8}$ ________
12. $17\frac{3}{8}$ more than $3\frac{3}{8}$ ________
13. the sum of $2\frac{3}{4}$, $4\frac{1}{2}$, $3\frac{2}{3}$, $1\frac{1}{6}$ ________
14. $40 - 19\frac{7}{8}$ ________
15. $5\frac{1}{8} + 5\frac{1}{8} + 5\frac{1}{8}$ ________
16. $20 - 5\frac{1}{8}$ ________
17. $4\frac{1}{4} + 3\frac{1}{2} + 4\frac{1}{4}$ ________

II. Look at your first answer (13) and find that number in the decoding chart. Right under this number you will find the letter it stands for (M). Write this letter (M) in space 1 in the hidden message. Do the same thing for all of your answers, and you will see the hidden message.

Decoder

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

M
1 2 3 4 5 6 7 8 9 10 11 12 13

14 15 16 17

HIDDEN MESSAGE

Teacher Commentary

A Recreational Activity Involving Equations

I. Materials: Student work sheets

II. Procedure:

  A. Distribute a work sheet to each student.

  B. Have each student decode the hidden message.

  C. Solution: O U T O F S I G H T

A HIDDEN MESSAGE

Secret messages are sometimes in code. Often the code will have numbers or letters in it. Today you will figure out a message by solving some equations.

I. Each solution stands for a letter.

1. $2x = 30$
2. $x + 4 = 25$
3. $3x - 5 = 55$
4. $x - 8 = 7$
5. $2x + 2 = 14$
6. $x - 0 = 19$
7. $4x - 17 - 19$
8. $7x = 49$
9. $6x + 1 = 49$
10. $2x + 19 = 59$

II. Find your solution to each equation in the decoder. Right under this number will be the letter it stands for. Put this letter in the space in the hidden message.

Decoder

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Hidden Message

This message tells you where the astronaut is.

___ ___ ___   ___ ___   ___ ___ ___ ___ ___
1 2 3   4 5   6 7 8 9 10

HIDDEN MESSAGE

Teacher Commentary

A Recreational Activity on Adding, Subtracting and Rounding Decimal Fractions

I. Materials: Student work sheets

II. Procedure:

A. Distribute a work sheet to each student.

B. Each student is to decode the hidden message.

C. Solution: Y O U R A N S W E R S A R E R I G H T

A HIDDEN MESSAGE

Secret messages are sometimes in code. Often the code will have numbers or letters in it. Today you will figure out a message by solving some examples.

I. Round the solutions to the nearest whole numbers. Each solution stands for a letter.

1. 16.9 + 8.2 ________
2. 7.4 + 6 + 2.04 ________
3. 32.1 - 10.7 ________
4. 3.3 + 6.5 + 8.24 ________
5. 11 - 9.9 ________
6. 20.7 - 6.9 ________
7. 26.32 - 7.23 ________
8. 7.6 + 7.45 + 7.867 ________
9. 12.86 - 8 ________
10. 9.004 + 9.163 ________
11. 8.63 + ____ = 27.13 ________
12. 6 - 4.69 ________
13. 1.6 + 11 + 3.4 + 2.27 ________
14. 8.9 - 4.3 ________
15. 5.9 + 5.9 + 5.9 ________
16. the difference between 26.867 and 35.4 ________
17. 18.94 minus 12.2 ________
18. 2.1 + 2.1 + 2.1 + 2.1 ________
19. 12.8 + 3.60 + 3.69 ________

II. Below is the decoder. Right under each number you have for an answer is a letter. Put this letter in the corresponding numbered space. Then you will see the message.

Decoder

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Hidden Message

___ ___ ___ ___   ___ ___ ___ ___ ___ ___ ___   ___ ___ ___
1 2 3 4   5 6 7 8 9 10 11   12 13 14

___ ___ ___ ___ ___
15 16 17 18 19

REVERSE-DIGIT PUZZLE

Teacher Commentary

A Recreational Activity on Subtraction and Addition with Reinforcement of Terms

I. Materials:

A. Paper

B. Pencil

II. Procedure:

A. Have each student write a 3-digit number on his paper. Each digit should be different.

B. Have them reverse the digits in the number.

C. Find the difference of the two numbers, smaller from larger.

D. Have a student tell you the units digit in his answer. You can tell him his answer.

E. Solution:

The sum of the units digit and the hundreds digit will be 9, the tens digit is always 9.

Example:		
	3-digit number	724
	reversed digits	427
	difference	297

Since units digit is 7, the answer is 297

F. If time allows, you may go one step further with this puzzle: Reverse the digits in the difference. If the difference is 99, put a zero in the hundred's place; then reverse the digits. Everyone should get the same sum: 1089

Example:		
	difference	297
	reversed digits	792
	sum	1089

THE FOURS GAME

Teacher Commentary

A Recreational Activity on Renaming Numbers Using Addition, Subtraction, Multiplication, and Division of Whole Numbers

I. Materials: Paper, pencil

II. Procedure:

Develop with the class the rules of the Fours Game.

The teacher may choose to omit certain rules, depending on the level of the students. Emphasize the importance of the order of operations when several operations are employed.

Once the students have mastered the rules, a long range (weekly or bi-weekly) assignment could be made. Example: Rename as many of the numbers from 1 to 20 with four fours as you are able. A bulletin board may be constructed to tabulate results daily.

The rules for renaming the numbers are as follows:

1. Use any of the numbers 1 to 9 exactly four times. Examples: 5, 5, 5, 5; 7, 7, 7, 7; etc.
2. These four numbers may be added, subtracted, multiplied, or divided. These processes may also be used in any combination. Example: $5 \times 5 + 5 \div 5$; $5 - 5 + 5 + 5$; $5 + 5 + 5 + 5$; etc.
3. The numbers may be placed together so that their place denotes different values. Examples: 5555; 55-55; 555-5; etc.
4. The number may be expressed as decimal examples: 55. 55; 555. 5; . 555 + . 5; etc.
5. The square root radical may be used. Examples: $\sqrt{5 \times 5} \div \sqrt{5 \times 5}$; $\sqrt{5 \times 5} + \sqrt{5 \times 5}$; etc.
6. The numbers may be expressed with the use of exponents. Examples: $5^5 - 5^5$; $5^5 \times 5^5$; $5^5 + 5^5$; etc.

Examples:

	TWOS	THREES	FOURS	NINES
0	$2 + 2 - 2 - 2$	$33 - 33$	$44 - 44$	$99 - 99$
1	$\frac{2 + 2}{2 + 2}$	$\frac{3 + 3}{3 + 3}$	$\frac{4 + 4}{4 + 4}$	$\frac{9 + 9}{9 + 9}$
2	$\frac{2}{2} + \frac{2}{2}$	$\frac{3}{3} + \frac{3}{3}$	$\frac{4}{4} + \frac{4}{4}$	$\frac{9}{9} + \frac{9}{9}$
3	$2 + 2 - \frac{2}{2}$	$\frac{3 + 3 + 3}{3}$	$\frac{4 + 4 + 4}{4}$	$\frac{9 + 9 + 9}{9}$
4	$2 + 2 + 2 - 2$	$\frac{3 \times 3 + 3}{3}$	$4 \times (4 - 4) + 4$	$\frac{9}{\sqrt{9}} + \frac{9}{9}$
5	$2 + 2 + \frac{2}{2}$	$3 + 3 - \frac{3}{3}$	$(4 \times 4 + 4) \div 4$	$\sqrt{9} + (\frac{9 + 9}{9})$
6	$2 \times 2 \times 2 - 2$	$\frac{3 + 3}{\frac{3}{3}}$	$4 + (4 + 4) \div 4$	$\frac{9}{\sqrt{9}} + \frac{9}{\sqrt{9}}$
7	$\frac{\frac{2}{2}}{.2} + 2$	$3 + 3 + \frac{3}{3}$	$4 + 4 - 4 \div 4$	$9 - (\frac{9 + 9}{9})$
8	$2 + 2 + 2 + 2$	$3 \times 3 - \frac{3}{3}$	$4 + 4 + 4 - 4$	$(9 \times 9 - 9) \div 9$
9	$\frac{22}{2} - 2$	$\frac{3 \times 3 \times 3}{3}$	$4 + 4 + \frac{4}{4}$	$\sqrt{9 \times 9} - 9 + 9$
10	$2 \times 2 \times 2 + 2$	$3 \times 3 + \frac{3}{3}$	$(44 - 4) \div 4$	$(9 \times 9 + 9) \div 9$

THE CASE OF THE FIBONACCI GERMS

Teacher Commentary

A Recreational Activity on Addition and Multiplication of Whole Numbers

I. Materials:

A. Story entitled "The Case of the Fibonacci Germs"

B. Direction sheet

C. Chart

II. Procedure:

A. Ask pupils if they are familiar with James Bond and other segret agents. Tell them that we have a secret agent we call James Bomb who has a problem we can help him solve.

B. Distribute the stories of "The Case of the Fibonacci Germs." Because this material may be difficult for some pupils, it will be necessary to direct the reading in the following manner.

1. You already have stimulated interest and gained some idea that the pupils have an adequate background of experience in "James Bond" type secret agent stories. Be sure the pupils realize that the purpose of the reading is to be able to identify and solve a particular problem.

2. Because the characters have names which are not within the children's basic sight vocabulary, it would be a good idea to write those words on the board before beginning the reading. Make sure the pupils can identify these names when they reach them in the story.

3. Ask questions to check comprehension of the problem they are to solve. They should at this point be able to state the problem to be solved and the basic facts needed to solve the problem.

4. Reread, either orally or silently, those parts of the story that might need review or reinforcement.

C. Distribute the two work sheets.

   1. The Direction Sheet is for students' use in filling out the chart.
   2. The Growth Chart of Fib Germs is to be the record of the students' work.

D. Help children who have difficulty in filling out the chart.

E. The answer to the problem is Yes. There were 13 germs alive at 5:00 P. M.

F. Suggest that there is a pattern in the numbers recorded on the chart. Can you find it? Let the class try several possibilities and then suggest that they look at any 3 consecutive numbers on the chart. Is there any relationship? (The sum of the first 2 numbers is equal to the third number.)

G. Try this pattern with other groups of three consecutive numbers on the chart.

H. Work with the class using this pattern to determine the number of germs which would have been alive at 6:00 P. M. and 7:00 P. M. if Bomb had not destroyed them.

I. Tell the pupils that this fascinating series of numbers was discovered in 1202 by Leonardo of Pisa, also known as Fibonacci. These are called Fibonacci numbers.

J. Here is another example:

   1. Pick any three numbers in order.
   2. Square the middle number.
   3. Multiply the first and third numbers.
   4. Compare this product with your first product.
   5. Do this a number of times until the pupils discover that the products always differ by 1.

Additional examples may be found in the following book:
Adler, Irving. The Giant Golden Book of Mathematics. Golden Press, N. Y. p. 30 ff

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THE CASE OF THE FIBONACCI GERMS

James Bomb is a secret agent. He found out that an enemy spy was trying to destroy America. The enemy spy had brought a deadly germ into the country. The germ was called Fibonacci, but James Bomb called it Fib. James Bomb knew he had to find the Fib germ and destroy it before it killed people. He had to work very fast because the germ was growing and making new germs. The Fib germ grew like this:

1. After one hour the first Fib germ made a new Fib germ. Then there were 2.
2. After two hours the first Fib germ made another Fib germ.

There would have been three Fib germs, but when the third one was made, the first Fib germ died. A Fib germ always made two new Fib germs and then died itself. Each new Fib germ made two more Fib germs before it died.

James Bomb jumped into his sports car and sped off in search of the Fib germs. The germs were hard to find. It took 5 hours for Bomb to find them. The first Fib germ was let loose at 12:00 noon. James Bomb found 13 Fib germs at 5:00 P. M. and destroyed them. Were all of the Fib germs destroyed or had some escaped?

James Bomb was very worried. If he did not destroy all of the germs he must find the rest. Maybe you can help James to find out if he destroyed all of the Fib germs.

THE CASE OF THE FIBONACCI GERMS

Direction Sheet

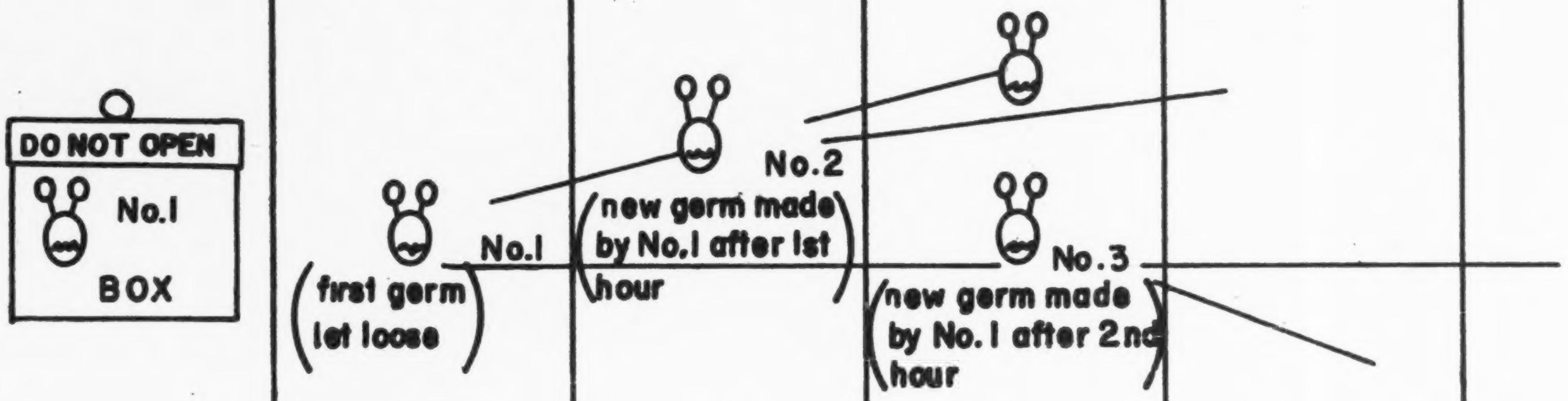
Facts

1. Each germ made 2 new germs - one after the first hour and the other in the second hour.
2. After each germ made 2 new germs it died.
3. The first germ was let loose at 12:00 noon. Call this germ #1.
4. At 1:00 P. M. a new germ was made by #1. Call this germ #2.
5. At 2:00 P. M. another germ was made by #1. Call this germ #3.
6. When germ #3 was made, germ #1 died.
7. Also at 2:00 P. M. a new germ was made by germ #2.
8. This process continued until all germs were destroyed.

How to Fill in the Chart

1. Use the fact sheet.
2. Part of the chart is filled in for you. You are to fill in the rest.
3. Each time a germ is made, put a picture of it in the correct hour column.
4. Keep your lines straight and not too far apart.
5. Keep in mind that every hour new germs were made. Also remember the germs which were made two hours before are now dead. For example, at 4:00 P. M. all of the germs in the 2:00 P. M. column are dead. At 5:00 P. M. all of the germs in the 3:00 P. M. column are dead. No germ can live more than two hours.
6. Do not let any germ make more than one germ the next hour and one germ the following hour.
7. When you have your chart filled in, count the number of Fib germs still alive at 5:00 P. M. before James Bomb arrived. James Bomb destroyed 13 Fib germs.
8. Did he destroy all of them?

GROWTH CHART OF "FIB" GERMS


DO NOT OPEN
No.1
BOX
No.1
(first germ let loose)
No.2
(new germ made by No.1 after 1st hour)
No.3
(new germ made by No.1 after 2nd hour)

Time of Day→	12:00 NOON	1:00 P.M.	2:00 P.M.	3:00 P.M.	4:00 P.M.	5:00 P.M.
total number of fib germs made each hour →	1	1	2			

BARRY, CLYDE, AND THE BRIDGES

Teacher Commentary

A Recreational Activity on Topology

I. Materials:

- A. Tape recorder and take-up reel
- B. Eight-station listening post
- C. Pencil and eraser
- D. Work sheet entitled "Barry, Clyde, and the Bridges", and accompanying tape-recorded presentation

II. Procedure:

- A. Equipment
    1. A tape recorder and an eight-station listening post is to be assembled.
    2. The tape recorder should be set at $7\frac{1}{2}$ speed setting.
    3. The tape will run approximately 25 minutes, not counting the one stop.
    4. Each student should receive a work sheet with the following changes on it.
        - a. Page 4 must be traced in color as follows:
            - (1) Section 1 - red
            - (2) Section 2 - green
            - (3) Section 3 - blue
            - (4) Section 4 - black
        - b. Page 5 must be traced in color as follows:
            - (1) Section 1 - red
            - (2) Section 2 - green
- B. Teacher
    1. Preview the tape recorded presentation.
    2. On the basis of this previewing, determine if additional information is needed for your individual group. Perhaps in some cases, more explanation will be needed to determine what is meant by the term "follow a path".

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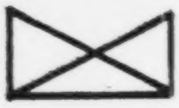
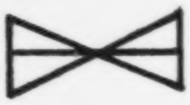
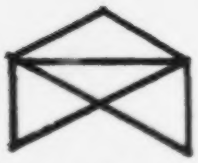
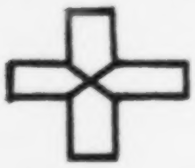
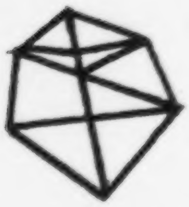
- B. Teacher
  1. Preview the tape recorded presentation.
  2. On the basis of this previewing, determine if additional information is needed for your individual group. Perhaps in some cases, more explanation will be needed to determine what is meant by the term "follow a path".

3. Each student should have in his possession a pencil and an eraser.

4. Each student should have a headphone, and should know how to adjust its volume.

5. One student should be appointed technician. He is to be the sole operator of the tape recorder. He should be fully instructed in how to turn the tape recorder off and on.

6. The answers to the five path problems on page 11 are as follows:

a.		3 even, 2 odd, yes
b.		5 even, 2 odd, yes
c.		6 even, 0 odd, yes
d.		9 even, 4 odd, no
e.		9 even, 2 odd, yes

Name ______________________________

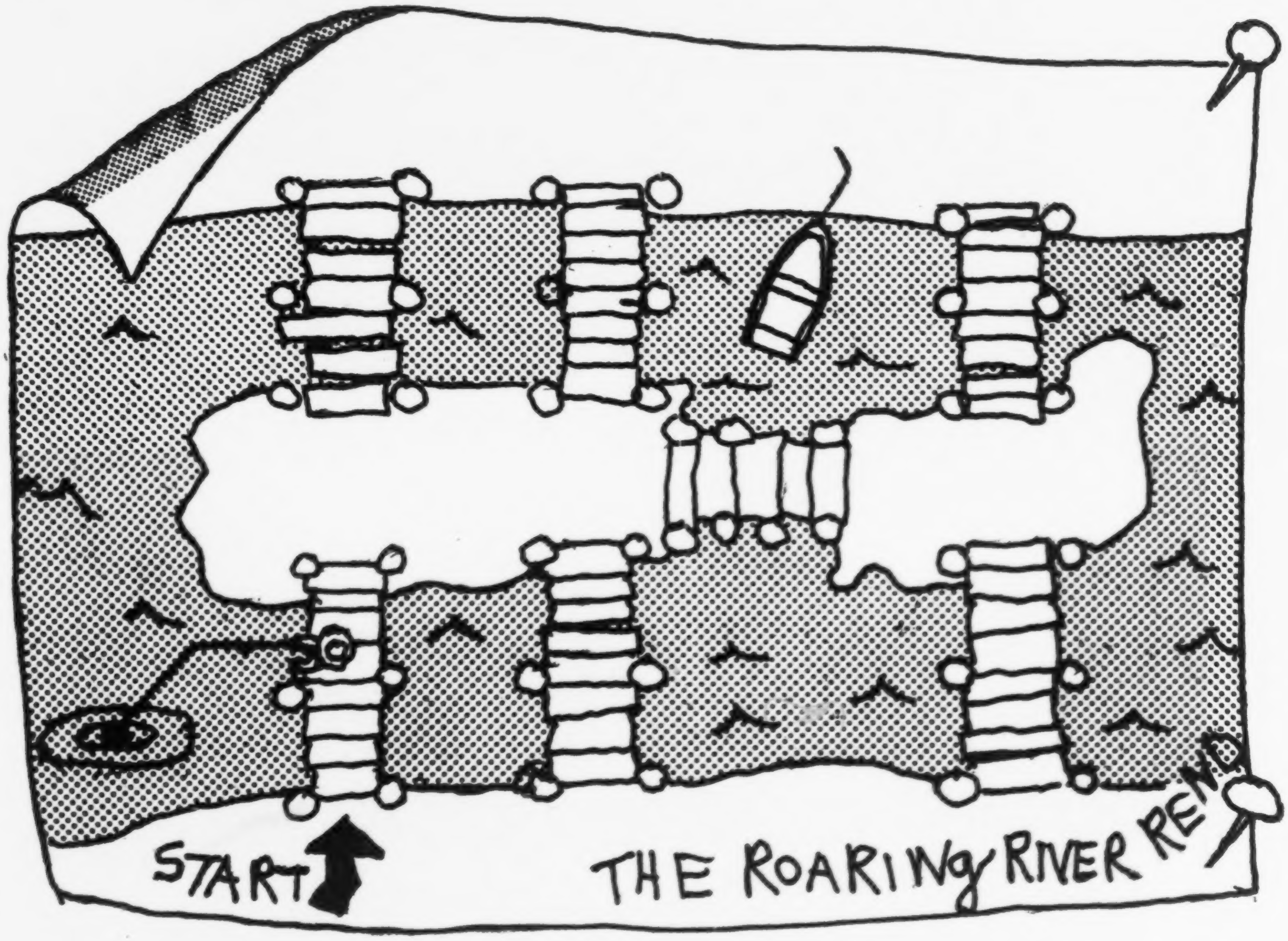
Date ______________________________

Barry, Clyde, and the BRIDGES

barry baa
clyde crumb

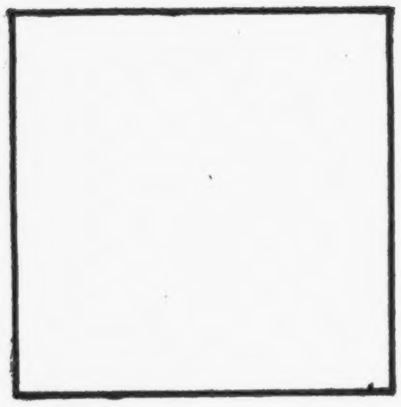
?
BUMBLING
BEND HIGH SCHOOL
?

Barry's Map


START
THE ROARING RIVER BEND

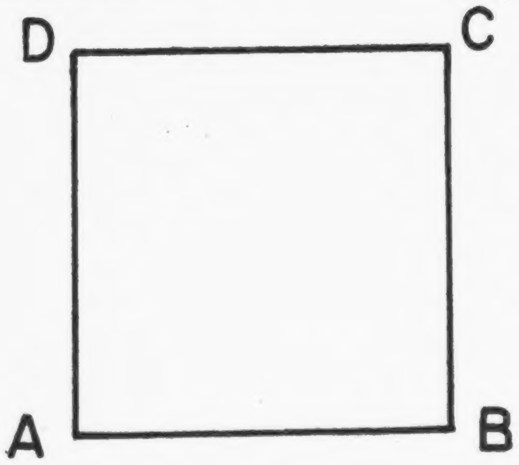
__________ YES, I think the bridges can be traveled only once.

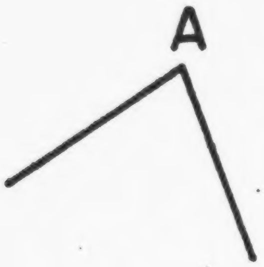
__________ NO, I do not think the bridges can be traveled only once.



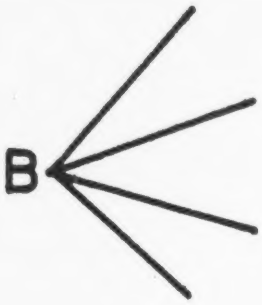
_____ YES, each side can be traveled only once.

_____ NO, each side cannot be traveled only once.

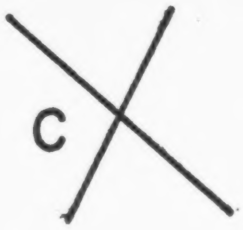




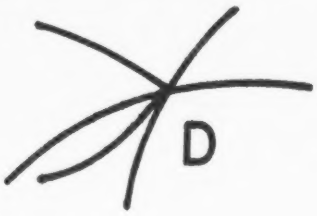
_____ paths



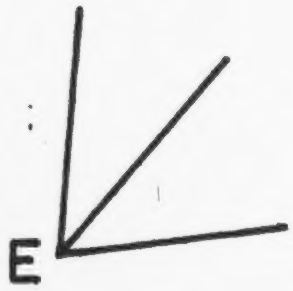
_____ paths



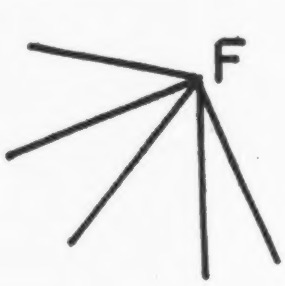
_____ paths



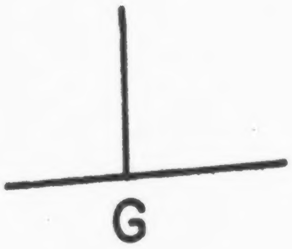
_____ paths



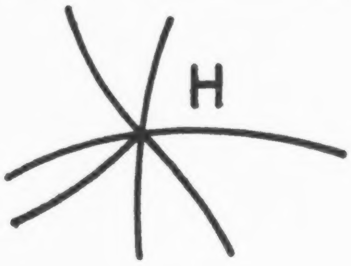
_____ paths



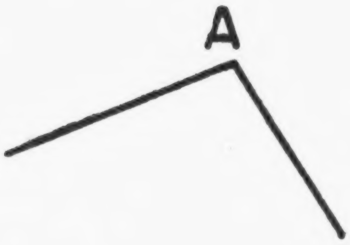
_____ paths



_____ paths



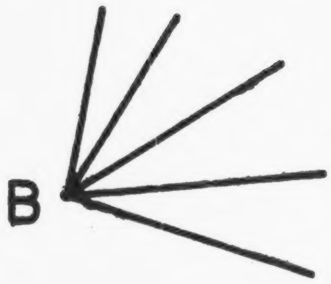
_____ paths



Number of Paths _____

Even _____

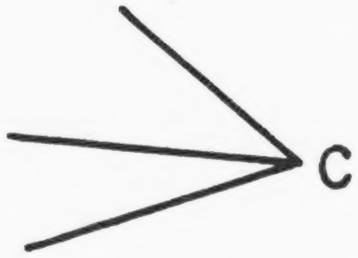
Odd _____



Number of Paths _____

Even _____

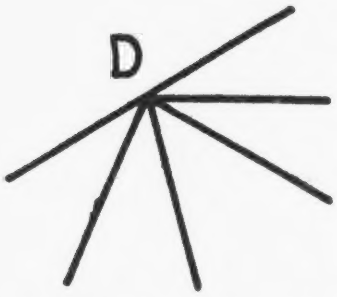
Odd _____



Number of Paths _____

Even _____

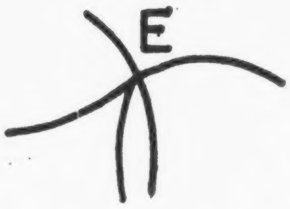
Odd _____



Number of Paths _____

Even _____

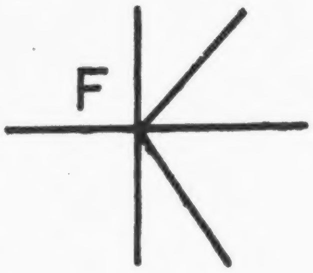
Odd _____



Number of Paths _____

Even _____

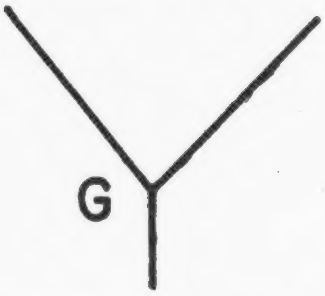
Odd _____



Number of Paths _____

Even _____

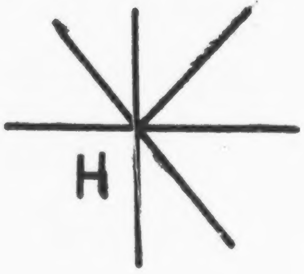
Odd _____



Number of Paths _____

Even _____

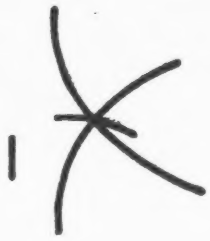
Odd _____



Number of Paths _____

Even _____

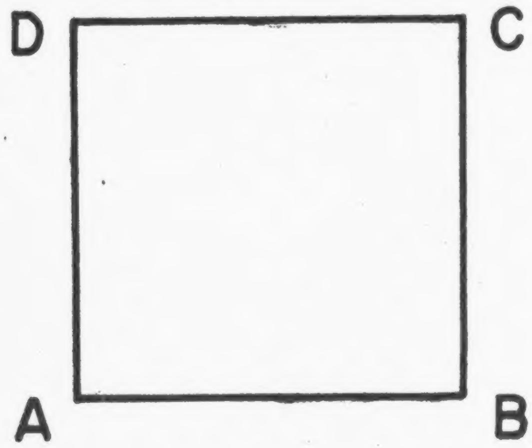
Odd _____



Number of Paths _____

Even _____

Odd _____

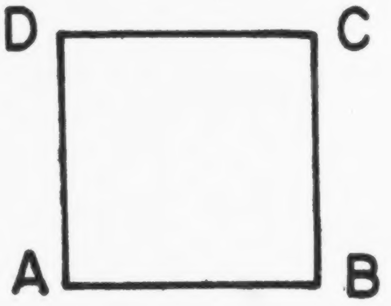
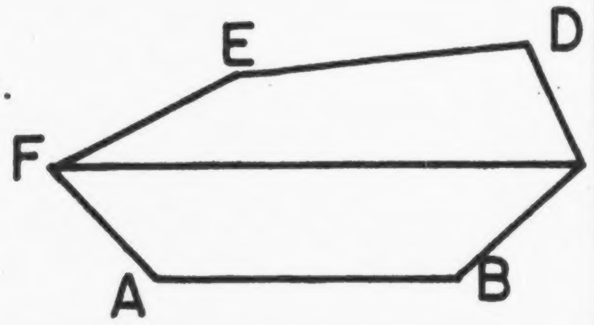
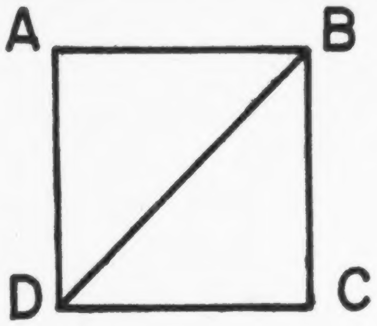
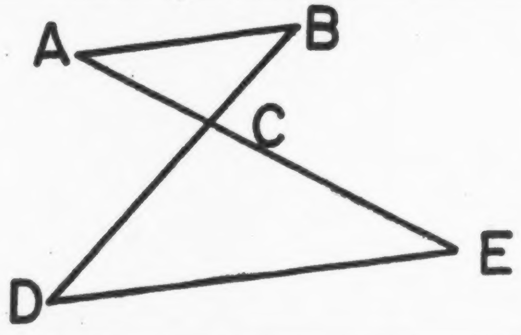
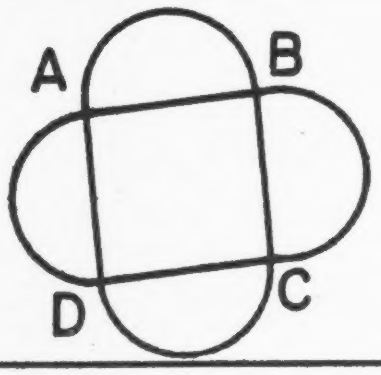


Corner A
Even _____
Odd _____

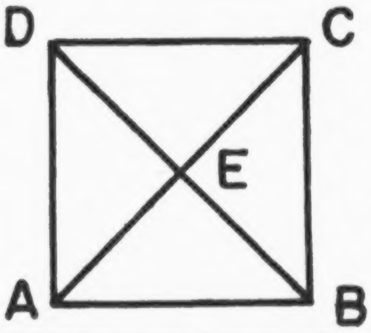
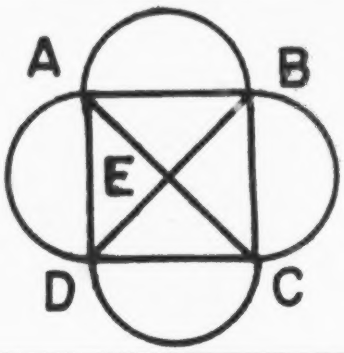
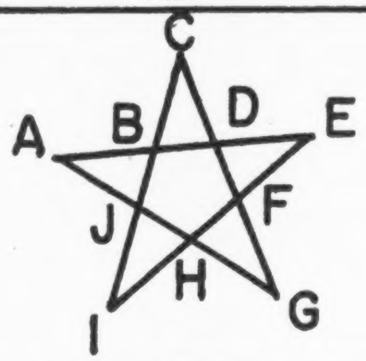
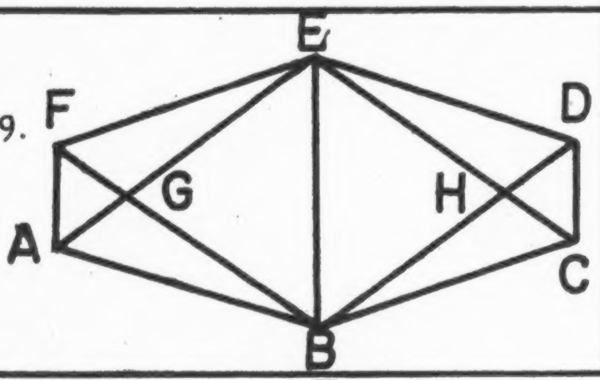
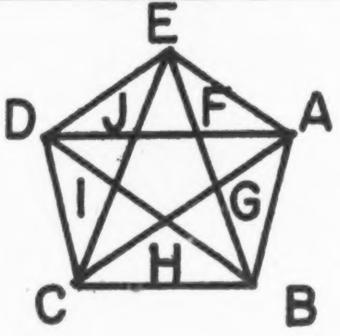
Corner C
Even _____
Odd _____

Corner B
Even _____
Odd _____

Corner D
Even _____
Odd _____

COLUMN 1	COLUMN 2 Number of even corners	COLUMN 3 Number of odd corners	COLUMN 4 Can it be traveled?
1. 	4	0	Yes
2. 			
3. 			
4. 			
5. 			

Go on to the next page.

COLUMN 1	COLUMN 2 Number of even corners	COLUMN 3 Number of odd corners	COLUMN 4 Can it be traveled?
6. 			No
7. 			No
8.  C A B D E J F H I G			Yes
9. 			No
10.  E D J F A I G H C B			Yes

QUESTION 1

Can all of the figures with zero odd corners be traveled?

_____ YES _____ NO

QUESTION 2

Can all of the figures with two odd corners be traveled?

_____ YES _____ NO

QUESTION 3

Can any figure with more than two odd corners be traveled?

_____ YES _____ NO

CONCLUSION 1

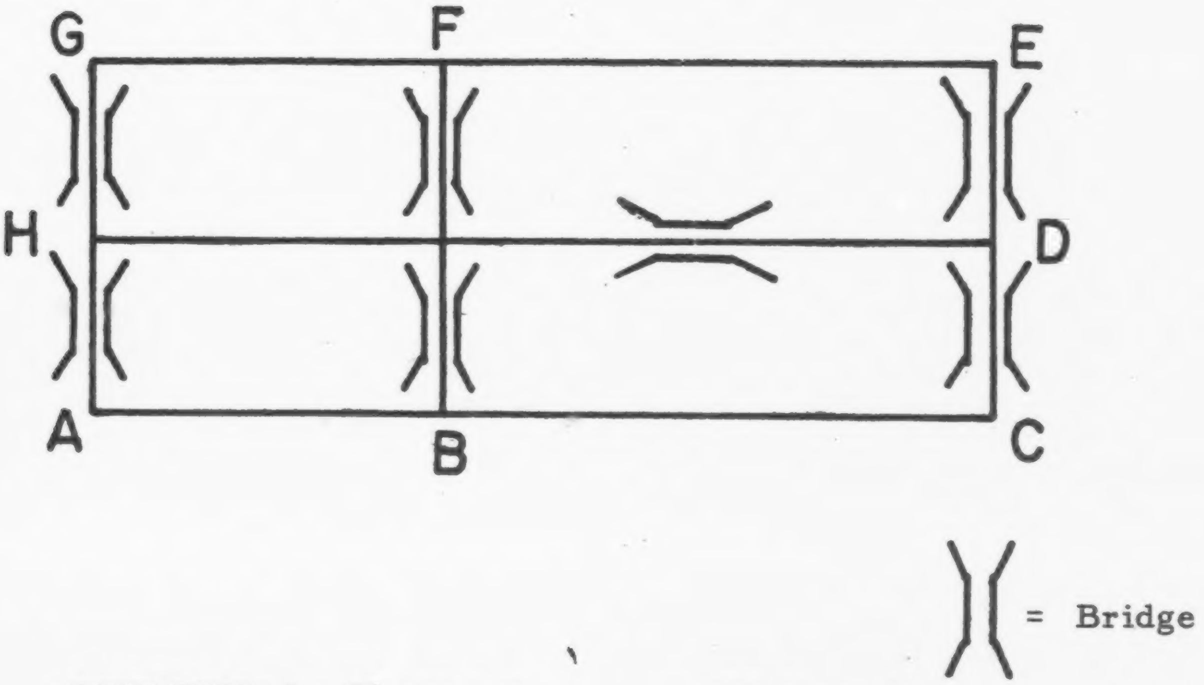
always
If a figure contains all even corners it can be traveled.
never

CONCLUSION 2

always
If a figure contains only two odd corners it can be traveled.
never

CONCLUSION 3

always
If a figure contains more than two odd corners it can be traveled.
never



QUESTION 1: There are ______ even corners.

QUESTION 2: There are ______ odd corners.

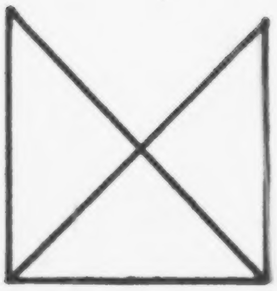
_____ YES, it is possible to travel this path problem.

_____ NO, it is not possible to travel this path problem.

The conclusion marked below is the conclusion which states why it is not possible to travel this path problem.

_____ Conclusion 1 _____ Conclusion 2 _____ Conclusion 3

1.



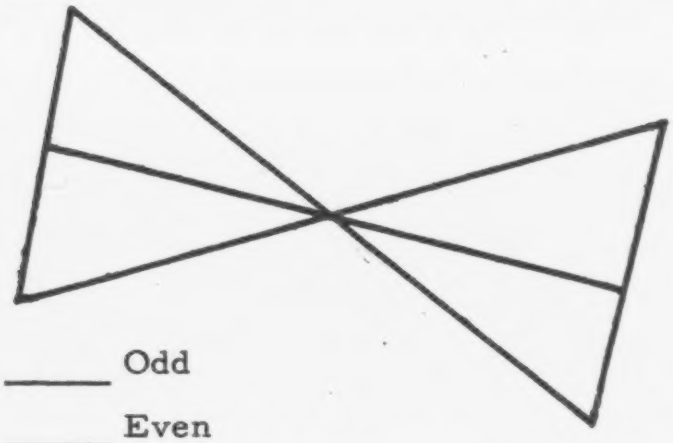
_____ Odd

_____ Even

_____ Possible

_____ Impossible

2.



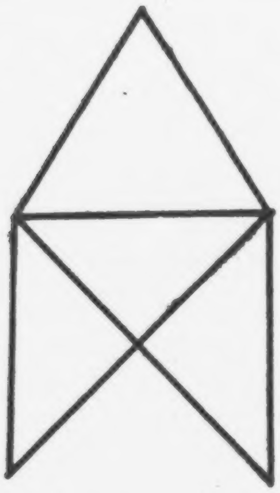
_____ Odd

_____ Even

_____ Possible

_____ Impossible

3.



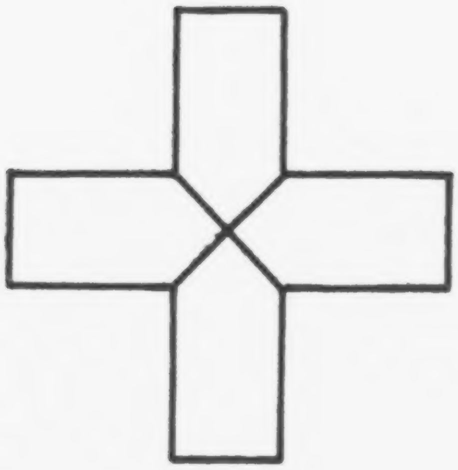
_____ Odd

_____ Even

_____ Possible

_____ Impossible

4.



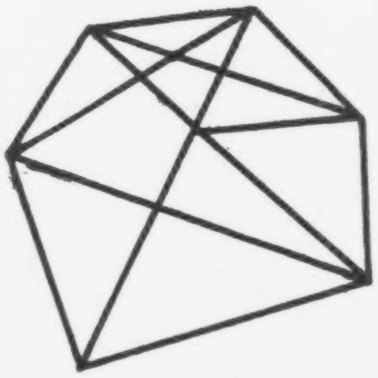
_____ Odd

_____ Even

_____ Possible

_____ Impossible

5.



_____ Odd

_____ Even

_____ Possible

_____ Impossible

PAUL BUNYAN AND THE CONVEYOR BELT

Teacher Commentary

A Recreational Activity on the Moebius Strip

I. Materials:

A. Two strips of paper per student. Each strip should be 2-3 inches wide and 18-24 inches long. Adding machine tape works very nicely for this purpose.

B. Either tape or paste for constructing the Moebius strip.

C. One pair of scissors per student.

II. Procedure:

A. Give each student two pieces of paper suitable for constructing a Moebius strip, tape or paste, a pair of scissors, and a ditto entitled "Paul Bunyan and the Conveyor Belt."

B. Develop the story using the steps of a directed reading activity.

C. When the story has been read, demonstrate the construction of the Moebius strip.

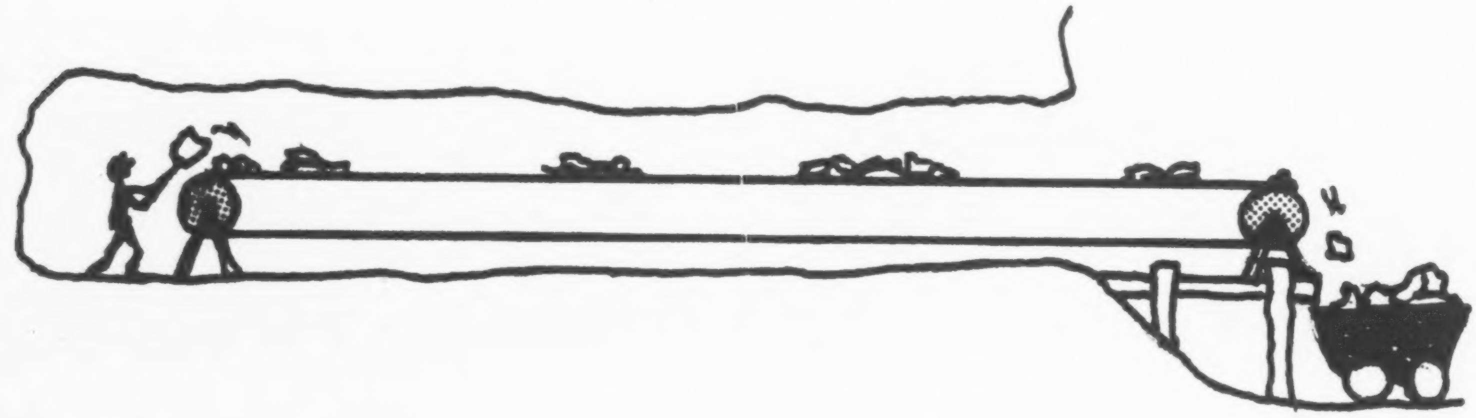
D. Each student should construct the Moebius strip, and should cut it as the work sheet suggests. It may be desirable to construct the Moebius strip when it is first discussed in the story, or, the story may be completed before beginning construction.

E. After each student has constructed the Moebius strip and cut it in the various ways, discuss each outcome.

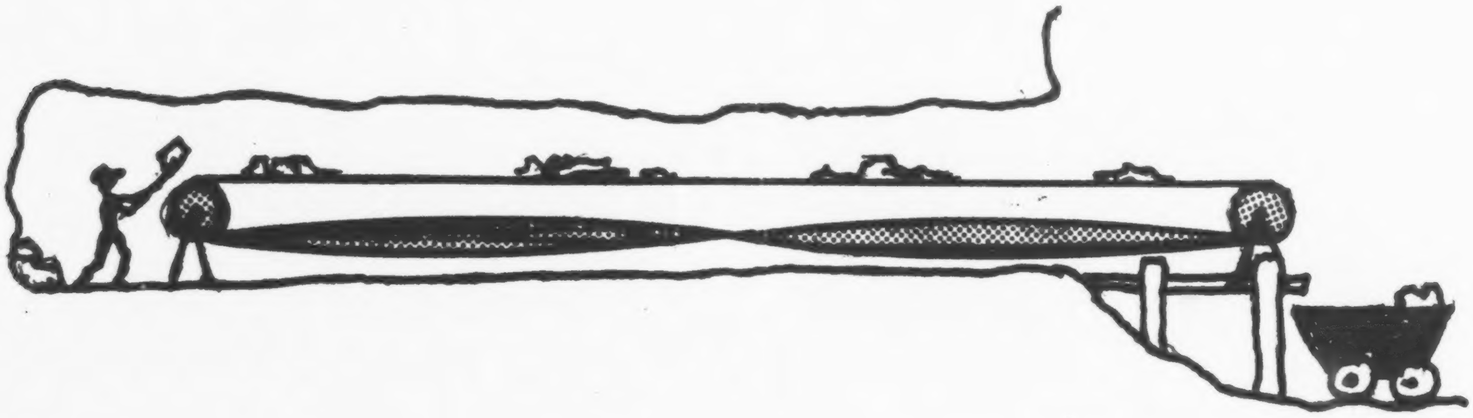
PAUL BUNYAN AND THE CONVEYOR BELT

Perhaps you have heard of Paul Bunyan before. He was a man who was so big and so strong he could cut giant trees with one blow of his axe. Everything he used had to be big. His axe was 10 feet long and his hands could hold 3 bushels of apples at one time.

Paul Bunyan had a gold mine. The mine was dug one mile into a mountain. Paul needed a conveyor belt to bring out the gold. If he made a regular belt, it would wear out only on one side.



Paul wanted the belt to wear out the same on both sides. He made one a mile long with a twist in it. He called it a moebius [mē - bē - us] strip. This belt would wear out evenly on both sides.



After a long time, Paul dug 2 miles into the mine. Now he needed a new conveyor belt which was twice as long as the first moebius strip. The old belt was still good and it was 4 feet wide. The new belt had to be 2 feet wide. Paul decided to cut the old moebius strip longwise without cutting the strip across the narrow part.

What do you think happened? Did Paul Bunyan get 2 belts? Did Paul get one belt 2 feet wide and 2 miles long?

Let's see if we can construct a moebius strip to solve this problem.

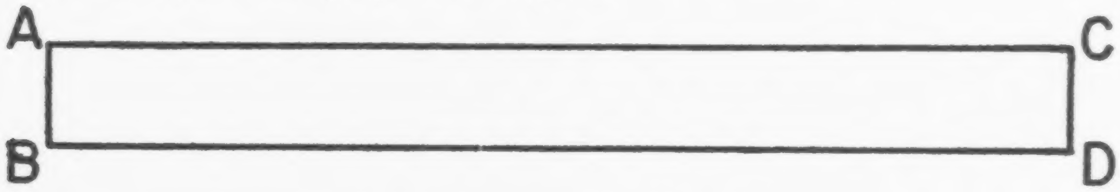
Adapted from: Johnson, Donovan A. and Glenn, William H. Topology: The Rubber-Sheet Geometry. Atlanta: Webster Publishing Company. 1960. pp. 12-13

THE MOEBIUS STRIP

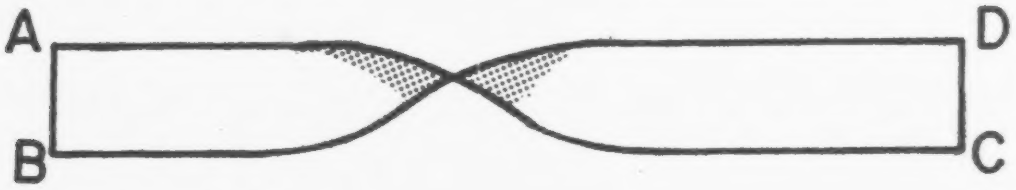
You have read about Paul Bunyan's problem, and how he tried to solve it by using the Moebius strip. Let's see if we can make a Moebius strip. Then we can find out whether Paul Bunyan's new Moebius strip conveyor belt worked

You all have been given a strip of paper about 2 inches wide and 18 to 24 inches long. You should also have a piece of tape and a pair of scissors.

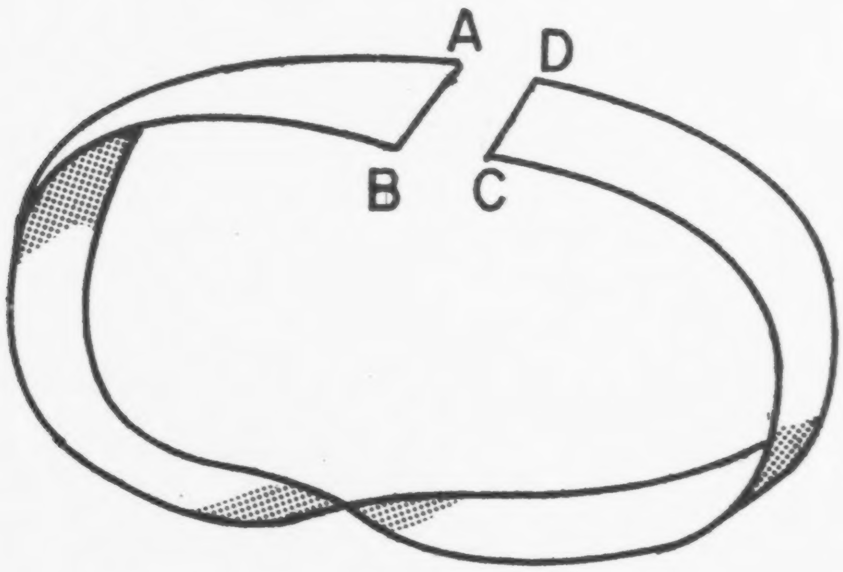
1. Hold the piece of paper straight.



2. Now give one end of the paper a half-twist.

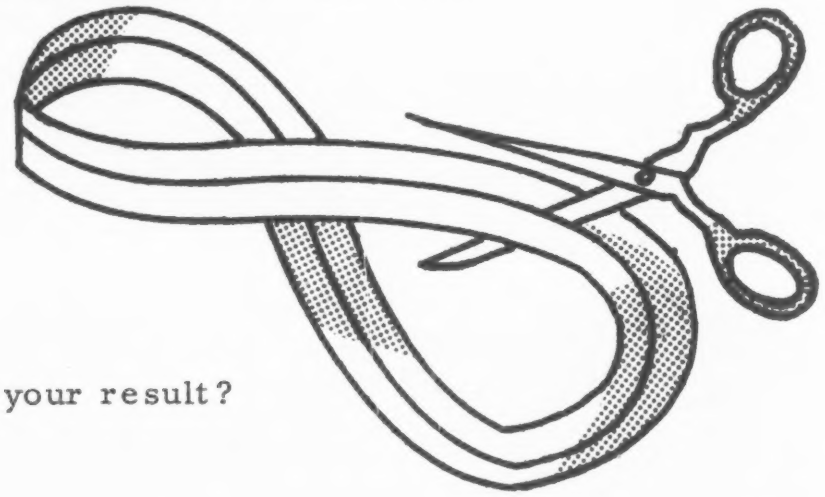


3. Put the two ends together and tape them. Be sure to leave the half twist in the paper.



4. Some mathematicians say the Moebius strip has only one side. To show this they draw a pencil mark on the strip lengthwise. No matter how far they draw they never run off the strip. See if this happens to your strip by drawing a line down the center.

5. Now that you have the pencil mark down the center, cut the strip down the middle lengthwise.

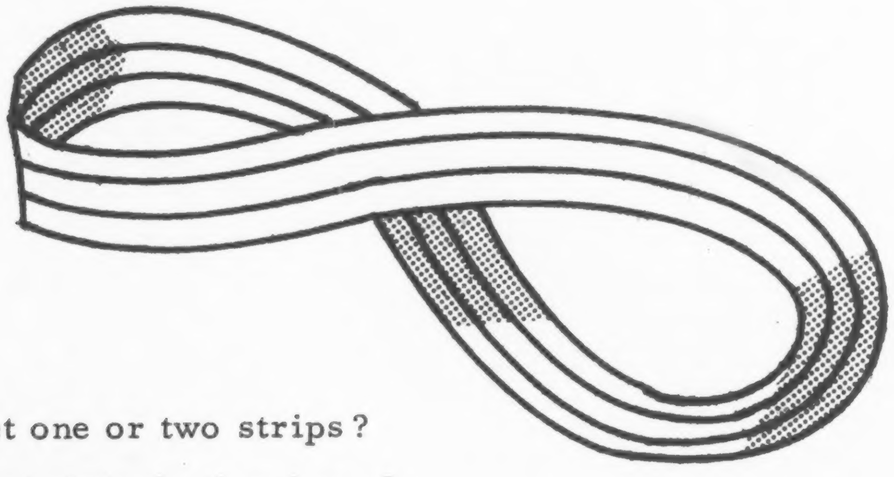


6. What is your result?

7. Cut your Moebius strip down the middle again. How many strips do you now have?

8. Would Paul Bunyan's new conveyor belt be long enough to go 2 miles back into the mine?

9. Here is another interesting experiment with the Moebius strip.
   a. Construct another Moebius strip.
   b. Instead of cutting the strip in half, cut it into thirds.



   c. Do you get one or two strips? How many twists do they have?

PAPER FOLDING A REGULAR PENTAGON

Teacher Commentary

A Recreational Activity on Pentagons

I. Materials:

A. Two strips of paper at least 11" long

B. Pencil

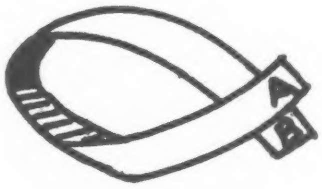
II. Procedure:

Distribute ditto and have student follow the instructions in building models.

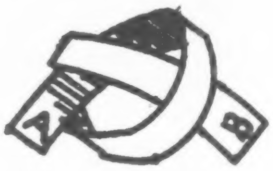
PAPER FOLDING A REGULAR PENTAGON

1. Cut a rectangular-shaped strip of paper about $1\frac{1}{2}$ inches wide and at least 11 inches long. Mark both sides of one end A and both sides of the other end B.
2. Form a loop with the strip by bringing end A over across end B as shown in figure I.
3. Then bring end A under end B and through the loop as shown in figure II.
4. Pull ends A and B outward until your model looks like figure III.
5. Press the model tightly to crease the edges.
6. Then trim off the A and B ends of the strip so your model looks like figure IV.

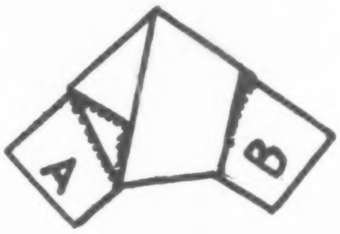
I.



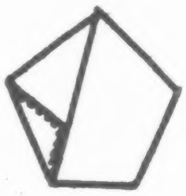
II.



III.



IV.



THE RED CUBE

Teacher Commentary

A Recreational Activity Concerning Patterns in a Cube

I. Materials:

A. Models of 6 painted cubes of the following dimensions. Each of the surfaces should be marked off in one inch squares

1. 2 x 2 x 2
2. 3 x 3 x 3
3. 4 x 4 x 4
4. 5 x 5 x 5
5. 6 x 6 x 6
6. 7 x 7 x 7

B. Student work sheet entitled, "The Red Cube"

C. Overlay - Code BEBCO J22

II. Procedure:

A. Introduce the problem by using the overlay.

B. Answer the questions on the overlay.

C. Distribute the student work sheets.

D. Record the results of the overlay in the chart on the student work sheet.

E. Divide the class into 6 groups.

F. Distribute one model of a cube to each group. The models which were used for the total and lateral surface area of cube may be used for this exercise.

G. Have students pass the cubes to the next group when they have completed the information for the chart.

H. After each group has had each cube, then discuss the results of the chart. The students should see patterns in the chart. The general patterns are:

1. $(x - 2)^3$ cubes not painted at all.
2. $6(x - 2)^2$ cubes painted on one side.
3. $12(x - 2)$ cubes painted on two sides.
4. 8 cubes painted on three sides.
5. 0 cubes painted on more than three sides.
6. x^3 = total number of small cubes.

(The students are not expected to discover the formulas without the help of the teacher.)

I. Discuss the chart.

J. Students should complete the exercises.

THE RED CUBE

1.

	Size of Cube						
Small Cubes with	2·2·2	3·3·3	4·4·4	5·5·5	6·6·6	7·7·7	n·n·n
no red sides							
one red side							
two red sides							
three red sides							
Total number of small cubes							

2. If you had an 8" by 8" by 8" painted cube, then how many of the 512 small cubes would have:

___________ a. no red sides?
___________ b. one red side?
___________ c. two red sides?
___________ d. three red sides?

3. Check the results of problem 2 by adding the answers to a, b, c, and d. What should your answer be?

4. If the size of a cube is doubled on each side, does the number of smaller cubes double? If not, then what is the change in the number of smaller cubes?

TRIANGLE KITE

Teacher Commentary

A Recreational Activity on Kite Building

This activity gives practice in measurement and shows a practical use of geometry.

I. Materials:

A. $\frac{3''}{16} \times \frac{3''}{8}$ spruce - 36" long

B. $\frac{3''}{8} \times \frac{3''}{8}$ spruce - 36" long

C. Gummed caps

D. String or cord

E. Lightweight paper or lightweight cloth

F. Scissors

G. Glue, brad

H. Student directions and diagram

II. Procedure:

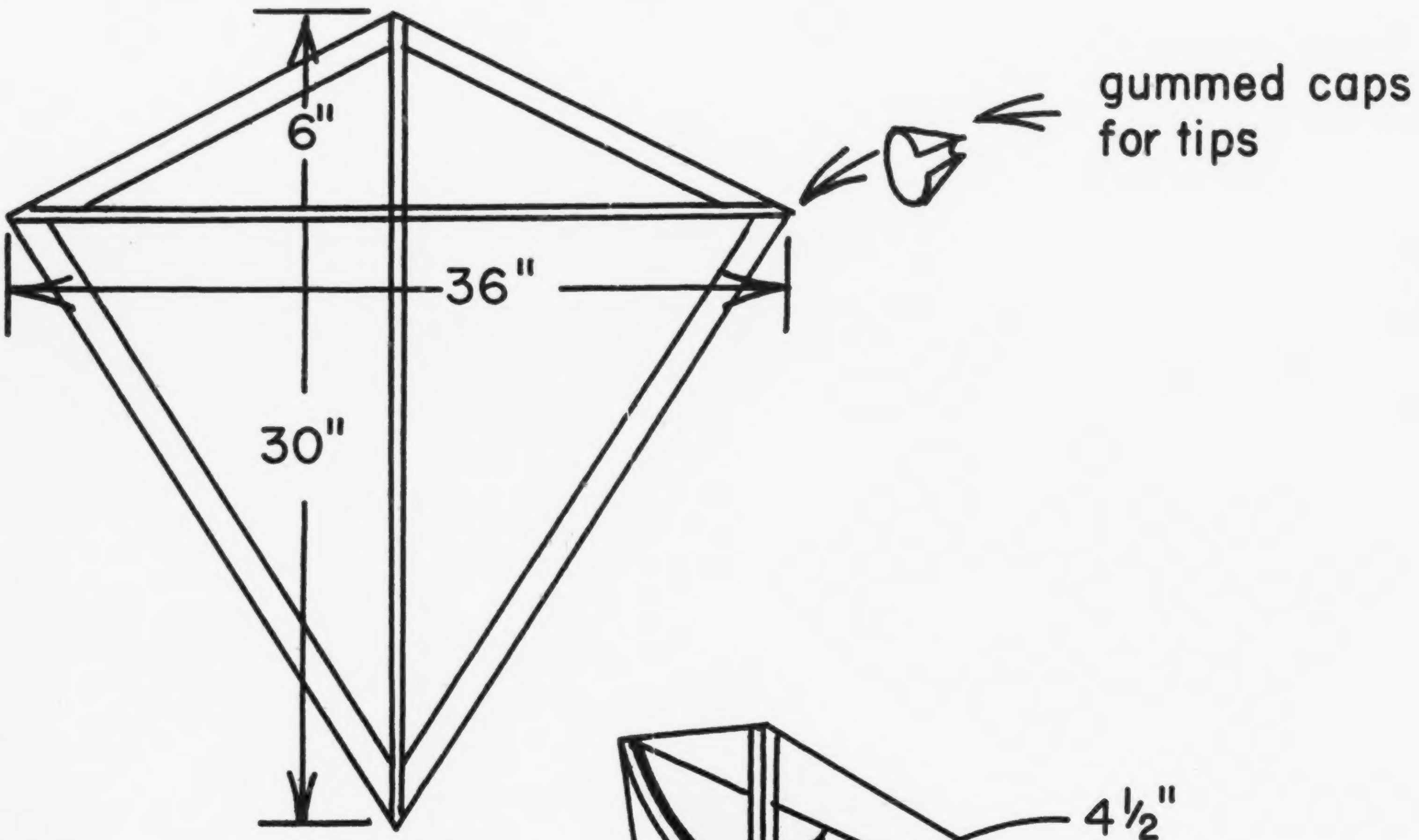
A. Comments to the teacher

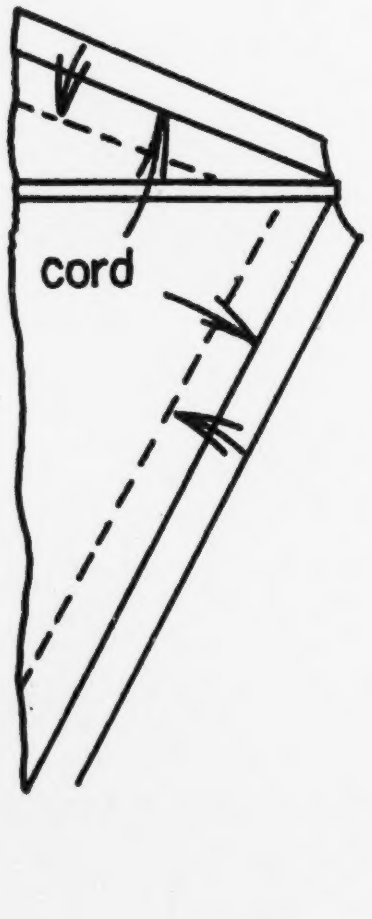
A project which may interest a student and does not involve complicated arithmetic manipulation is kite building. This activity gives detailed instructions on how to build a triangle kite. The example of the kite is given only to show the kind of project that is possible where the arithmetic is at a minimum (although it is still there) and the enjoyment at the maximum.

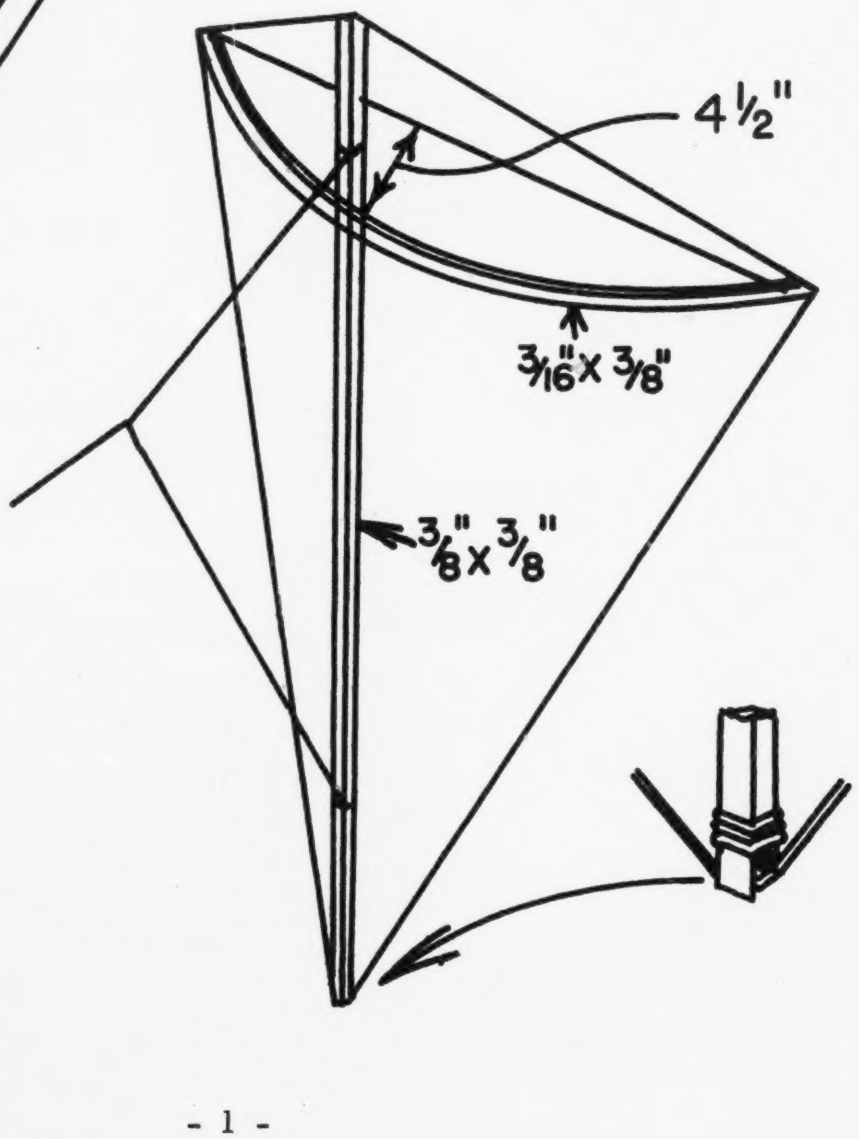
B. Directions for building the triangle kite

1. Glue and nail the two pieces of wood together as is shown in the diagram. Then tie them together with heavy thread.
2. Draw a string taut over the ends of the pieces of wood.
3. Bow the cross-stick as is shown in the diagram.
4. Cover the kite with lightweight paper or cloth.
5. Cover the tips with gummed caps or tape.
6. Tie the center stick with string for flying.

TRIANGLE KITE


6"
36"
30"
gummed caps for tips




4½"
3/16" x 3/8"
3/8" x 3/8"

BUILDING THE FRAMEWORK FOR A MODEL HOUSE

Teacher Commentary

A Recreational Activity on Constructing a Model House

Reviews constructing 45° and 90° angles, addition and subtraction

I. Materials: For each student

   A. Four strips of tag board 2" x 10".

   B. Piece of tag board larger than $9\frac{1}{4}$" x 11".

   C. Pencil

   D. 12" ruler

   E. Scissors

   F. Protractor

   G. About 12" of masking tape

   H. Quick-drying cement

   I. 40 wood splints ($\frac{1}{4}$" x $4\frac{1}{2}$") from Science Department.

II. Procedure:

   A. In this unit your students will frequently be reading printed materials. In order to insure their readiness to read these materials, review the use of those vocabulary words on the sheet entitled, "Building the Frame for a Model House - Vocabulary." The underlined words which are on this sheet are found within the student materials. On this page they are used in context, so that their meanings are clarified. Use this sheet for oral reading and discussion to make sure the students will be able to recognize the words and know their meanings.

   B. Part I

      1. Give each student all of Part I.

      2. Give each student a box containing the following items:

         a. a 12" ruler

         b. a pencil

         c. 12" of masking tape

         d. two strips of tag board 2" x 10"

         e. a protractor

3. The teacher should help students where needed.

4. Check the students' work as they finish.

Additional Practice. Do this with strips still in place as outlined in Part I.

1. On strip #1 measure 6" and make a mark. Call this mark "F".

2. On strip #2 measure 8" and make a mark. Call this mark "G".

3. If the corner is square what should you have for the distance from F to G? 10"

Discuss with the class the accuracy of each method.

1. Which method is more accurate? (6" 8" 10")

2. Why? A small error using the 6" 8" 10" principle will have less effect on the angle than a similar error using the 3" 4" 5" principle. Point out that in the actual construction of a house, feet are used instead of inches.

C. Part II

1. Give each student all of Part II.

2. Make sure that each student has all of the materials from Part I, plus:

   a. 2 more tag board strips 2" x 10"

   b. a pair of scissors

3. Give help and suggestions where needed.

4. Check the students' work as they finish. Have them search for the other solutions.

   Answers: 1. equal; 2. right; 2-a. 90

D. Part III

1. Give each student all of Part III.

2. Make sure that each student has all of the materials listed on the first page of the teacher's guide with the exception of the tag board strips. Tape can be used to hold the floor in place.

3. Give help where it is needed. It may be necessary to check all answers.

   Answers: 2a - 9", 2b - $7\frac{1}{4}$", 2c - 1", 2d - square,

   5a - $8\frac{1}{2}$", 5b - 5, 5c - 1", 5d - $7\frac{1}{2}$", 5e - $1\frac{1}{2}$", 5f - yes,

   6a - $6\frac{3}{4}$", 6b - 4, 6c - $\frac{3}{4}$", 6d - 6", 6e - $1\frac{1}{2}$",

   6f - parallel to corner studs

E. Assessment: Check the model houses for accuracy.

F. Suggest that interested students complete the models on their own.

BUILDING THE FRAME FOR A MODEL HOUSE - VOCABULARY

1. Jim built a model airplane.
2. The framework of the house will be built of wood.
3. Studs are used to make the frame for the house.
4. An angle is formed when two straight lines meet.
5. A protractor is used to measure angles.
6. Diagrams will help you build your model house.
7. Opposite sides of the model house must be the same.
8. You must construct your model house very carefully.
9. Train tracks are parallel.
10. A diagonal is a straight line. It connects the opposite points of a polygon.
11. In order to cut paper, you need a pair of scissors.
12. You will need to make careful measurements in order to make your model house.
13. Glue will be used to hold your model house together.
14. The studs will be made of wood splints.
15. Tag board will be used to make the bottom of the house.

BUILDING THE FRAMEWORK FOR A MODEL HOUSE

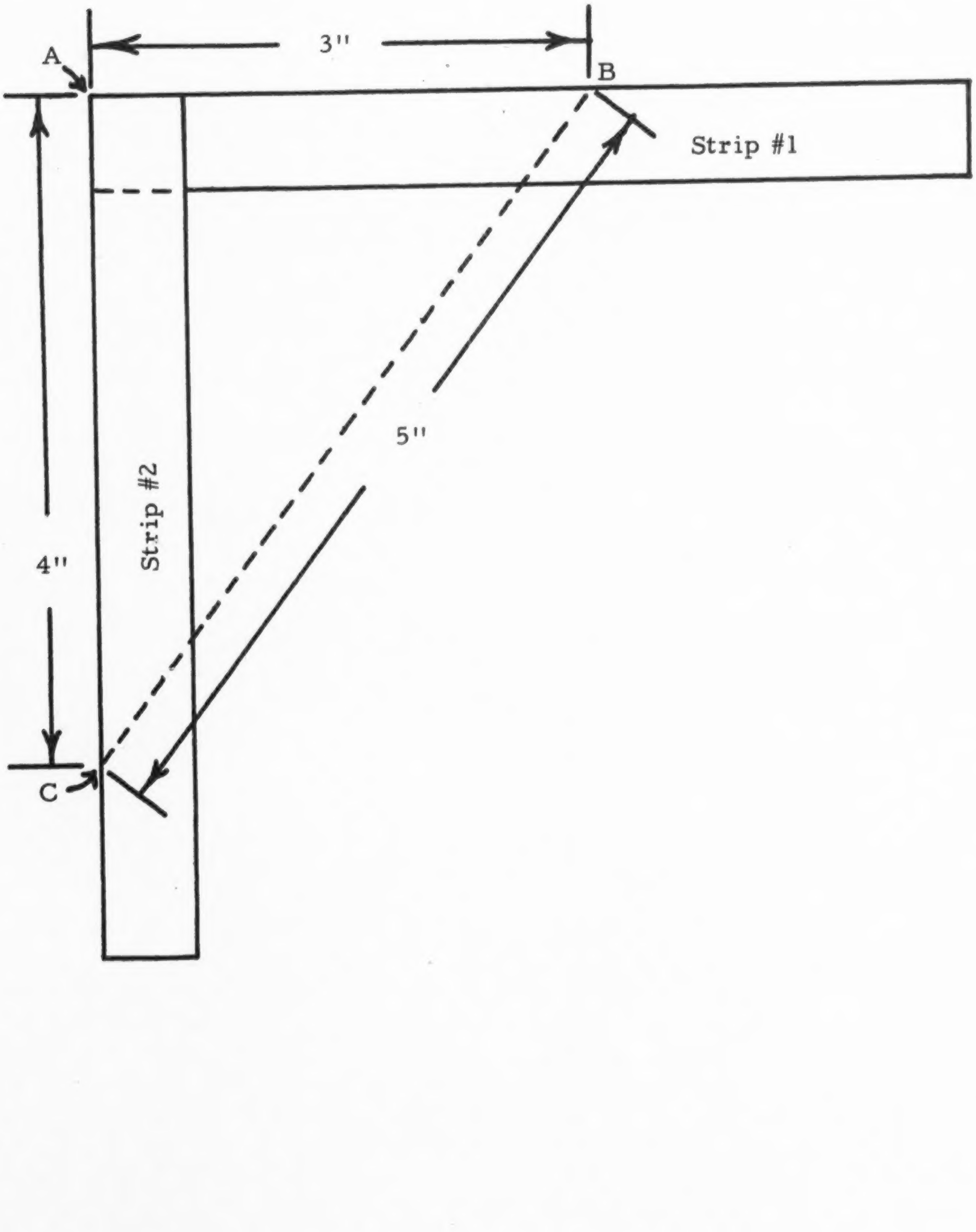
PART I

Gene Smith was interested in building a model house. He knew that all of the posts or studs (studz) for the model house frame should be placed at right angles (ăng' ŭls) to the floor. The only tools he had to plan his work were a pencil and a ruler. He went to a neighbor who was a carpenter and asked for help. Gene's friend told him to get two pieces of tag board which measured 2 inches by 10 inches (2" x 10"). The friend showed him how to make a right angle using just his ruler and a pencil. Let us follow along step by step and do just what he did. When you have finished, you can use a protractor (prō trăk' ter) to check your work.

A. Steps to follow: Look at diagram 1 (dī' ŭh grăm) on the next page as you follow these steps.

1. Choose two strips of tag board from your desk.
2. Mark your strips #1 and #2.
3. Tape strip #1 to your desk.
4. Place strip #2 over the end of strip #1 to form what looks like a right angle.
5. From point A measure 3 inches on strip #1.
   a. Place a mark on the outside edge.
   b. Call this mark "B".
6. From point A measure 4 inches on strip #2.
   a. Place a mark on the outside edge.
   b. Call this mark "C".

DIAGRAM 1


3"
A
B
Strip #1
5"
4"
Strip #2
C

7. Measure from the outside edge of B to the outside edge of C.
   a. If the distance is exactly 5 inches, tape strip #2 to your desk. The corner is square.
   b. If the distance is NOT exactly 5 inches, move strip #2 to make the angle bigger or smaller until the distance from B to C is exactly 5 inches. Then tape strip #2 in place.

8. Check the angle (∠BAC) with a protractor.
   a. Was Gene's work correct? ____________________

PART II

Gene wanted the frame for his model house to be 10 inches by 8 inches. His neighbor told him that he could plan to build this with just two more strips and no other equipment. See if you can follow what Gene did. Diagram 2 on the next page should help you.

1. Choose two more tag board strips, each strip being 2" x 10".
2. Cut the two strips, so that the two strips are 8 inches long.
3. Mark the new strips #3 and #4.
4. Build a right angle just the way you did in Part I. Use strip #1 and strip #3. See diagram 1 on page 3.
5. Place strip #2 over the end of strip #3 opposite point A.
    a. Call this point "D".
    b. Place strip #2 so that it looks of equal distance from or parallel (păr' ŭh lel) to strip #1.
6. Place strip #4 over the end of strip #1 opposite point A.
    a. Call this point "E".
    b. Place strip #4 so that it looks parallel to strip #3.
7. The place where strips #2 and #4 come together should be marked "F".

The statements below will show what Gene learned from his work. Fill in the blanks.

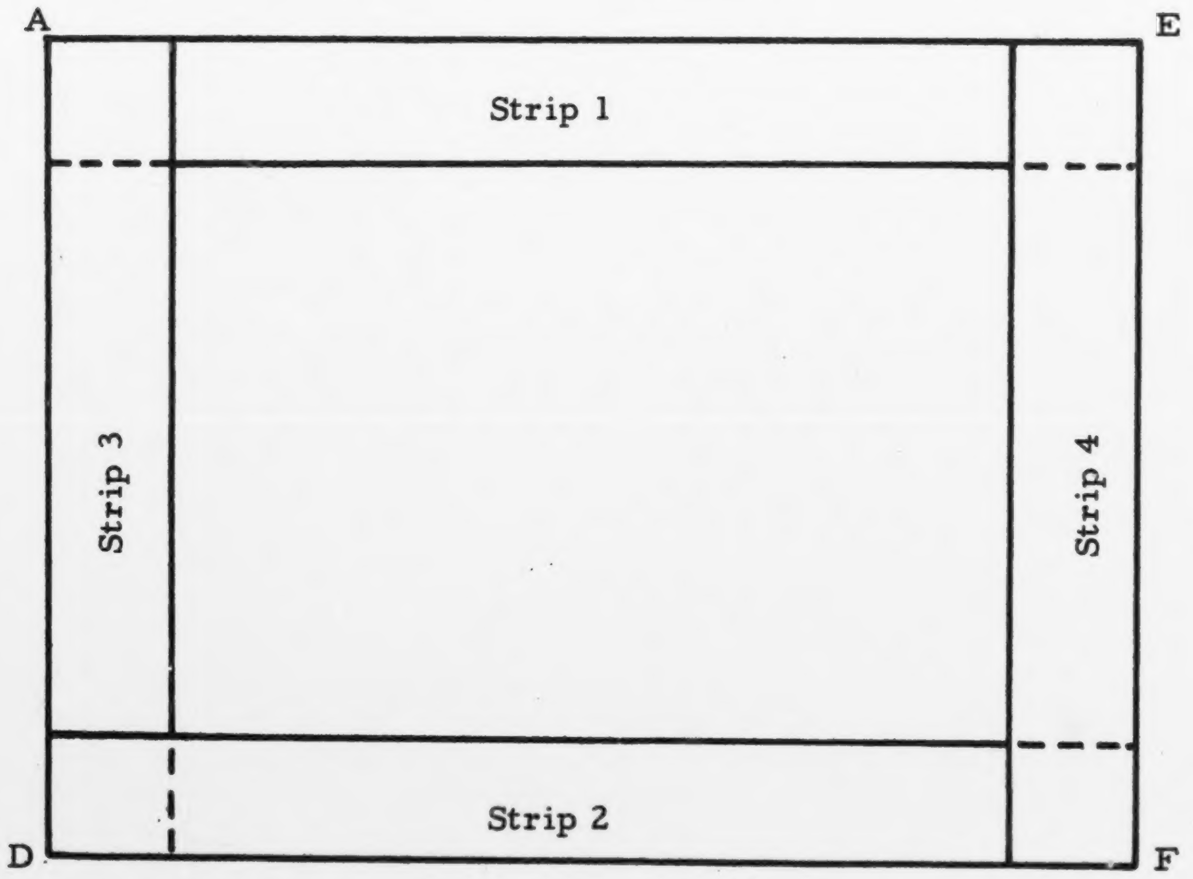
1. In a rectangle opposite (ŏp' pō zĭt) sides are ________________________.
2. All angles in a rectangle are ____________________ angles.
    a. Each of these angles measures ___________ degrees.
3. A line drawn from one corner of a rectangle to the opposite corner of a rectangle is called a diagonal (dī ăg' ĕn nul). If you measure the two diagonals in a rectangle, they will be equal. Measure the diagonals to see if this is true.

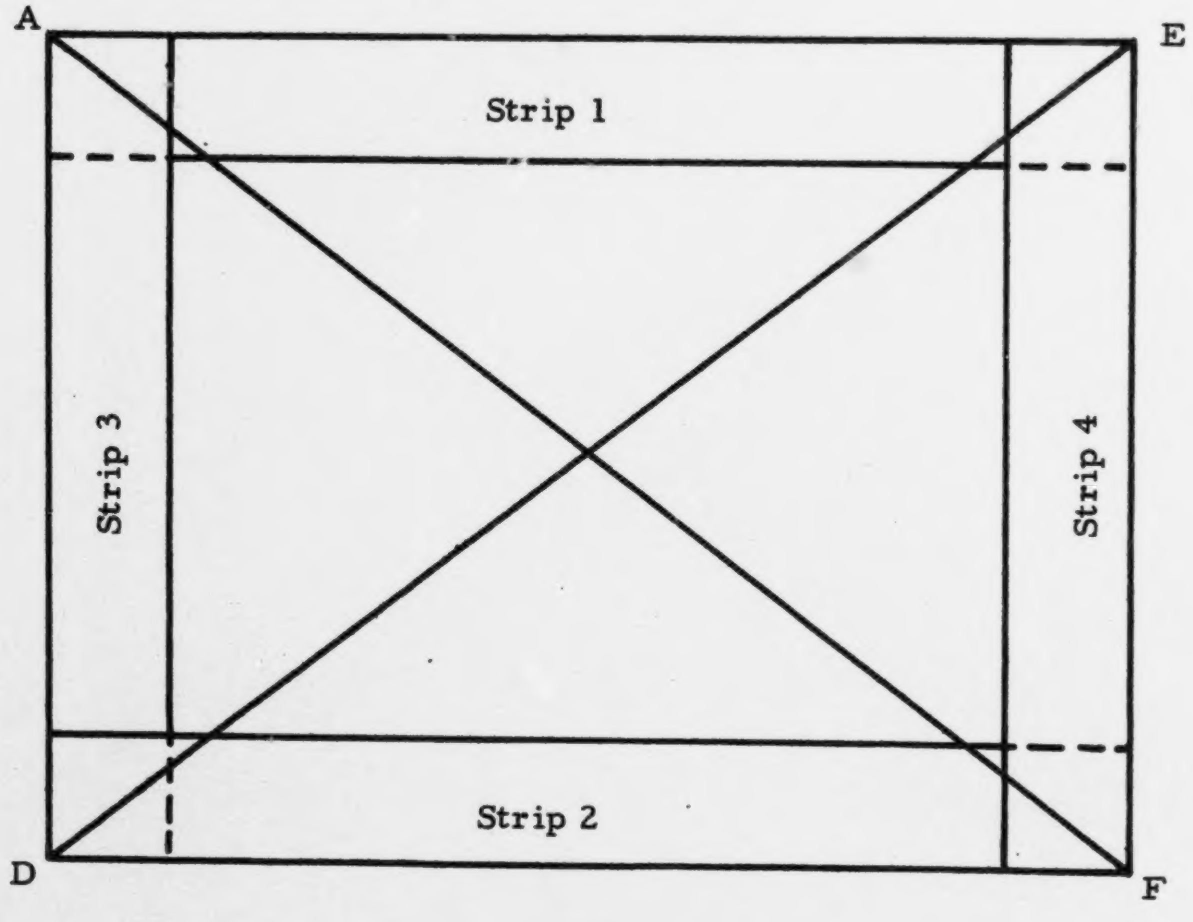
To check your plan for the **frame:** Diagonal AF (------) must equal diagonal DE (------). You **should find** that all the angles are right angles. Check the angles with a **protractor** to see if this is true.

Answers to questions on page 5.

1. equal and parallel
2. right, a. 90°

DIAGRAM 2


A
E
Strip 1
Strip 3
Strip 4
Strip 2
D
F


A
E
Strip 1
Strip 3
Strip 4
Strip 2
D
F

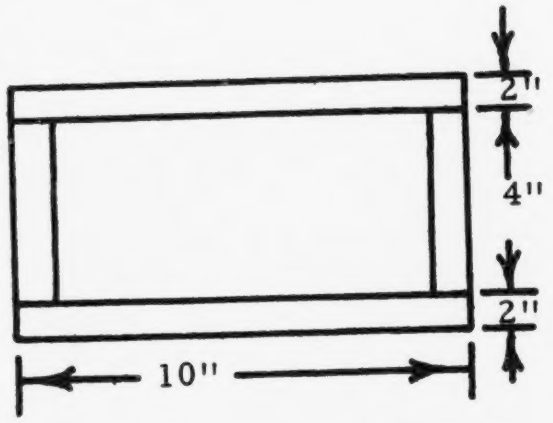
When Gene was finished, he found that he had a frame that was exactly 10" x 8". This frame did not suit him because it would not lay perfectly flat. This happened because he placed one strip on top of the other.

After thinking it over carefully, he found that there were three ways he could fix the frame so that it would lay flat. See how many ways you can find for him to fix the frame for his model house.

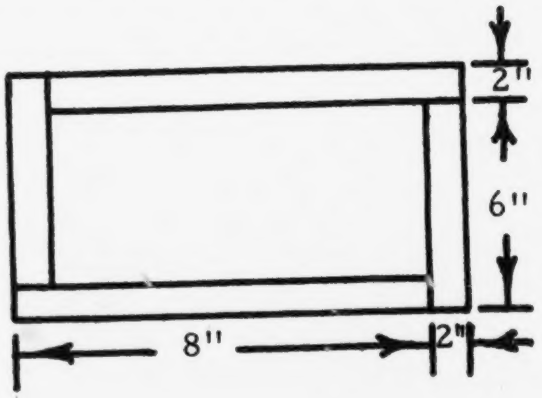
If you have tried, but cannot find a way for Gene to get his house to lay flat, see what he did on the following page.

ANSWERS TO QUESTIONS ON PAGE 8

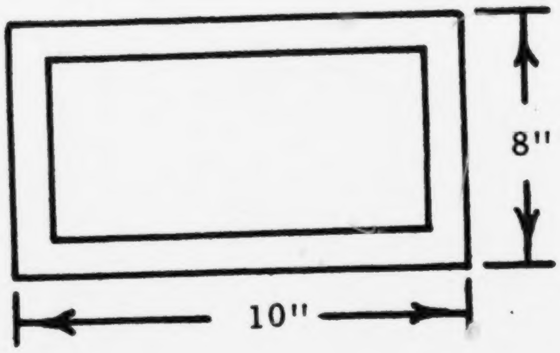
1. Cut 4 inches from two of the strips and place them to the inside of the other strips.



2. Cut 2 inches from each strip. Place them so that only one width (2") adds to each length.



3. Cut each end of all strips on a 45° angle and put them together like a picture frame.

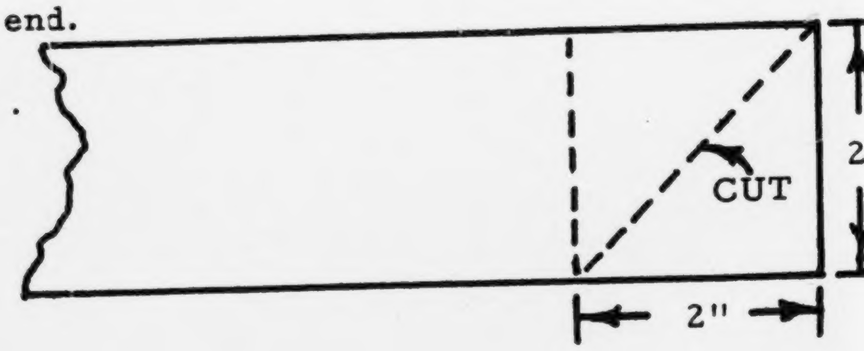


To cut a 45° angle

a. Construct a square on the end.

b. Connect two opposite points with a straight line.

c. Cut this line.



PART III

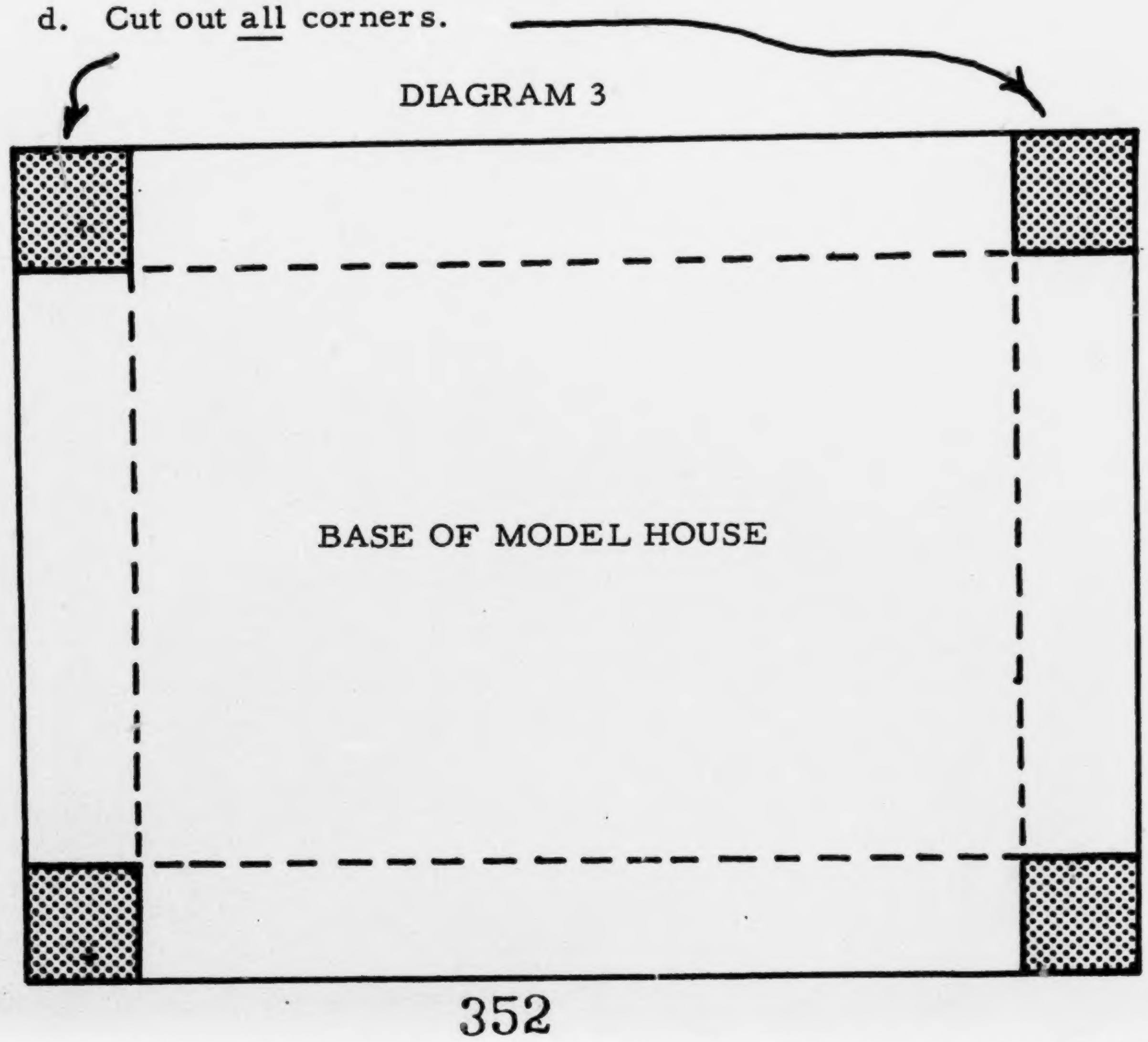
Gene has now learned how a carpenter builds a right angle, a 45° angle, and a rectangle with just a ruler and a pencil. He is now ready to build his model house. Since many houses are pre-made or "pre-fabs," he thinks this will be a good way to build his house.

If you follow along, you can build a model just like his.

Steps to follow:

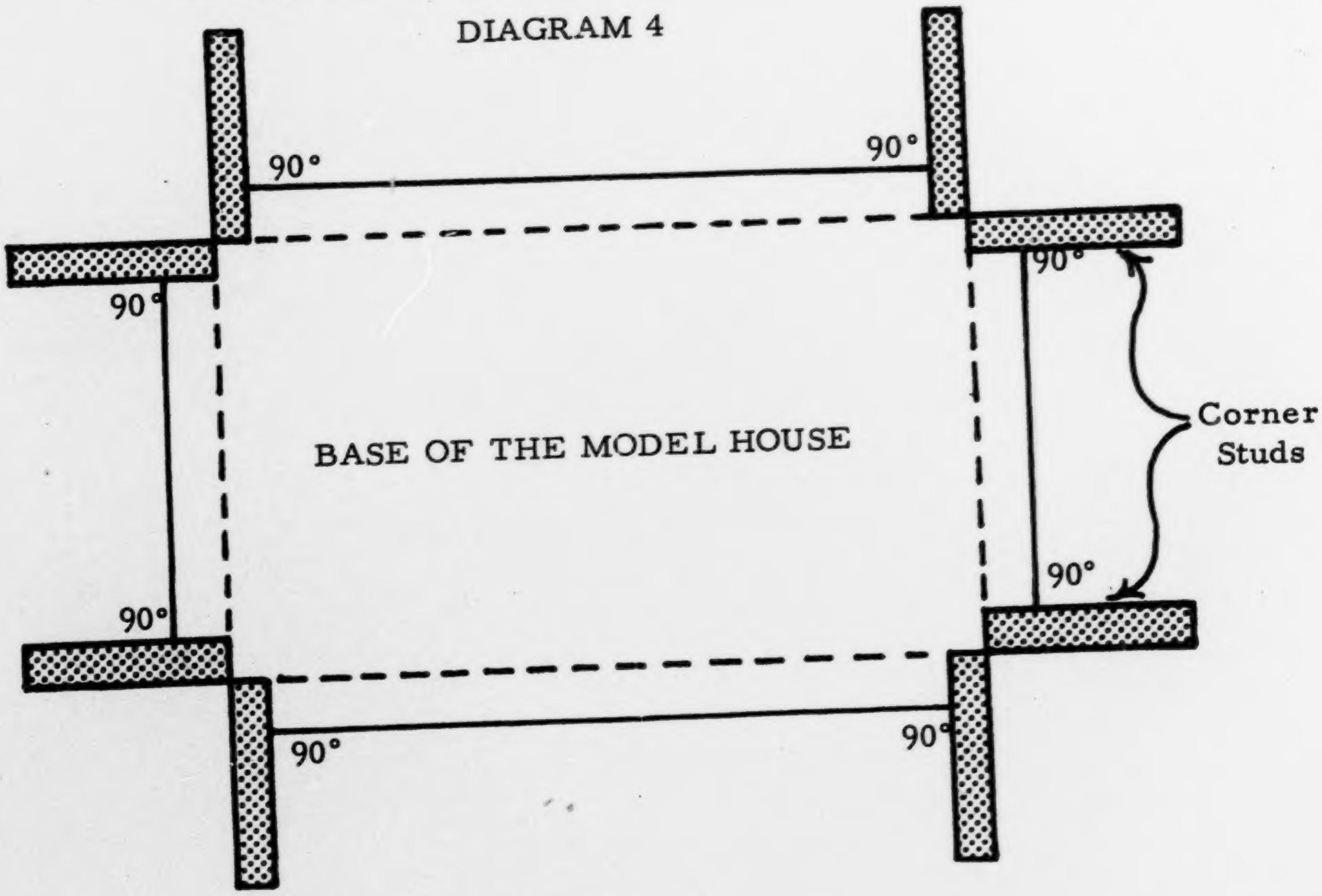
1. For the floor or base:
   a. Take a piece of tag board $9\frac{1}{4}"$ x 11".
   b. Measure in 1 inch from each side and draw a line on each side as shown in the diagram. You may use a solid line instead of the broken line shown in diagram 3.
   c. Score the lines you have drawn on each side of the tag board. (To score: Hold a ruler on each line and run the point of your scissors (siz' ers) down the line. Press down hard on the ruler as you are scoring to keep it on the lines.)
   d. Cut out all corners.

DIAGRAM 3



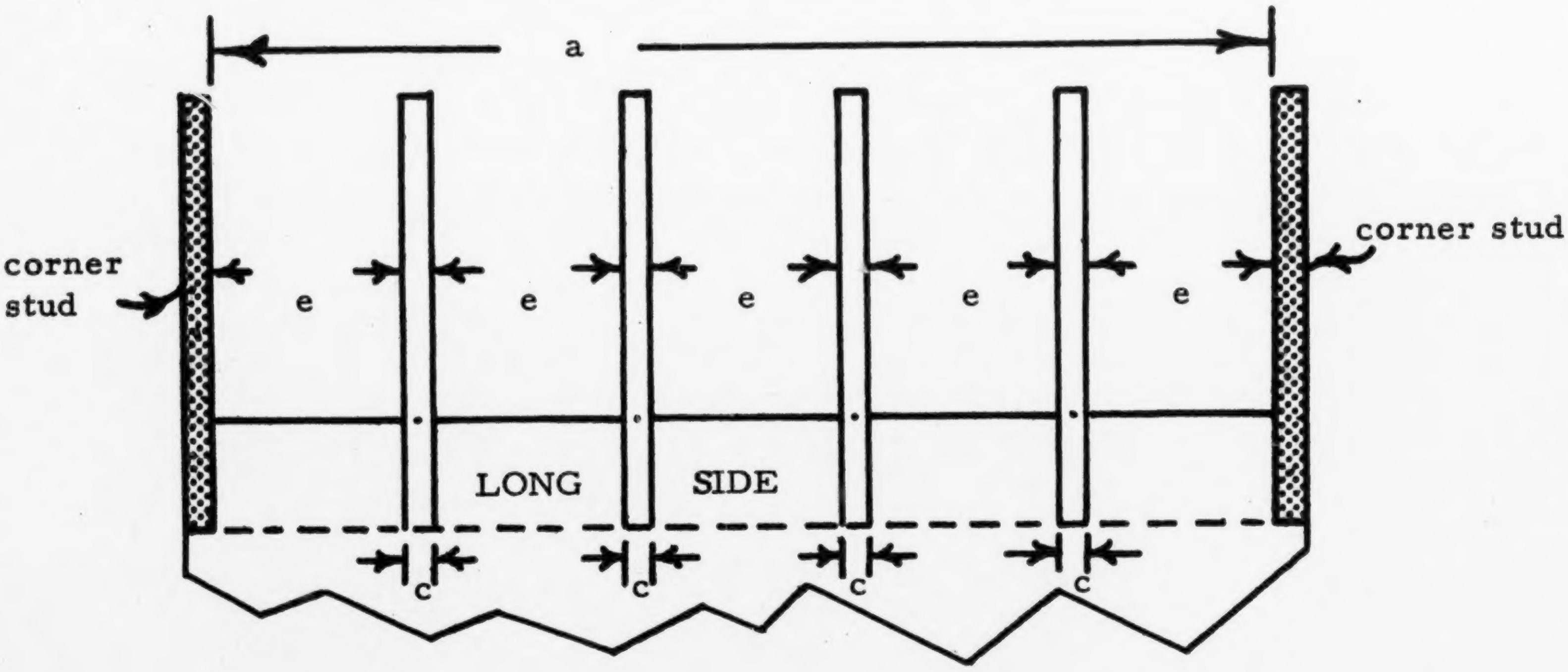
2. The part inside the scored lines will be the floor of the house.
   a. What will the length of the house be? (distance between the dotted lines) ____________________
   b. What will the width of the house be? ____________________
   c. What is the size of the cut out from each corner? ________ x ________
   d. What is the shape of the cut outs? ____________________
3. Turn the floor over.
   a. Fold each side up along the scored lines and then lay flat.
      (1) The crease or fold now shows on the working side.
4. You now need to place corner studs at each of the corners. See diagrm 4 on the bottom of this page.
   a. Choose 8 wood splints. (When these splints are glued to the corners, they are called corner studs.)
   b. These splints must form a 90° angle where they meet the floor.
      (1) Use the 3", 4", 5" method to get the 90° angle. (You used this method in part I, using strips #1 and #2.)
      (2) Use the protractor to check your angle.
   c. Glue the corners in place.

DIAGRAM 4



5. On the longer sides you are going to place more studs between the corners at equal distances apart. To help you get them in the right places, answer the questions below. Use your model base and follow the diagram as you find the answers to the questions.

DIAGRAM 5



Questions:

a. What is the distance between the corner studs? ________________

b. Using 4 more studs, how many spaces will we have? __________

c. How much space will be taken up by the 4 middle studs? (Remember - each stud measures $\frac{1}{4}$ inch.) __________________

d. What measure of space will be left? (a - c) __________________

e. How much space between studs? (d ÷ b) __________________

f. If each of these studs are parallel to the corner studs, will they form a right angle at the base? __________________

g. Fill in the measurements in diagram 5.

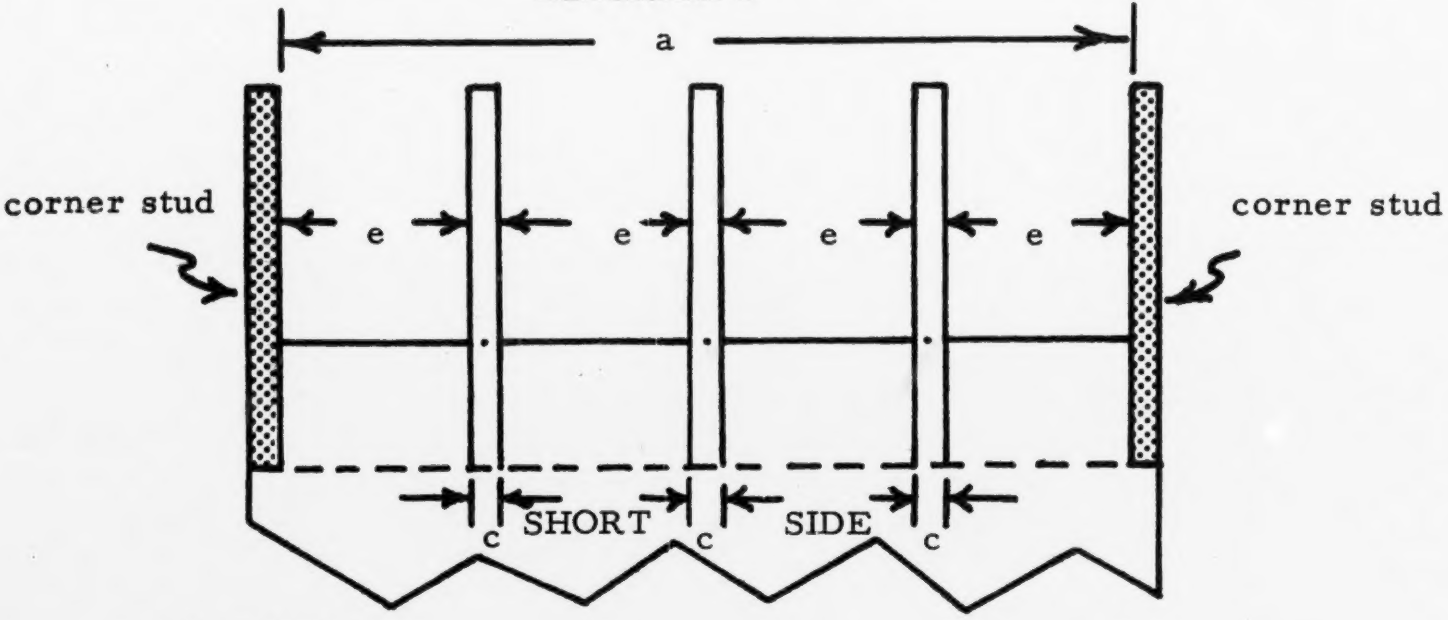
h. Place pencil marks on the long side of your model base where the studs will be glued.

i. Glue the studs in place.

j. Follow the same steps for the other long side of your model base.

6. On the shorter sides you are going to place 3 more studs between corners. To help you get them in the right places, answer the questions below. Use your model base and follow the diagram below as you find the answers to the questions.

DIAGRAM 6



Questions:

a. What is the distance between the corner studs? ______________

b. Using 3 inside studs, how many spaces will you have? ____________

c. How much space will be taken up by the three studs?
(Remember - each stud measures $\frac{1}{4}$ inch.) ____________________

d. What measure of space will be left? (a - c) ___________________

e. How much space is there between the studs? (d ÷ b) __________

f. How can we get these studs to form a right angle at the base?

____

g. Fill in the measurements in diagram 6.

h. Place pencil marks on the short side of your model base where the studs will be glued.

i. Glue the studs in place.

j. Follow the same steps for the other short side of your model base.

7. **Corner Braces (These are necessary to add strength to the model house framework.)**

   A. **Choose a wood splint.** (When wood splints are used to add strength to the corner studs, they are called corner braces.) Do the following with your wood splint:

      (1) Measure how wide the splint is.

      (2) From the end of the splint, measure the same distance as the width of the splint.

      (3) Mark the distance across the splint. You should have a perfect square.

      (4) Connect (join) the two opposite corners with a straight line. (See diagram.)

      (5) Cut off the triangular (tri ang u ler) corner (A). You now should have a perfect 45° angle (B).

      (6) You have now finished one end of a corner brace. Place this end on the tag board base as shown in diagram 7.

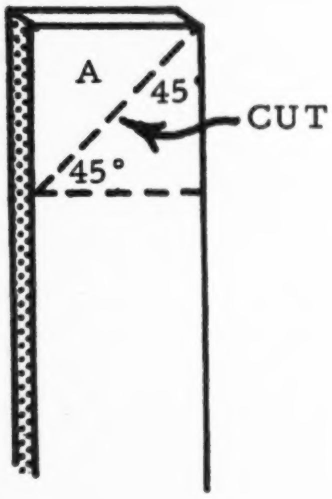
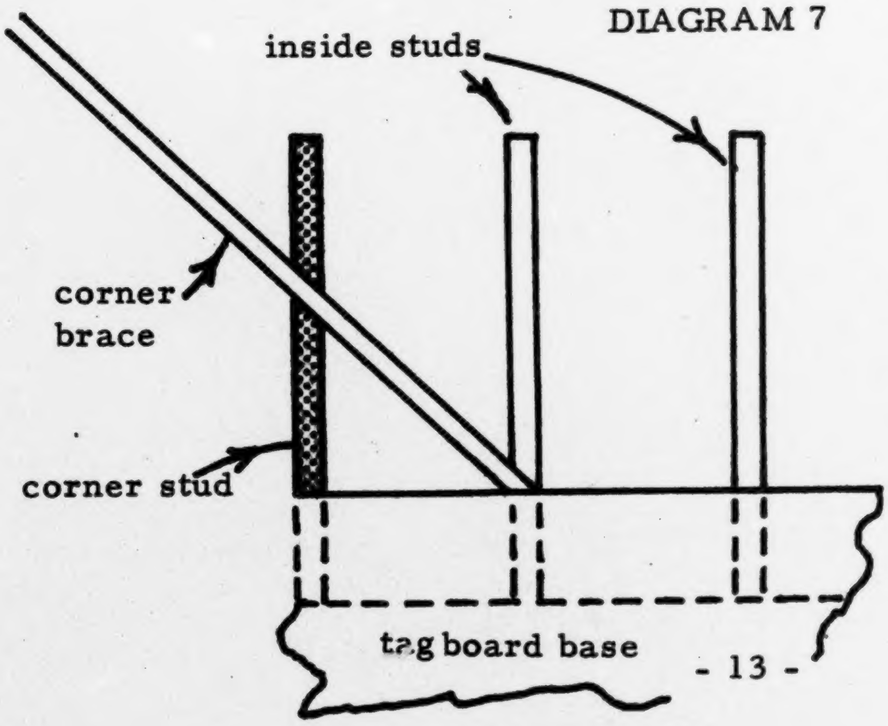
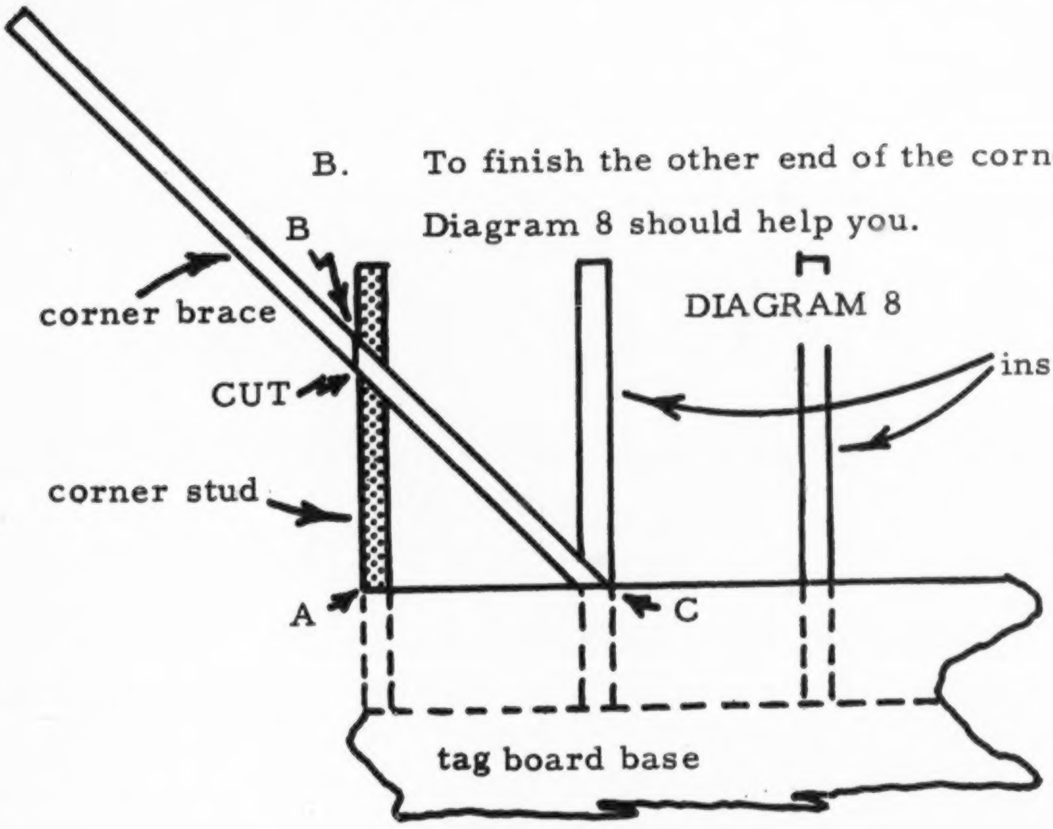


DIAGRAM 7



B. To finish the other end of the corner brace, do the following: Diagram 8 should help you.

DIAGRAM 8



(1) Measure from A to C.

(2) From the tag board base measure this same distance (A to C) on the corner stud. Mark this and call it "B".

(3) Have the top edge of the corner brace match the mark (B) on the corner stud.

(4) Use the edge of the corner stud as a guide and mark the corner brace angle.

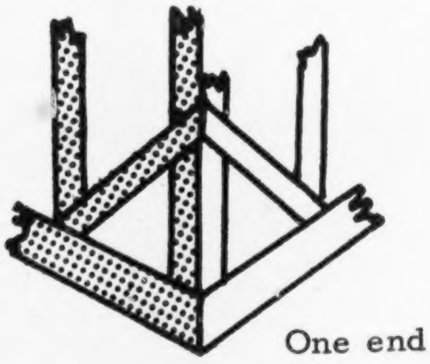
(5) Draw a line on the corner brace where this angle is to be cut.

(6) Carefully cut the corner brace to form the other end. This angle is a 45° angle.

C. Make seven (7) more corner braces. Use the one you just made as a model.

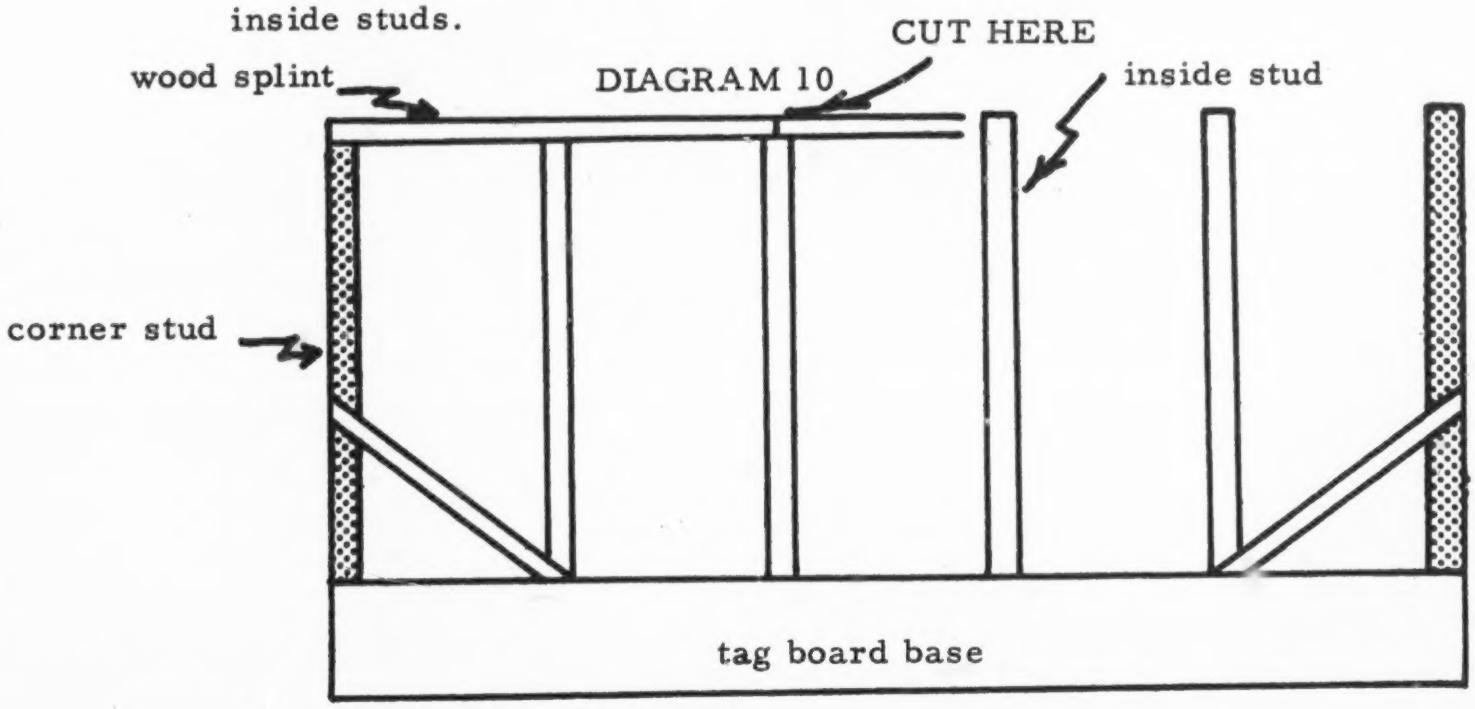
D. Now glue the corner braces to all four corners. See diagram 9.

DIAGRAM 9



8. To brace the inside studs on the long sides of your model house, do the following: Use the diagram below to help you brace the inside studs.

DIAGRAM 10


wood splint
CUT HERE
inside stud
corner stud
tag board base

A. Start at a corner stud.

B. Place the first wood splint across the studs at the very top.

C. If the splint goes beyond an inside stud, cut it so that it ends in the center-top of an inside stud. (See diagram 10 above.)

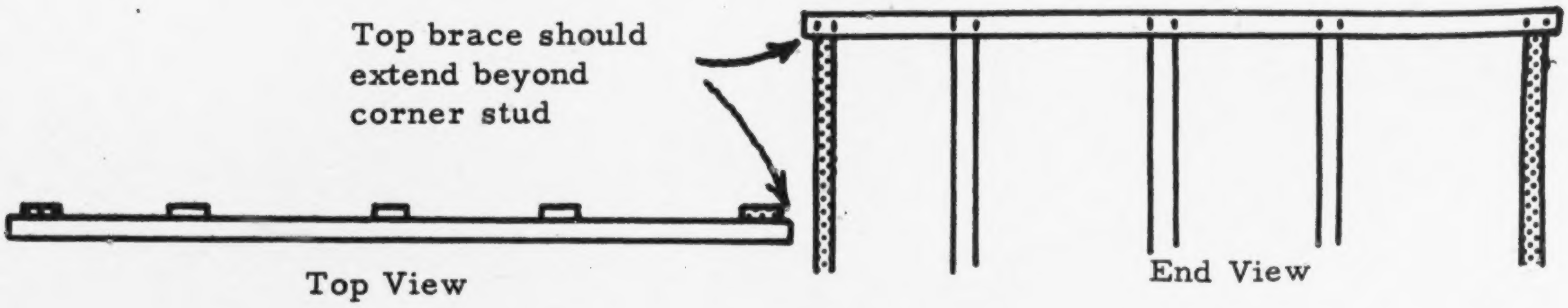
D. Glue this splint in place.

E. Follow the same steps in placing the other splints across the top of the long sides. (Make sure the splints end at the center-top part of an inside stud.)

9. To brace the inside stud on the short side of the model house, do the following:

A. Start at the outside of a corner stud.

B. Place the first wood splint across the studs at the very top. This first wood splint should be placed a little beyond the corner stud.


Top brace should extend beyond corner stud
Top View
End View

C. If this splint goes beyond an inside stud, cut it so that it ends in the center-top of an inside stud.

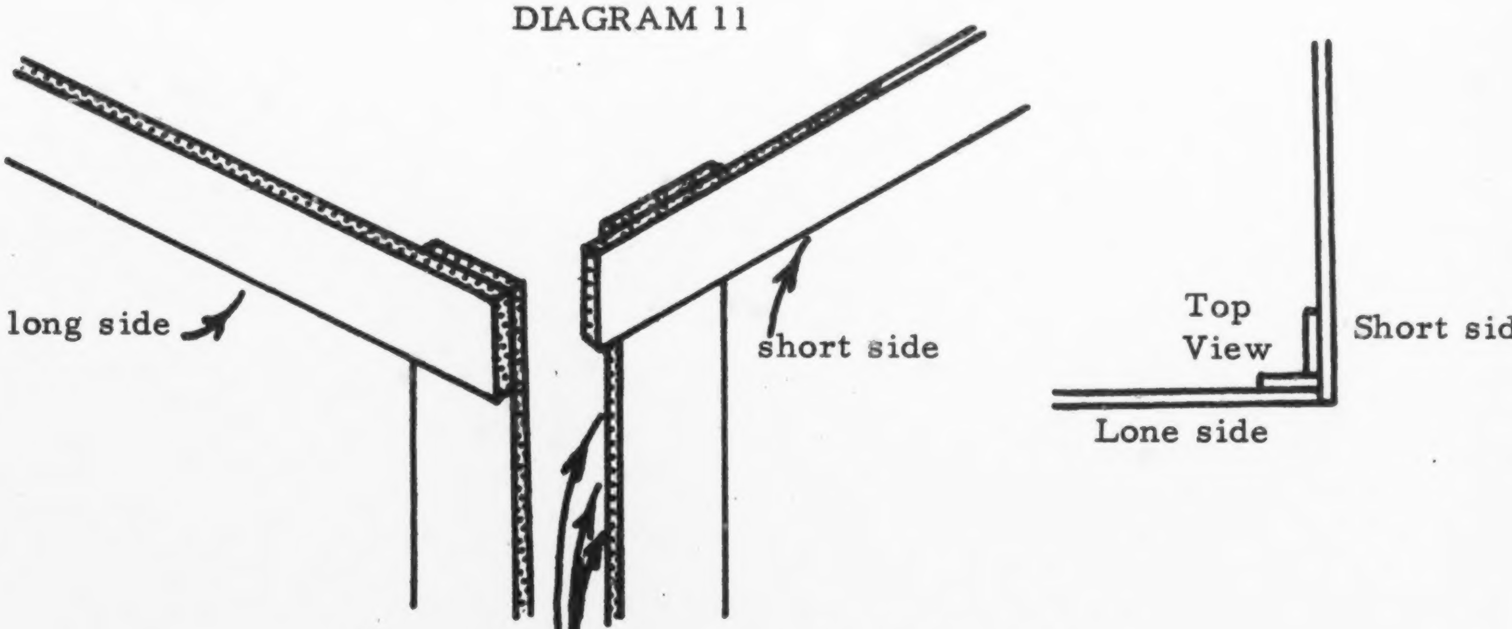
D. Glue this splint in place.

E. Follow the same steps in placing the other splints across the top of the short sides. (Make sure the splints end at the center part of an inside stud.)

10. NOW BEND THE SIDES UP TO FORM THE WALLS OF THE HOUSE.

11. Now glue the long side corner studs to the short side corner studs. Work with one end at a time. See diagram 11.

DIAGRAM 11



Place glue on corner studs of short sides

12. You have now finished building the framework for a model house. All it needs now is a roof?

ANSWERS TO PART III

Answers to questions on page 10.

2. a. 9"
   b. $7\frac{1}{4}$"
   c. 1" x 1"
   d. a square

Answers to questions on page 11.

5. a. $8\frac{1}{2}$"
   b. 5 spaces
   c. 1 inch
   d. $7\frac{1}{2}$"
   e. $7\frac{1}{2}" \div 5 = \frac{15}{2} \times \frac{1}{5} = \frac{3}{2} = 1\frac{1}{2}"$
   f. yes

Answers to questions on page 12.

6. a. $6\frac{3}{4}$"
   b. 4 spaces
   c. $\frac{3}{4}$"
   d. 6"
   e. $6 \div 4 = 4\overline{)6} = 1\frac{1}{2}"$
   f. (1) Use the 3", 4", 5" method
      (2) Use a protractor

GAINING A SQUARE INCH

Teacher Commentary

A Recreational Activity on the Area of a Rectangle and a Square

I. Materials:

   A. Student work sheets entitled, "Gaining a Square Inch"

   B. Scissors

II. Procedure:

   A. Review the procedure for finding the area of a square and a rectangle.

   B. Distribute student work sheet.

   C. Distribute scissors.

   D. Have students complete work sheet.

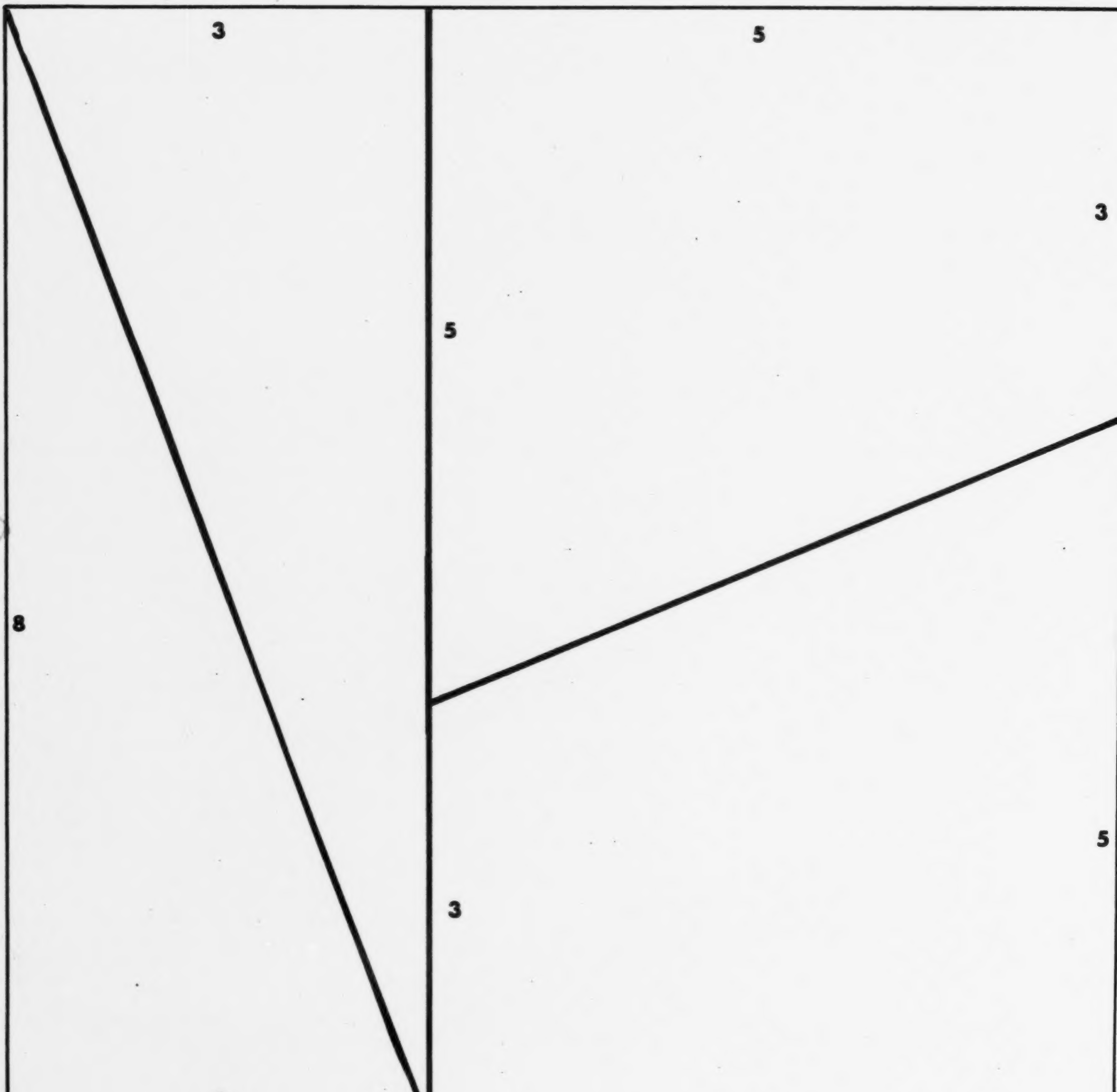
   E. Discuss the results.

      1. Both figures must have the same area.

      2. If you notice where the pieces are put together in the rectangle, you will see that they do not fit together. (Look along the diagonal.) This accounts for the discrepancy in the areas.

      3. Measure the angles on the diagonals and you will see that they do not form a linear pair.

GAINING A SQUARE INCH



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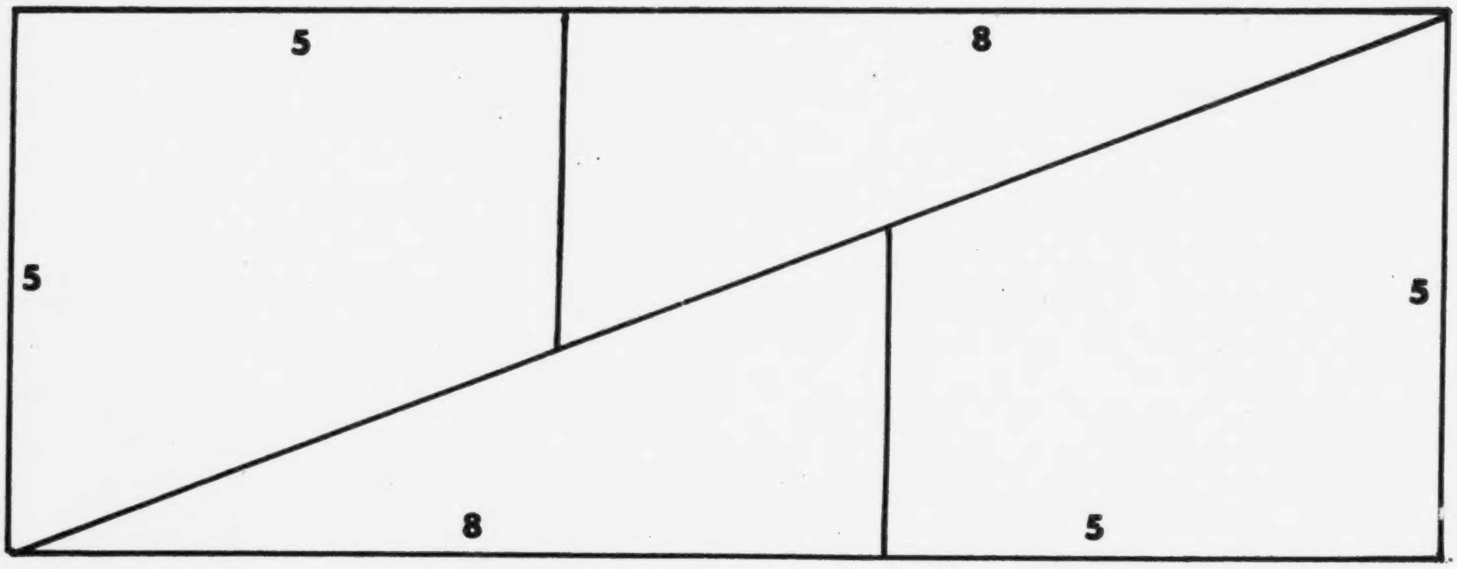
GAINING A SQUARE INCH

1. What is the area of the square? ____________________

2. Cut out the square. Also cut the large square into the four indicated regions.

3. Rearrange the regions to form a rectangular shaped figure as shown below.



4. What is the area of the rectangle? ________________

5. Compare the area of the rectangle with the area of the square.

6. How do their areas compare?

7. Should their areas be the same? Why?

8. What does this problem indicate about drawing mathematical conclusions from experiments?

GETTING BOARD

Teacher Commentary

A Recreational Activity on Linear Measurement

I. Materials:

A. Student work sheet "Getting Board"

B. Ruler

C. A 2 x 4 piece of wood

II. Procedure:

A. Distribute work sheet "Getting Board."

B. Explain the problem to the students using a piece of 2 x 4.

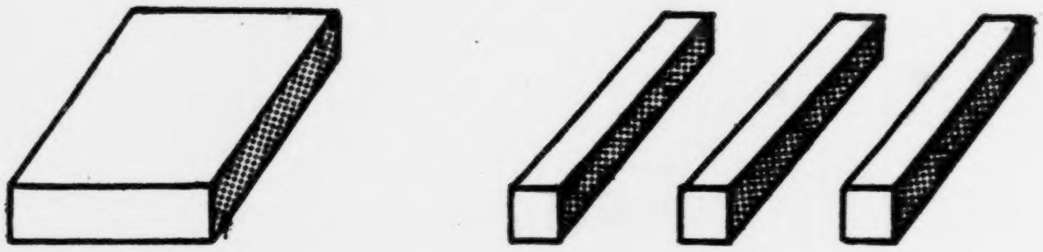
C. Have students guess how to divide the board into three equal parts.

D. Have students complete "Getting Board" independently or under teacher's guidance.

E. Evaluation exercises:

1. Have students draw a line $4 \frac{1}{2}$ inches long and divide it into 3 equal parts.
2. Have the students make a chart $6 \frac{7}{8}$ inches wide. The chart must contain four columns of equal width. Suggest rotating their ruler to 8 inches instead of 6 inches, and using marks at 2, 4, and 6 inches.

GETTING BOARD

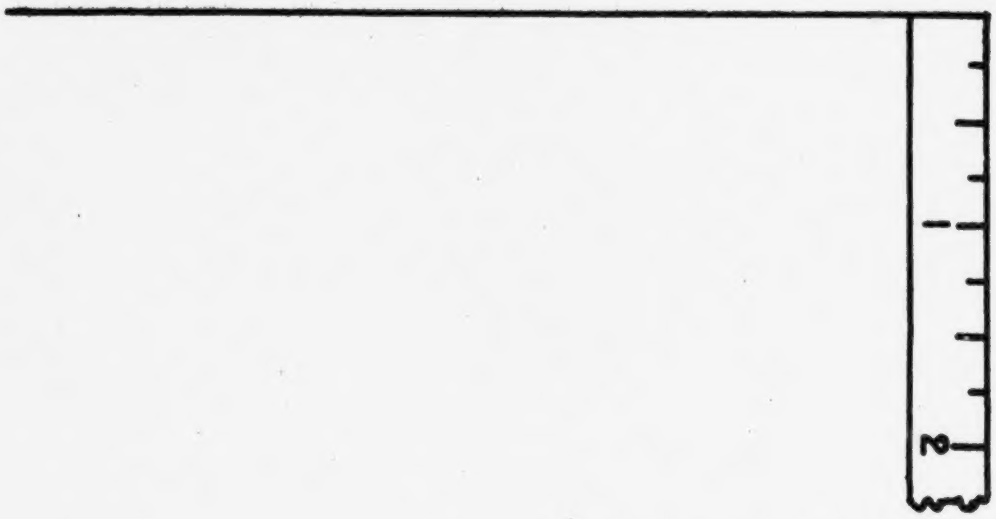


Mr. Morton is trying to place molding around the top of a shed he built. He went to the lumber yard and ordered one 2 x 4 (read 2 by 4). This means that when the wood is cut at the mill it measures 2 inches by 4 inches.

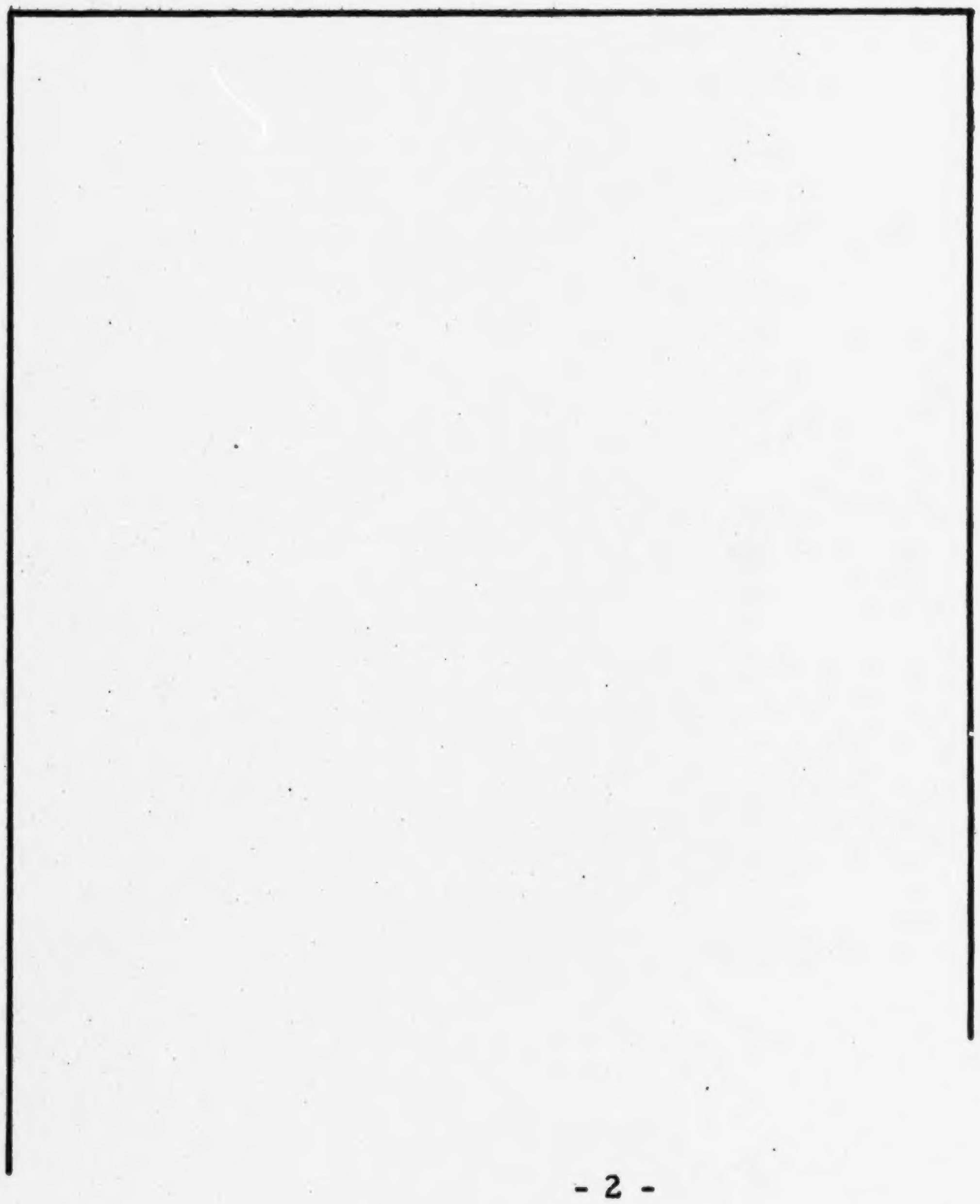
When they sand the wood to make it smooth, it measures $1 \frac{5}{8}$ inches by $3 \frac{5}{8}$ inches.

Mr. Morton has a problem. The wood is thick enough ($1 \frac{5}{8}$ inches) but it is too wide ($3 \frac{5}{8}$ inches). He decides to cut the wood into 3 equal parts but he doesn't know how to divide $3 \frac{5}{8}$ by 3. Can you discover a way of helping him? Let's see.

Draw a line $3\frac{5}{8}$ inches long with your ruler. Place the end of your ruler on each end of the line and draw a line so that it forms a square corner.



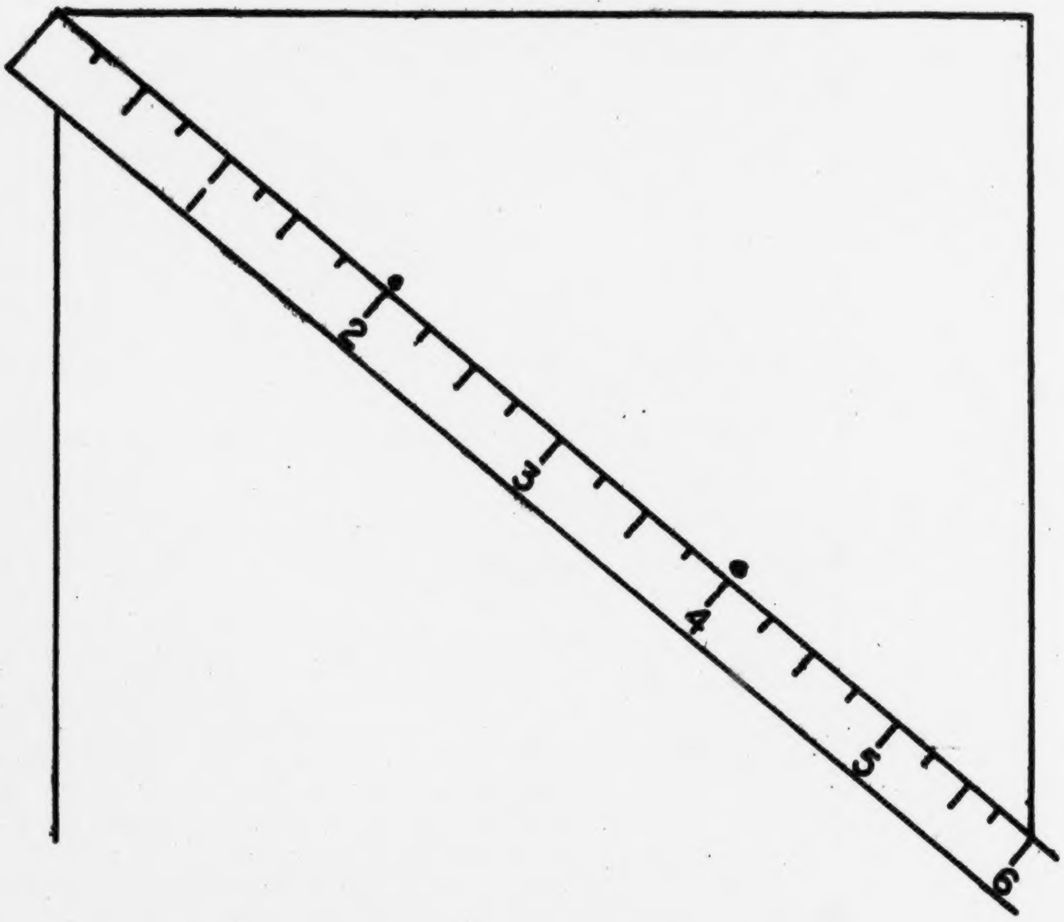
Your lines could look like this.



Next place your ruler on the line, and rotate one end of the ruler until your ruler reads 6 inches.

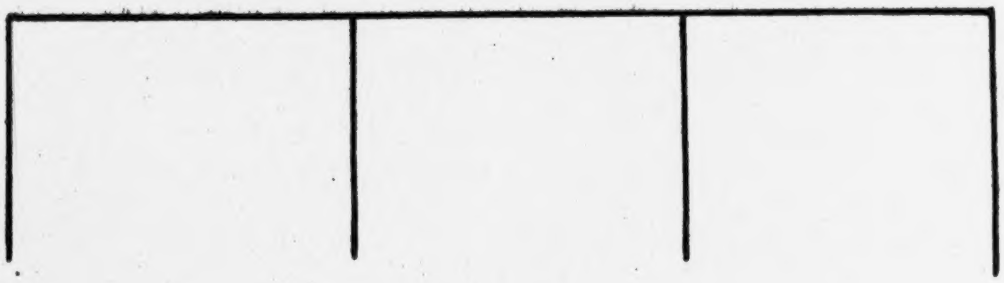
Place a dot at the 2 and 4 inch marks.

Your ruler should look like this.



Now place your ruler so that it makes a square corner with the line, and draw a line through each dot.

Your drawing should look like this.



You have now divided the $3\frac{5}{8}$ inch line into 3 equal parts.

By using the same method, Mr. Morton was able to divide his 2 x 4 into 3 equal parts.

PATTERNS WITH ORDERED PAIRS

Teacher Commentary

A Recreational Activity on Finding Patterns Using Ordered Pairs

I. Materials: Student work sheet entitled, "Patterns of Ordered Pairs"

II. Procedure:

A. Distribute student work sheet

B. Allow students to find the patterns on their own after the directions have been discussed

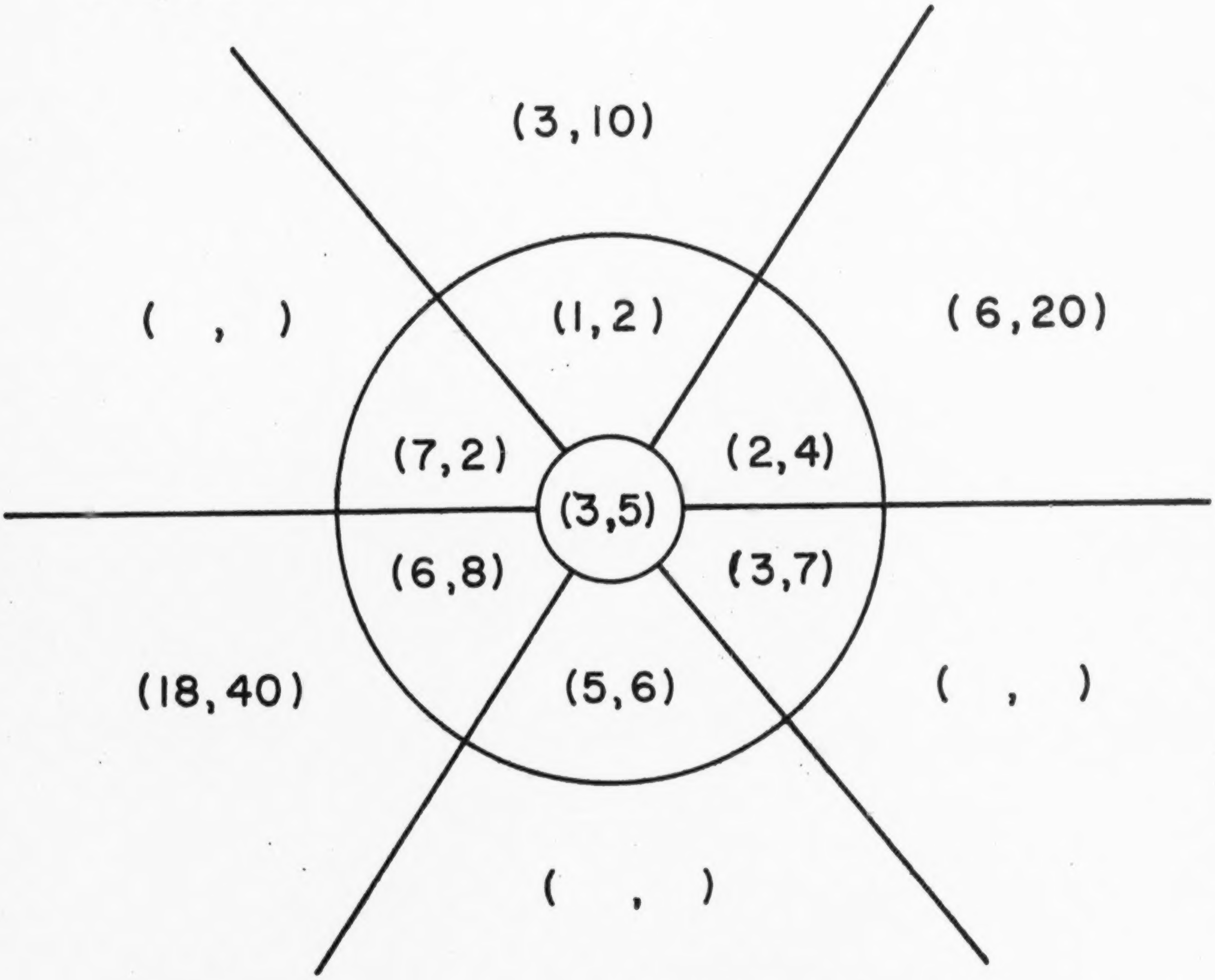
C. Students can make additional patterns of their own using ordered pairs

D. Solutions:

A.	B.
(3, 10)	(11, 10)
(6, 20)	(22, 20)
(9, 35)	(36, 35)
(15, 30)	(43, 30)
(18, 40)	(54, 40)
(21, 10)	(41, 10)

PATTERNS OF ORDERED PAIRS

A. Operation #

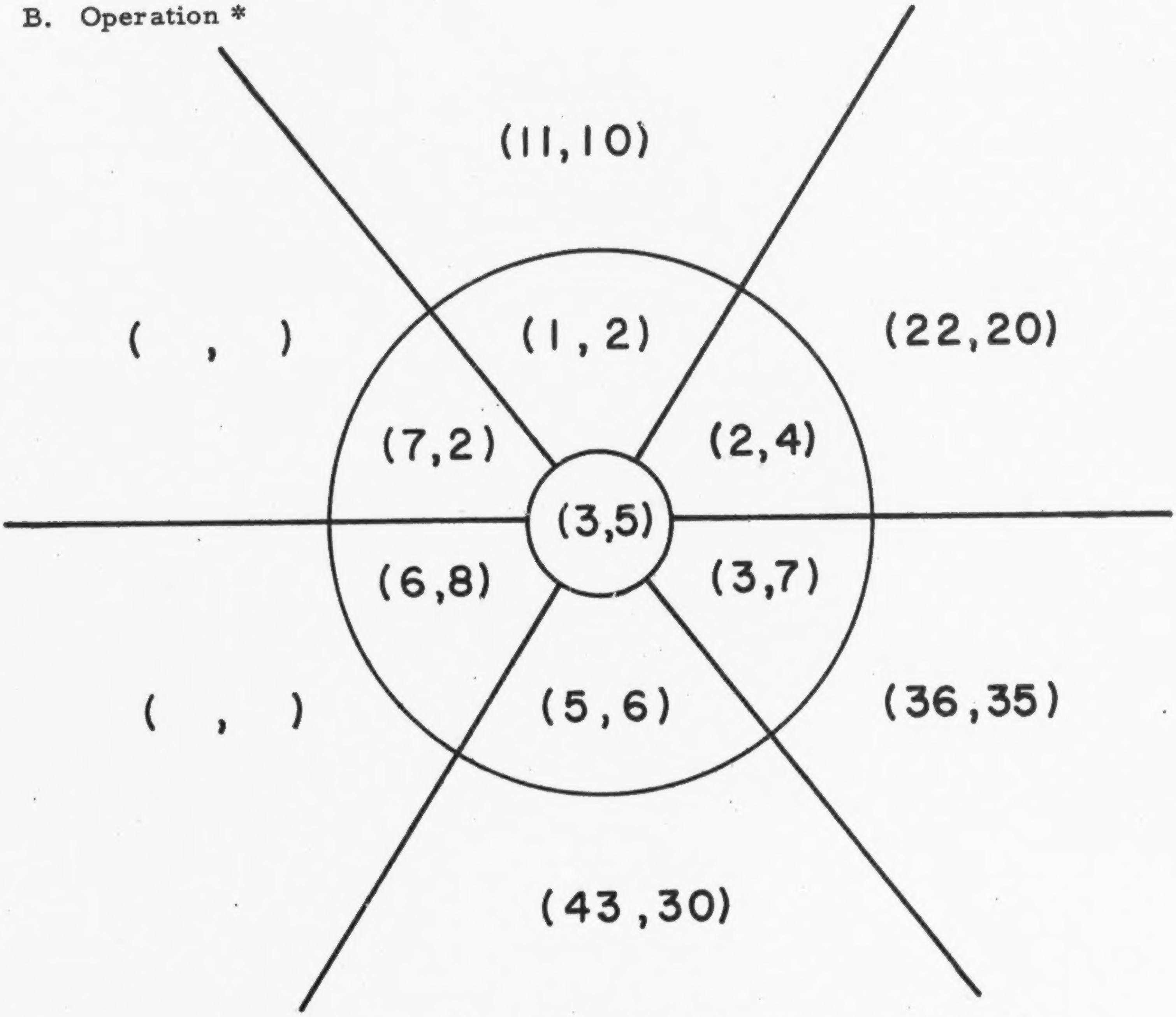


DIRECTIONS: Find the pattern that exists between the number pair in the inner region and the number pairs in the middle region to discover and complete the number pairs in the outer region.

(3, 5) # (1, 2) = (3, 10)
(3, 5) # (2, 4) = (6, 20)
(3, 5) # (3, 7) = (,)
(3, 5) # (5, 6) = (,)
(3, 5) # (6, 8) = (18, 40)
(3, 5) # (7, 2) = (,)

PATTERNS OF ORDERED PAIRS

B. Operation *


(11,10)
(,)
(1,2)
(22,20)
(7,2)
(2,4)
(3,5)
(6,8)
(3,7)
(,)
(5,6)
(36,35)
(43,30)

DIRECTIONS: Find the pattern that exists between the number pairs in the inner region and the number pairs in the middle region to get results in the outer region.

(3, 5) * (1, 2) = (11, 10)
(3, 5) * (2, 4) = (22, 20)
(3, 5) * (3, 7) = (36, 35)
(3, 5) * (5, 6) = (43, 30)
(3, 5) * (6, 8) = (,)
(3, 5) * (7, 2) = (,)

TOWER OF HANOI

Teacher Commentary

A Recreational Activity on Arrangement of Discs

I. Materials:

A. A piece of $\frac{1}{2}$" wood, $12\frac{1}{2}$" long and at least $4\frac{1}{2}$" wide.

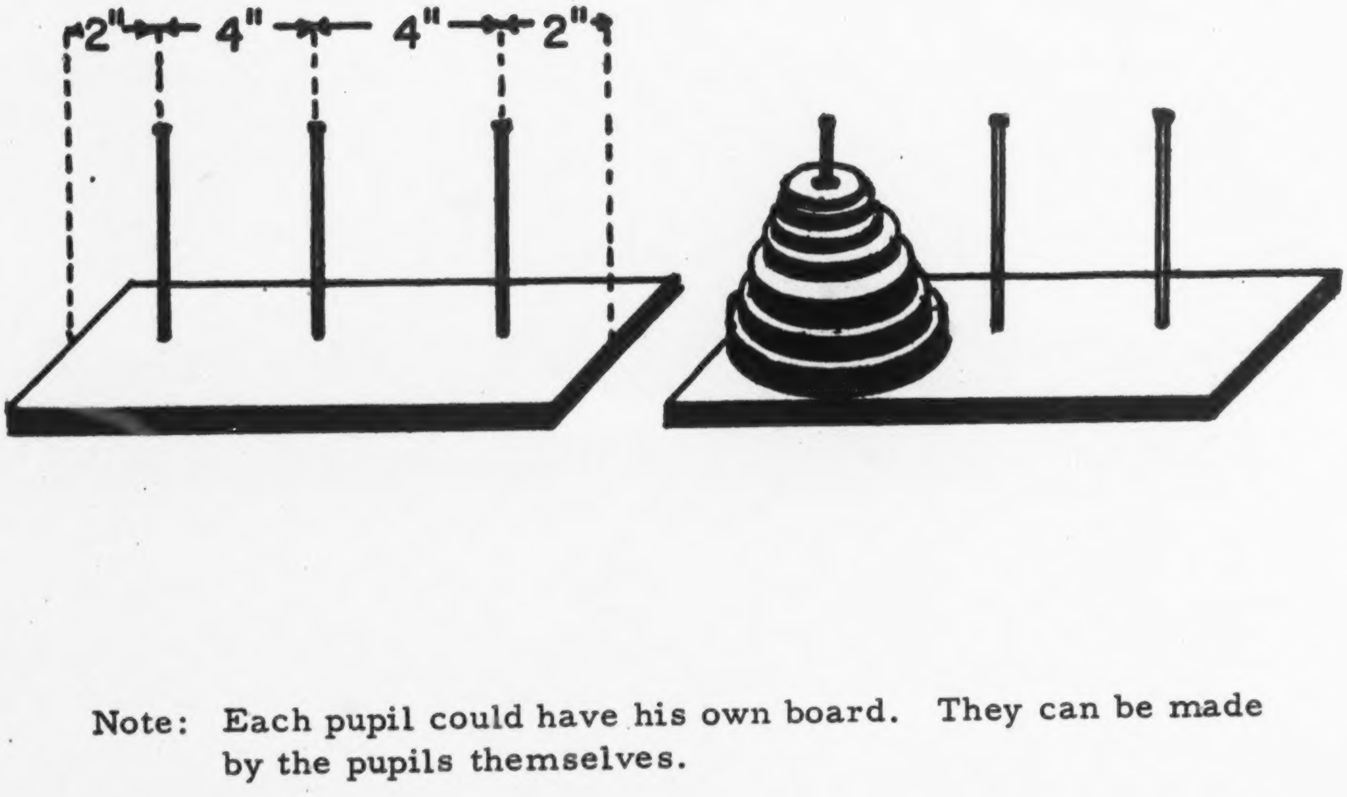
B. Three long finishing nails.

C. At least seven discs (these should be $\frac{1}{4}$" thick and have radii of 2", $1\frac{3}{4}$", $1\frac{1}{2}$", $1\frac{1}{4}$", 1", $\frac{3}{4}$", and $\frac{1}{2}$").

II. Procedure:

A. Setting up the board

Set up the board as shown in the sketches.



Note: Each pupil could have his own board. They can be made by the pupils themselves.

B. Directions for the game.

1. Start with all the discs used in the game on one peg. Place the largest on the bottom, the next largest on that, and so on up to the smallest disc on top. (You may start the game with as few discs as you desire.)
2. Transfer the entire set of discs to one of the other two pegs. Move only one disc at a time, and make certain that no disc is allowed to rest on one smaller than itself.
3. If a disc is transferred to any other it is counted as a move.

C. Classroom use of the Tower of Hanoi game.

1. Use the game for recreational purposes only rather than a lesson per se.
2. One student or a small group could play the game while the teacher works with the rest of the class on other topics.

D. Example of how to use the game.

1. Let the pupil(s) begin the game with a given number of discs and allow them to play the game according to the rules.
2. After the game has been played, tell the pupil(s) the fewest number of transfers that can be made with the given number of discs. Let the pupil(s) try to finish the game with the fewest number of moves.
3. Repeat steps 1 and 2 above using a different number of discs. For example, you may allow the pupils to use 2, 3, 4, or 5 discs for different versions of the game.
4. Have the pupil(s) keep a record of the fewest number of moves required for each game using a different number of discs.

Answer:

Number of discs	1	2	3	4	5
Number of moves	1	3	7	15	31

From this table the pupils may see the pattern:
Formula for number of discs to number of moves is:
$2^n - 1$.

5. Ask the students to use the chart to determine the fewest moves to transfer 6 discs. Suggest that the number may be found by playing the game closely or by a mathematical pattern.

HIDDEN WORD PUZZLE

Teacher Commentary

A Recreational Activity on Vocabulary Words From Statistics

I. Materials: Student work sheet with square array of letters

II. Procedure:

   A. Give each student a duplicated sheet.

   B. Have him find as many math words as he can.

   C. Solution:

		M	E	D	I	A	N				S	
			R	A	N	G	E				C	
			A	T		R					O	
M	E	A	N	A	T	A	L	L	Y		R	
			K			P		M	O	D	E	
						H						
			S	T	A	T	I	S	T	I	C	S
F	R	E	Q	U	E	N	C	Y				

Check List:

1. Data
2. Range
3. Rank
4. Graph
5. Frequency
6. Statistics
7. Tally
8. Score
9. Mean
10. Mode

LANGUAGE OF STATISTICS PUZZLE

See if you can find words from statistics in the puzzle. Some words run across and some run down. Altogether there are 10 words. How many can you find?

K	L	M	E	D	I	A	N	N	C	T	S	I
O	J	H	R	A	N	G	E	G	Z	G	C	F
P	L	Q	A	T	F	R	E	Q	O	M	O	Y
M	E	A	N	A	T	A	L	L	Y	B	R	J
F	T	N	K	H	T	P	G	M	O	D	E	C
P	U	Y	P	C	G	H	R	L	U	N	D	Q
A	L	L	S	T	A	T	I	S	T	I	C	S
F	R	E	Q	U	E	N	C	Y	R	E	A	T

WORD MAZE

Teacher Commentary

A Recreational Activity on Vocabulary Words From Algebra

I. Materials: Work sheet with maze

II. Procedure:

A. Give each student a work sheet

B. Have each student try to find the twelve math words hidden in the maze

C. You may give the words to be found or you may allow students to find them without hints

Hidden Words:

1. ratio
2. rational
3. line
4. plane
5. equation
6. equal
7. quart
8. part
9. one
10. rate
11. pair
12. ten

WORD MAZE

There are at least twelve math words hidden in the large square below. Can you find them? Here is what to do.

You may start in any square and move in any direction to the square next to it. You may continue to move one square in any direction until a word is made. You may not enter the same square twice while spelling a word.

R	T	I	O
U	A	L	N
Q	P	A	E
E	R	I	T

SCRAMBLED WORDS

Teacher Commentary

A Recreational Activity on Vocabulary for Geometry

I. Materials: Work sheet of scrambled words

II. Procedure:

A. Give each student a work sheet.

B. Have each student unscramble the letters to form vocabulary words.

C. Solution:

1. pentagon
2. obtuse
3. octagon
4. right
5. perpendicular
6. circle
7. parallel
8. acute

SCRAMBLED WORDS

Unscramble the letters to form math words:

1. tgannepo
2. obsuet
3. cotgoan
4. grith
5. reppdnecuiarl
6. recilc
7. lapaelrl
8. ctaue

SCRAMBLED WORDS

Teacher Commentary

A Recreational Activity on Vocabulary Words for Algebra

I. Materials: Work sheets of scrambled words

II. Procedure:

- A. Hand out work sheets to students.
- B. Have students unscramble words.
- C. Solution:
    1. constant
    2. order
    3. negative
    4. inverse
    5. formula

SCRAMBLED WORDS

Unscramble the letters to form algebra words:

1. staconnt
2. deror
3. vegenati
4. verinse
5. alufrom